IIIS Discussion Paper

No.40 / November 2004

Are International Equity Markets Really Skewed?

Colm Kearney and Margaret Lynch

School of Business Studies
Institute for International Integration

IIIS Discussion Paper No. 40

Are International Equity Markets Really Skewed?

Colm Kearney and Margaret Lynch

Are International Equity Markets Really Skewed?

Colm Kearney and Margaret Lynch

School of Business Studies,
and
Institute for International Integration Studies,
Trinity College Dublin.

September 2004

Abstract

Although the extreme tails of the distributions of equity returns tend to exhibit more negative than positive returns, very few studies have analysed how pervasive is skewness across entire distributions. We use daily returns on 6 international stock market indices from Britain, France, Germany, Italy, Japan and the United States over 24 years from January 1978 to February 2002 to search for skewness in the tails, in different intervals, and in the entire distributions using binomial distribution tests and two distribution free tests, the Wilcoxon Rank Sum Test and the Siegel Tukey test. We find limited evidence of statistically significant skewness in the tails, with more skewness closer to the means.

Keywords: Asymmetric returns, skewness, international equity markets
JEL Classifications: C14, G15.

Colm Kearney acknowledges financial support from the Irish Research Council for the Humanities and Social Sciences Senior Research Fellowship grant.

Contact Details

Colm Kearney, Professor of International Business, School of Business Studies, Trinity College, Dublin 2, Ireland. Tel: 353-1-6082688, Fax: 353-1-6799503, Email: colm.kearney@tcd.ie.
Margaret Lynch, School of Business Studies, Trinity College, Dublin 2, Ireland.
Fax: 353-1-6799503, Email: Margaret.lynch@tcd.ie.

1. Introduction

The finance literature has known for many decades that the empirical distributions of daily equity market returns have higher peaks and fatter tails than the standard normal distribution (see Mandelbrot (1963), Fama (1965) for early work on this issue), and leptokurtosis is now a universally recognized property of short horizon equity market returns. With regard to the third moment of returns, however, there is less agreement about the nature, extent and implications of skewness. Using updated data from Fama (1965), Simkowitz and Beedles (1980) found evidence that the returns on individual securities are positively skewed. Subsequent work by Singleton and Wingender (1986), Aggarwal, Rao and Hiraki (1989), Alles and Kling (1994), Pierro (1994, 1999, 2002), Aggarwal and Schatzberg (1997), Cont (2001) and Jondeau and Rockinger (2003) have all found varying degrees of skewness in national and international equity markets. Alongside this research into the nature and extent of skewness in equity returns, others have investigated the implications of asymmetry in equity markets. In this vein, asset pricing models that incorporate a preference for positive skewness have been advanced by Kraus and Litzenberger (1976, 1983), Friend and Westerfield (1980), Lim (1989), Harvey and Siddique (1999, 2000), Chen, Hong and Stein (2000) and Barone-Adesi (2003). The implications of skewness for portfolio selection have been investigated by Conine and Tamarkin (1981), Lai (1991), Chunhachinda *et al* (1997), Ait-Sahalia and Brandt (2001), Engle and Patton (2001), Ang and Chen (2002), Prakash, Chang and Pactwa (2003), Sun and Yan (2003) and Patton (2004).

Researchers who analyse the implications of skewness for asset pricing theory and portfolio selection often take it as given that equity returns are asymmetrical. For example, Harvey and Siddique (1999) state that 'Skewness, asymmetry in distribution, is found in many important economic variables such as stock index returns and exchange rate changes…'. Similarly, Chen, Hong and Stein (2000) state that 'Aggregate stock-market returns are asymmetrically distributed….the very largest movements in the market are usually decreases, rather than increases—that is, the stock market is more prone to melt down than to melt up. For example, of the ten biggest one-day movements in the S&P 500 since 1947, nine were declines.' Cont (2001) observes 'large drawdowns in stock prices and stock index values but not equally large upward movements.' Finally, Engle and Patton (2001) report that daily returns on the *Dow Jones Industrial Index* from 1988 to 2001 are 'substantially negatively skewed; a common feature of equity returns.' As Piero (1999, 2002) points out, however, many studies of the nature and extent of skewness in equity markets have used the asymptotic distribution of the common skewness statistic to test for symmetry, and this could lead to the incorrect conclusion that returns are asymmetric when the parent distribution is perfectly symmetric but non-normal. Using daily stock index returns from 9 international equity markets in Europe, Japan and the United States over the 14 year period from January 1980 to September 1993, Peiró (1999) uses a range of distribution-free test statistics to show limited statistically significant evidence of skewness in either the entire distributions of in the 10 percent tails.

In this paper, we use daily log price changes from 6 international equity markets (Britain, France, Germany, Italy, Japan and the United States) for almost 25 years from January 1978 to February 2002. Although these markets were also investigated by Peiro (1999), we update his dataset by almost 10 years and extend his analysis in two ways. *First*, having established that international equity returns do not follow the normal probability distribution on the basis of excess kurtosis levels, studentised range and Jarque-Bera statistics, we implement the methodology of Fama (1965) to construct frequency distribution histograms across the entire distributions to shed more light on the location of asymmetry. *Second*, we extend the binomial distribution tests and the Wilcoxon rank sum and Siegel-Tukey distribution free tests that Peiro (1999) applied to the entire distribution and to the 10 percent tails, to all the other intervals in the distributions. Our main findings are that although international equity returns are unambiguously non-normal, there is much less evidence of skewness. Contrary to what many financial researchers seem to believe, we find very little statistically significant skewness in the tails of the distributions. These findings concur with those reported by Peiro (1999). When we do find skewness, it tends to be closer to the centre rather than to the tails of the distributions. Furthermore, these asymmetries tend to arise because of a higher probability of positive rather than negative excess returns. This finding differs from Peiro (1999), and it is also opposite to what many financial researchers seem to believe about asymmetries in the tails.

The rest of our paper is organized as follows. Section 2 provides a detailed analysis of normality in log price changes. Section 3 describes the binomial distribution test and presents our results. Section 4 describes the distribution-free tests, the Wilcoxon Rank Sum test and the Siegel-Tukey test, and provides the corresponding results. Section 5 draws together our main findings and conclusions.

2. Testing for Normality

Our data comprises daily log price changes from 1 January 1978 to 2 February 2002 from 6 international markets, the *Financial Times All Share* index (Britain), the *DS Total French Market*, the *Frankfurt Faz* (Germany), the *Milan Comit* (Italy), the *Nikkei 225* (Japan), and the *New York Stock Exchange* (United States). The data has been downloaded from *Datastream International Ltd*. Days on which the log price change was zero have been excluded, and the number of observations varies from 5898 for Germany, to 6058 for France, 6082 for Britain, 6008 for Italy, and 5944 for Japan and the United States. Descriptive statistics in Table 1 indicate a clear rejection of the normality of the daily returns for all markets. The returns are leptokurtic, confirming the findings of other researchers using daily data. The absence of normality is also shown in the Jarque-Bera (JB) test results. This is a simple test of normality based on the fact that a normal probability density function has skewness = 0 and kurtosis = 3. Under the null hypothesis of normality, the JB statistic is distributed as a χ^2 statistic with 2 degrees of freedom. The studentised range statistic also clearly rejects normality.

To further examine the distribution of log price changes, we construct frequency distribution tables using the methodology of Fama (1965). For each market, the proportions of price changes within given standard deviations of the mean are computed and compared with what would be expected if the distributions were standard normal. In Panel A of Table 2, the first column on the left divides the distributions into 8 intervals containing observations within 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0 standard deviations from the mean. The second column gives the proportions of observations that would appear in each interval for the standard normal distribution. In the next 6 columns, each market has 2 numbers in each interval; the top number is the proportion of observations in the interval, and the bottom number is these proportions minus the standard normal proportions. For example, the bottom entry in interval 0.5S for Japan, 0.160, is computed by subtracting 0.383 (the unit normal entry) from 0.543. A positive number in the bottom entry for each country indicates an excess of relative frequency in the empirical distribution over what would be expected for the interval if the distribution was normal. It is clear from the column on the right of the Table that the empirical distributions contain, on average, over 11 percent more frequency within a half standard deviation from the mean than would be expected if they were normal. These figures vary from just under 5 percent for Britain to 16 percent for Japan, and they indicate a significant degree of leptokurtosis in all markets in our sample. This pattern is continued in the 1.0S and 1.5S intervals with an average excess of relative frequency of 9 and 2.8 percent respectively. Indeed, all entries for each market in the first three intervals up to 1.5 standard deviations from the mean are positive. Although there is a general excess of relative frequency for all markets within 1.0 standard deviations it is not as great as that reported for the 0.5 standard deviation interval. This indicates that the empirical relative frequency between 0.5 and 1.0 standard deviations is less than would be expected under the standard normal. In the empirical distributions, there is a consistent deficiency of relative frequency within 2.0, 2.5, 3.0, 4.0 and 5.0 standard deviations from the mean, implying that there is too much relative frequency beyond these intervals. This excess of relative frequency beyond these intervals is commonly referred to as long tails. Fama (1965) reported similar results in his analysis of these intervals for the *Dow Jones 30* stocks, the main difference being that we have consistently higher average percentages.

A negative number in an interval implies scarcity of relative frequency. For example, the entry in the 2.5S interval for Britain (-0.035) implies that we observe 3.5 per cent less relative frequency within 2.5 standard deviations of the mean than would be expected under the standard normal. From interval 2.0S to 5.0S inclusive, all entries are negative except for two very small positive entries in interval 2.0S for France and the United States, indicating a general deficiency of relative frequency within any of the intervals from 2 to 5 standard deviations from the mean. This means that there is a general excess of relative frequency beyond these points. In particular there is an excess of relative frequency greater than five standard deviations from the mean.

Panel B of Table 2 compares the expected number of observations greater than 2, 3, 4 and 5 standard deviations from their means, when sampled from a normal distribution, with the actual numbers observed in the empirical distributions. The results are consistent in recording departures from normality for all markets in the study. Beyond 3 standard deviations, there should be on average about 16 observations for each market. The actual numbers range from a minimum of 57 for the United States to a maximum of 187 for Italy. Beyond 4 standard deviations, there should be no observations in a normal distribution given our sample sizes. The actual numbers, however, range from a minimum of 22 for Britain to a maximum of 149 for Italy. Of particular significance in the study of asymmetry is the long tails already described as an excess of relative frequency beyond 3.0, 4.0, and 5.0 standard deviations from the mean. The actual numbers of observations more than 5 standard deviations from their means range from a minimum of 9 for Britain to a maximum of 124 for Italy.

To summarise our findings thus far, we have unambiguous evidence of non-normality in all markets in our sample. This suggests that a similar analysis of the intervals of the distributions for asymmetry should be insightful. Before doing this, however, it is also instructive to construct histograms of the empirical distributions of the excess returns. The histograms are presented in Figure 1. Following Peiró (1999), if the distribution of returns is symmetric, then the median must necessarily be the axis of symmetry and coincide with the mean. As we have 6 different time series in our analysis, we subtract the mean of each series from each individual return in the series, thus shifting the axis of symmetry to zero for all series of excess returns. This means that symmetry of returns will be reflected in the symmetry of the histograms about zero. The intervals are the same for all markets in the study, ranging from values of strictly less than –0.055 to values strictly greater than 0.055. To provide for ease of observation, each interval is denoted by an integer value ranging from 1 to 13. The histograms in Figure 1 do not present clear evidence of asymmetry in the probability density function of any market. We do not need to rely on these graphical representations in order to make firm conclusions, however, because we now proceed to conduct a formal Binomial distribution test on each interval that is symmetric about zero.

3. Binomial Distribution Tests

We can use the binomial distribution test to formally examine the probabilities of obtaining negative versus positive excess returns in all intervals of the distributions. Following Peiró (1999, 2001), logarithmic returns will be considered to be symmetric if two conditions hold: *first*, if the probability of obtaining a positive excess returns equals the probability of obtaining a negative excess return after zero excess returns have been excluded, and *second*, if the distribution of negative excess returns in absolute values is equal to the distribution of positive excess returns. If the probability of a negative excess return is the same as the probability of a positive excess return, the returns can be considered to follow a binomial distribution with parameters *n* equal to the number of observations and $p = 0.5$. The test statistic *T* is the number of negative excess returns for each series. The null distribution of *T* is the binomial distribution with parameters p = p* (0.5) and n = number of observations. Because the values of *n* are large, the normal approximation is used. Approximate quantiles for x_q for *T* are given by

$$Xq = n.p + z_q \sqrt{n.p(1-p)} \quad (1)$$

Where z_q is the qth quantile of a standard normal random variable. The hypothesis takes the form of a two- tailed test:

$$H_0 : P = 1/2 \quad \text{and} \quad H_1 : P \neq 1/2$$

The rejection region of desired size α corresponds to the two tails of the null distribution of *T*, where the lower tail, denoted by α_1 is approximately half the size of the target α and the upper tail, denoted α_2 is also approximately half the size of α.

Because the number of observations is large, we use equation (1) to approximate the $\alpha/2$ quantile t_1 and the (1- α_2) quantile t_2 of a binomial random variable with parameters $p^* = 1/2$ and n = number of observations, t_1 is a number such that

$$P(Y \leq t_1) = \alpha_1 \quad (2)$$

And t_2 is a number such that

$$P(Y \leq t_2) = 1 - \alpha_2 \quad (3)$$

The p value is found using

$$P(Y \leq t_\gamma) \approx \frac{P(Z \leq t_\gamma - n.p^* + 0.5)}{\sqrt{n.p^*(1+p^*)}} \quad (4)$$

where t_y represents the choice of test statistic, in this case the number of negative excess returns, see Conover (1999) and Brown and Hollander (1977) for discussion.

The binomial distribution tests for the whole sample and for the 10 percent highest absolute excess returns are shown in Table 3. Looking first at the whole sample, we can see in the first two columns that all countries except Italy have a higher proportion of positive excess returns over the whole period. The null hypothesis of equal probability of obtaining a positive or a negative excess return is rejected for Britain, France, Germany and Japan. France records the highest percentage of positive returns at 53 percent and the lowest percentage of negative returns at 47 percent. We cannot reject the null hypothesis for Italy and the United States at the 5 percent level of significance. Looking now at the results for the largest 10 percent returns, Italy is the only country that has a higher percentage of negative returns, both in the whole sample and in the 10 percent highest excess returns. All other countries except France have more negative than positive excess returns in the 10 percent highest returns. France is the only country that has more positive than negative excess returns in the whole sample and in the 10 percent highest excess returns. There is a statistically significant difference in the probability of obtaining a negative versus a positive return for Germany and Japan at the 10 percent tails of the distributions, but not for any other country. While there are differences in the numbers of positive and negative excess returns at these extreme levels, the differences are not statistically significant for 4 of the 6 markets sampled. This finding is at odds with the writings of several authors who have observed that the greatest changes in returns are negative. This phenomenon, first noted by Black (1976), is well documented and appears to be the basis of theoretical assumptions concerning asymmetry in returns. Many researchers including Harvey and Siddique (1999), Cont (2001) and Chen, Hong, Stein (2000) have used this difference in proportions of negative price changes at extreme values to argue that price changes present clear evidence of asymmetry.

Table 4 presents the positive and negative excess returns for all markets using the same intervals as those used by Peiró (1999). For Britain, there is a statistically significant difference in the intervals 0-0.5 percent, 0.5-1.5 percent and 2.5-3.5 percent. For France, there is a statistically significant difference in the intervals 4.5-5.5 percent; for Germany in the 0.5-1.5 percent, 2.5-3.5 percent and >5.5 percent intervals; for Italy in the 0-0.5 percent, 1.5-2.5 percent and >5.5 percent intervals; and for Japan in the 0.5-1.5 interval. Consistent with Table 3, there is no interval in which there is a statistically significant difference in the probability of getting a negative or a positive return for the United States market. Overall, the most interesting result is that only 2 of our 6 markets, Italy and Germany, record differences in the probability of negative versus positive returns at greater than 5.5 percent

In the interval containing observations between 3.5 percent and 4.5 percent, Table 5 shows that for all 6 countries, the numbers of positive and negative excess returns are very close, and that beyond this interval, the numbers of negative returns dominate the numbers of positive returns, but not usually at statistically significant levels. In the case of France, for example, there is exactly the same number of positive and negative excess returns within the 4.5-5.5 percent interval. The exception to this is found in Germany and Italy, both of which reject the null hypothesis of equality in returns in the >5.5 percent interval. Only one third of the countries in our sample have asymmetry in the tails of their distributions. Looking at Britain, for example, in the interval that contains returns greater than 5.5 percent in absolute values, there are 4 negative and 2 positive returns and in the interval composed of returns between 4.5 percent and 5.5 percent, there are 2 negative returns and 1 positive return. This observation conforms to Harvey and Siddique (1999) who define negative skewness as the phenomenon whereby negative changes dominate positive changes of the same magnitude. But given that we are looking at a total of 3 observations out of 6082 in one interval and a total of 6 observations out of 6082 in the second interval, it does not offer clear justification for concluding asymmetry in the whole distribution based on this small number of returns. In addition to this, our formal binomial test does not reject the equal probability of obtaining a negative or a positive return in either of these two intervals. As we have already observed, there is one interval where the numbers of positive and negative excess returns are almost identical, 3.5 percent to 4.5 percent. Beyond this interval, further out in the tails, negative returns do appear to dominate positive returns, but for the most part not at levels that are significant. This lack of statistical significance is clearly related to the numbers of observations in these tails.

Consistent with our findings from the normality tests in Section 2, we find that there are differences in the probability of obtaining negative and positive excess returns for Britain, Germany and Japan at levels that are statistically significant in one interval of the distributions, 0.5-1.5 percent. This asymmetry occurs because the number of positive excess returns in all cases exceeds the number of negative excess returns in this interval.

The total percentages of all returns, both positives and negatives, in this one interval are 45 percent for Britain, 41 percent for Germany and 38 percent for Japan. There are also 38 percent of the total returns to the United States market in this interval. Although the difference in probability is not statistically significant for the United States, the p-value is 0.09, which is the nearest we come to any evidence of asymmetry for the United States market. This is the difficulty with tests for asymmetry that do not target individual intervals of the distribution. They may conclude that stock market returns are asymmetric because there are more negative than positive excess returns in the tails, and because the biggest observable changes are negative. Why is asymmetry in an interval that contains almost half of all returns, in the case of Britain for example, ignored in favour of the tail behaviour, where the numbers of positive and negative returns can be in single digits and almost negligible as a percentage of the entire distribution? In the 0.5-1.5 interval for Britain, there are 1,397 positive returns, which is 116 greater than the 1,279 negative returns, and the difference is statistically significant at the 2 percent level of significance. If the average positive return in this interval is approximately 1 percent (ie, in the middle of the interval), this positive asymmetry will cancel out the negative effect on returns of greater number of negatives in the tail of the distribution by 5 times if these average 10 percent. Focussing on the tails of equity return distributions as the most important source of asymmetries seems to be unwarranted.

4. Distribution Free Tests

The symmetry or asymmetry of equity return distributions is very often established by using the sample skewness statistic in (5).

$$\alpha = \frac{\sum_{t=1}^{T}(R_t - \overline{R})^3 / T}{\sigma^3} \tag{5}$$

As Peiró (1999, 2001) notes, however, the asymptotic distribution of this statistic is tied to the assumption of normality in the series under review, and its characteristic behaviour can be different under alternative distributions in the parent series. It is consequently unsafe to make conclusions about the symmetry or asymmetry of equity returns on the basis of results obtained using this statistic. We clearly reject the normality assumption for the 6 international equity markets in our study. We should therefore look to other measures of differences in location and dispersion between returns above and below the mean. Distribution free tests are appropriate because they require few if any assumptions about the distribution of returns in the underlying population. The two tests that we now employ are the Wilcoxon rank sum test and the Siegel-Tukey test. These are 2-sample tests designed to detect differences in location (mean) and dispersion about the mean. In each case, the null hypothesis establishes the equality of the underlying distributions.

The Wilcoxon rank sum test can be considered a test for symmetry if the only assumption made is that the random sample is drawn from a continuous distribution, (see Gibbons (1971)). In this test, the absolute values of positive and negative excess returns are combined into one ordered sample. The test statistic is the sum of the ranks of the absolute values of the negative excess returns in the combined ordered sample. The Wilcoxon test statistic is:

$$W_N = \sum_{i=1}^{N} iZ_i \tag{6}$$

where the Z_i are indicator random variables defined as follows. Let

$$Z = (Z_1, Z_2, \ldots., Z_N) \tag{7}$$

where Z_i = 1 if the *i*th random variable in the combined ordered sample is from the set of negative excess returns and Z_i = 0 otherwise. Under the null hypothesis of equal distributions the exact mean and variance of W_N are

$$E(W_N) = \frac{m(N+1)}{2} \qquad var(w_N) = \frac{mn(N+1)}{12} \tag{8}$$

where m is the number of negative excess returns and n is the number of positive excess returns, and $m + n = N$

In the Siegel-Tukey test, the absolute values of positive and negative excess returns are also combined in one ordered sample. Like the Wilcoxon Rank Sum test, the Siegel-Tukey belongs to a class of statistics called the linear rank statistic. The weights are constructed so that the higher weights are assigned to the middle of the combined sample and the smaller weights to the extremes. The Siegel-Tukey statistic is

$$S_N = \sum_{i=1}^{N} a_i Z_i \qquad (9)$$

where Z_i are the indicator random variables as defined in Equation (7)

and α_i is defined as

$$\begin{array}{ll} 2\,i & \text{for } i \text{ even, } 1 < i \leq \frac{N}{2} \\ 2i - 1 & \text{for } i \text{ odd, } 1 \leq i \leq \frac{N}{2} \\ 2(N-1)+2 & \text{for } i \text{ even, } \frac{N}{2} < i \leq N \\ 2(N-1)+1 & \text{for } i \text{ odd, } \frac{N}{2} < i < N \end{array}$$

Under the null hypothesis of equal distributions, the asymptotic distribution of S_N is the same as W_N.

$$S_N \rightarrow N\left(\frac{m(N+1)}{2}, \frac{mn(N+1}{12}\right) \qquad (11)$$

The results of these distribution-free tests are shown in Table 5. The equality of the distributions of the negative and positive excess returns cannot be rejected at the 5 percent level using either the Wilcoxon test or the Siegel- Tukey test on the whole sample distribution and on the 10 percent largest excess returns for the United States. This is consistent with Peiro (1999) who sampled the S&P500 and the Dow Jones Indexes of the

NYSE for a shorter time period and could not reject the null of equal distributions at the 5 percent level for either of these indexes. Looking at the results for the whole sample, the Wilcoxon test rejects the equality of the distributions for France and Italy, and the Siegel–Tukey test does likewise for Britain, Germany and Japan. Interestingly, both tests do not reject symmetry for the whole sample in any market. Looking next at the results for the 10 percent largest returns, we see that the null of equal distribution of positive and negative returns is rejected only in Italy using the Siegel-Tukey test. It seems, therefore, that the findings of asymmetry in the whole sample for all countries except the United States are not due mainly to asymmetries in the tails of the distributions.

A possible weakness of distribution free tests is that they may lack power when looking at asymmetry of entire distributions with a large numbers of observations and no assumptions about the underlying probability distribution function. The power of these tests is strengthened when they are performed on the intervals of the distributions with considerably less observations. It is therefore of considerable interest to conduct these same tests over different intervals to see whether they can reject the hypothesis of symmetry in any of the intervals, and if so, whether this tends to occur near the mean or further out at the tails. Table 7 provides the results. Britain shows very weak evidence of asymmetry, with only the Siegel-Tukey test rejecting symmetry at the 0.5-1.5 percent interval and the Wilcoxan test rejecting symmetry at the 4.5-5.5 percent interval. France is the only market that rejects symmetry in any interval on the basis of both the Wilcoxan and the Siegel-Tukey tests, and it does this at the 0.5-1.5 percent interval. Germany shows evidence of asymmetry in the 3.5-4.5 percent interval according to the Siegel-Tukey test (although it should be noted that the 29 positive excess returns exceed the 27 negatives), and in the 4.5-5.5 interval according to the Wilcoxan test. Italy rejects the hypothesis of symmetry using the Wilcoxan test in the 0.0-0.5 percent interval. We find no asymmetry in for any interval in the distributions for Japan or the United States. Overall, Table 6 reports 72 distribution free tests symmetry using the Wilcoxan and the Siegel-Tukey tests, for our 6 markets divided into 6 intervals. We have been able to reject symmetry at the 5 percent level in only 7 of these cases, and only 3 of these are in the 2 intervals closest to the tails of the distributions. The remaining 4 cases are in the 2 intervals closest to the means. To the extent that asymmetry occurs in the markets our sample, it is somewhat more likely to occur towards the centre of the distributions rather than at the tails.

It is interesting to compare our results with those obtained by Peiró (1999), and to highlight some differences in findings. Because Peiró (1999) did not find significant differences in proportions of negative and positive excess returns in his whole sample, he turned his attention to the 10 percent extremes. He found significant differences in probability in these extremes, in the same markets where the Siegel-Tukey tests had detected asymmetry. He attributed the rejection of equality in the Siegel-Tukey tests to the unequal dispersion of positive and negative extreme excess returns. To confirm this, he performed the test excluding these extreme values, and found that he could not reject the null hypothesis of equal distributions of positive and negative excess returns. He concluded that it seems reasonable to attribute the rejection of the equality of distributions in these markets to the different dispersion caused by the extreme returns, with more frequent negatives than positives.

We find that the Siegel-Tukey test rejects the null hypothesis for the markets in Britain, Germany and Japan. These markets also exhibit significant differences in proportions of negative and positive excess returns in the binomial test results. But the differences show higher proportions of positive rather than negative excess returns. The Siegel-Tukey standardized statistic in each case is negative, which following Peiró (1999) would have led us to expect the reverse. We therefore calculated the percentages of negative and positive excess returns in the 10 percent highest excess returns, and we find significant differences in the proportions of positives and negatives for the same markets as before; Britain, Germany and Japan. But in these extremes, the proportions of negative excess returns are higher. We therefore performed these tests again, excluding the 10 percent highest excess returns, and find that the Siegel-Tukey tests reject the null of equality of the underlying distributions for Britain and Japan. In both cases, the standardized statistic in the Siegel-Tukey tests is negative, but the proportion of positive returns exceeds the negative returns. Since we now have evidence of asymmetry for Britain and Japan in the whole sample, in the 10 percent highest excess returns, and in the remainder of the returns after we have excluded the top 10 percent, we cannot isolate the source of this asymmetry to the tails as Peiró (1999) did.

To substantiate the theory of Fogler and Radcliffe (1976) that asymmetry is influenced by the differencing interval, we also conducted a series of nonparametric tests on our data during the period from 3/1/1980 to 27/9/1993, which corresponds exactly to Peiro's (1999) dates. In the Wilcoxon tests, our results are almost identical, in that we reject the null hypothesis of equal distributions for Italy. When it comes to the Siegel –Tukey test results, both studies reject the null hypothesis of equal distributions for Britain and Japan. To verify that the asymmetry identified in these markets is attributable to extreme returns, Peiró (1999) repeated the tests excluding the 10 percent tails, and found that it is no longer possible to reject the null of equality for the remainder of the sample. We also repeat these tests, but we still find that the returns for Britain reject the null hypothesis of equal distributions at the 5 percent level of significance. Furthermore, in looking at the percentages of negative and positive excess returns, we find that a negative standardized Siegel-Tukey statistic is not necessarily an indicator of a higher percentage of negative excess returns.

5. Summary and Conclusions

Financial researchers have recognised for some time that the extreme tails of the distributions of equity returns tend to exhibit more negative than positive returns (for recent examples, see Harvey and Siddique (1999), Chen, Hong and Stein (2000), Cont (2001) and Engle and Patton (2001)). The literature, however, remains unclear about how pervasive is asymmetry in returns across the entire distribution, and very few studies to date have conducted a rigorous and systematic analysis of this question. A notable recent exception to this is the work of Peiró (1999, 2001) who finds little evidence of skewness in 9 developed international equity markets in Europe, Japan and the United States. In this paper, we have contributed to this effort by analysing daily returns on 6 international stock markets from Britain, France, Germany, Italy, Japan and the United States from January 1978 to February 2002.

We first examined excess kurtosis levels, studentised range and Jarque-Bera statistics, and using the methodology of Fama (1965) to analyse frequency distribution histograms, we established that international equity returns do not follow the normal probability distribution. Having established non-normality in all international equity markets in our sample, we then examined the behaviour of excess returns. We constructed a set of frequency distribution histograms using the same intervals as Peiró (1999), and we confirmed his finding of no clear evidence of asymmetry in the overall shape of the histograms. We then conducted binomial distribution tests to see if there is a difference in the probability of obtaining a negative versus a positive excess return in the extremes of the distributions. As a further test of asymmetry, we used two distribution free tests, the Wilcoxon rank sum test and the Siegel-Tukey test on the entire samples, on the 10 percent highest excess returns, and on the individual intervals that form the rectangles of the histograms.

We find that while there are more negative than positive returns in the extreme tails of the distributions 5 of the 6 countries studied, the differences are generally not statistically significant. Distribution-free tests show no statistically significant evidence of asymmetry in most intervals of the distributions. Our findings call into question some perceptions about the asymmetry of daily stock market returns. More specifically, our analysis demonstrates that asymmetry is not a statistically significant stylistic feature of stock returns. We do not find that there is a higher probability of obtaining a negative rather than a positive excess return of the same magnitude in the returns to the stock markets in our study. When we do find asymmetry, it is more likely to be related to the higher probability of obtaining a positive rather than a negative excess return of the same magnitude.

Table 1.
Descriptive Statistics

	Britain	***France***	***Germany***	***Italy***	***Japan***	***US***
Observations	6082	6058	5898	6008	5944	5944
Mean	0.0004	0.0005	0.0003	0.0006	0.0001	0.0004
Standard deviation	0.0090	0.0114	0.0118	0.0137	0.0124	0.0101
Student range	19.660	15.986	18.677	18.447	23.114	35.677
Skewness	-0.896	-0.502	-0.857	-0.660	-0.171	-1.496
Kurtosis	11.020	6.160	9.797	8.208	10.682	54.607
Jarque-Bera	170658	2776	11353	7227	14645	661817

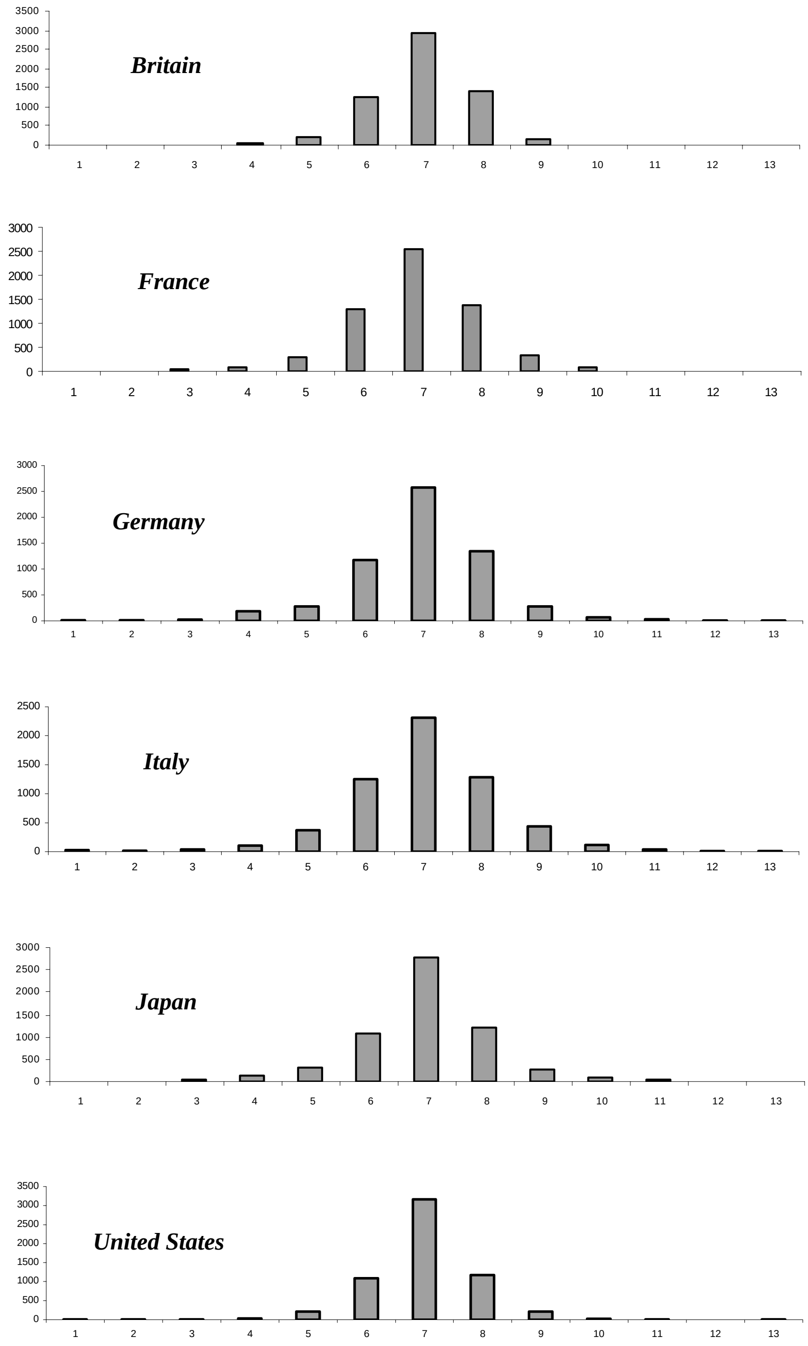
Notes. The skewness statistic is measured as m_3 / s^3, and the kurtosis statistic is m_4 / s^4. The Jarque-Bera statistic is $T(\text{Skewness}^2 / 6 + (\text{Kurtosis-3})^2 /24)$, where m_k is the central moment of order k, and s^2 is the sample variance. $\overline{R}$ is the sample mean of returns and T is the number of observations.

Table 2
Interval Analysis of Asymmetry in International Equity Markets

Intervals	*Unit Normal*	*Britain*	*France*	*Germany*	*Italy*	*Japan*	*United States*	*Average*
				Panel A: Proportions of observations				
0.5S	0.383	0.431	0.485	0.496	0.494	0.543	0.534	0.114
		0.048	0.102	0.113	0.111	0.160	0.151	
1.0S	0.683	0.726	0.768	0.789	0.764	0.789	0.798	0.090
		0.044	0.086	0.106	0.081	0.107	0.115	
1.5S	0.866	0.877	0.901	0.900	0.882	0.892	0.914	0.028
		0.011	0.035	0.033	0.016	0.026	0.048	
2.0S	0.955	0.928	0.956	0.950	0.937	0.943	0.966	-0.088
		-0.026	0.001	-0.004	-0.018	-0.011	0.011	
2.5S	0.988	0.952	0.976	0.973	0.957	0.969	0.984	-0.019
		-0.035	-0.011	-0.014	-0.030	-0.019	-0.004	
3.0S	0.997	0.960	0.987	0.984	0.968	0.985	0.990	-0.018
		-0.037	-0.010	-0.014	-0.029	-0.013	-0.008	
4.0S	1.000	0.966	0.995	0.993	0.975	0.994	0.994	-0.014
		-0.034	-0.005	-0.007	-0.025	-0.006	-0.006	
5.0S	1.000	0.968	0.996	0.997	0.979	0.998	0.996	-0.011
		-0.032	-0.004	-0.003	-0.021	-0.002	-0.004	
				Panel B: Numbers of observations				
2.0S	274	275	263	289	375	333	199	289
3.0S	16	58	73	93	187	87	57	93
4.0S	0	22	28	35	149	32	30	49
5.0S	0	9	16	16	124	10	19	32

Notes. Panel A provides proportions of all observations within intervals of the distributions. Panel B converts these proportions to numbers of observations. Interval 0.5S refers to observations within one half standard deviation of the mean. The Unit Normal column gives the percentages of total observations that would be found if the distributions were normal. In the columns for each market, the top number is the proportion of observations within 0.5,1,1.5,2,2.5,3,4 and 5 standard deviations of the mean, and the bottom number is calculated by subtracting the corresponding unit normal entry from the top number. The averages column on the right of the Table gives the average proportion of observations in all markets relative to the standard normal distribution.

Figure 1
Interval Distribution Charts



There are thirteen intervals as follows: (-∞, -o.055), (-0.055, -0.045), (-0.045, -0.035), (-0.035, -0.025), (-0.025, -0.015), (-0.015, 0.005), (0.005, 0.015), (0.015, 0.025), (0.025, 0.035), (0.035, 0.045), (0.045, 0.055), (0.055, ∞).

Table 3
Binomial Tests on the Whole Sample
and on the 10 Percent Largest Excess Returns

	Percent positive	***Percent negative***	***rho value***	***Percent positive***	***Percent negative***	***rho value***
	Whole sample			***10% highest excess returns***		
Britain	51.69	48.31	0.01	46.22	53.78	0.06
France	53.03	46.97	0.00	51.15	48.85	0.60
Germany	51.29	48.71	0.05	44.24	55.76	0.01
Italy	49.83	50.17	0.61	48.17	51.83	0.39
Japan	51.58	48.42	0.02	44.95	55.05	0.02
US	50.98	49.02	0.14	47.81	52.19	0.31

Table 4
Binomial Test on Intervals of the Distributions

		0-0.5	***0.5-1.5***	***1.5-2.5***	***2.5-3.5***	***3.5-4.5***	***4.5-5.5***	***>5.5***
Britain	Negative returns	1409	1279	198	37	9	2	4
	Positive returns	1538	1397	179	20	7	1	2
	p-value	0.02	0.02	0.35	0.03	1.00	0.50	0.34
France	Negative returns	1309	1277	312	75	27	5	11
	Positive returns	1247	1376	327	67	15	5	5
	p-value	0.21	0.25	0.09	0.56	0.58	0.05	0.23
Germany	Negative returns	1281	1173	277	187	27	13	15
	Positive returns	1291	1344	280	69	29	7	5
	p-value	0.86	0.00*	0.93	0.00*	0.69	0.13	0.02
Italy	Negative returns	1207	1250	369	105	39	15	29
	Positive returns	1104	1281	435	117	37	9	11
	p-value	0.03	0.55	0.02	0.46	0.91	0.31	0.01
Japan	Negative returns	1337	1053	316	116	35	13	8
	Positive returns	1425	1209	282	93	34	12	11
	p-value	0.10	0.00*	0.18	0.13	1.00	1.00	0.32
US	Negative returns	1555	1088	215	31	8	5	11
	Positive returns	1609	1170	210	24	10	2	5
	p-value	0.35	0.09	0.85	0.42	0.81	0.45	0.21

Notes. The Table provides the numbers of positive and negative excess returns in each interval of the distributions for the 6 markets in our sample, along with the binomial test of difference in their number. The '*' indicates statistical significance at the 1 percent level.

Table 5
Distribution Free tests for Asymmetry

	W	*W**	*p-value*	*ST*	*ST**	*p-value*
			Whole sample			
Britain	9014637	1.15	0.13	8742443	-2.828	0.00
France	8440961	-2.684	0.00	8569520	-0.793	0.21
Germany	8425043	-0.748	0.23	8068392	-1.953	0.03
Italy	8866264	-2.816	0.00	8977512	-1.161	0.12
Japan	8640132	-1.538	0.10	8270158	-4.306	0.00
US	8670647	0.133	0.45	8631193	-0.464	0.32
			10 percent largest returns			
Britain	102777	1.484	0.93	97725	-0.855	0.20
France	95065	2.138	0.98	88521	-0.885	0.19
Germany	99118	0.923	0.82	97046	-0.084	0.47
Italy	96402	1.389	0.92	88996	-2.102	0.02
Japan	94207	-1.478	0.07	97711	0.206	0.58
US	93986	0.843	0.80	90954	-0.608	0.27

Notes. W is the Wilcoxon rank sum statistic. ST is the Siegel-Tukey statistic. W* is the standardized Wilcoxon rank sum statistic and ST* is the standardized Siegel-Tukey statistic. The p-values are marginal significance levels.

Table 6
Distribution Free Interval Tests

		0.0-0.5	***0.5-1.5***	***1.5-2.5***	***2.5-3.5***	***3.5-4.5***	***4.5-5.5***
Britain	Negative returns	1409	1279	198	37	9	2
	Positive returns	1538	1397	179	20	7	1
	W*	-1.386	1.173	0.197	0.752	-0.159	-2.449
	p-value	0.08	0.88	0.58	0.77	0.44	0.01
	ST*	-0.869	-1.812	-0.606	-1.254	-0.794	1.225
	p-value	0.19	0.04	0.27	0.11	0.21	0.89
France	Negative returns	1309	1277	312	75	27	5
	Positive returns	1247	1376	327	67	15	5
	W*	-1.228	-1.943	1.679	0.676	0.328	0.731
	p-value	0.11	0.03	0.95	0.75	0.63	0.77
	ST*	-0.609	-2.298	-0.724	-0.607	0.853	-0.104
	p-value	0.27	0.01	0.24	0.27	0.80	0.46
Germany	Negative returns	1281	1173	277	87	27	13
	Positive returns	1291	1344	280	69	29	7
	W*	-0.493	-1.096	2.752	1.029	-0.664	0.04
	p-value	0.31	0.14	1.00	0.85	0.25	0.52
	ST*	0.183	-0.351	-0.646	-0.512	-1.812	-0.436
	p-value	0.36	0.36	0.26	0.30	0.04	0.33
Italy	Negative returns	1207	1250	369	105	39	15
	Positive returns	1104	1281	435	117	37	9
	W*	-1.731	-0.584	1.421	0.918	-0.348	-0.468
	p-value	0.04	0.28	0.92	0.82	0.36	0.32
	ST*	0.636	1.196	-1.564	0.738	1.065	-0.351
	p-value	0.74	0.88	0.06	0.77	0.86	0.36
Japan	Negative returns	1337	1053	316	116	35	13
	Positive returns	1425	1209	282	93	34	12
	W*	-1.418	2.831	1.42	-0.382	-0.084	1.088
	p-value	0.08	1.00	0.92	0.35	0.47	0.86
	ST*	-0.932	-0.276	0.487	-0.552	0.468	-0.326
	p-value	0.18	0.39	0.69	0.29	0.68	0.37
US	Negative returns	1555	1089	215	31	8	5
	Positive returns	1609	1170	210	24	10	2
	W*	0.059	-0.26	0.605	0.187	-0.8	0.387
	p-value	0.72	0.40	0.73	0.57	0.21	0.65
	ST*	1.633	-0.408	0.398	0.56	1.155	-0.387
	p-value	0.95	0.34	0.66	0.71	0.88	0.08

Notes. W* is the standardized Wilcoxon rank sum statistic and ST* is the standardized Siegel-Tukey statistic. The p-values are marginal significance levels.

Bibliography

Aggarwal, R., R.P Rao and T. Hiraki (1989) Skewness and Kurtosis in Japanese Equity Returns: Empirical Evidence, *Journal of Financial Research*, **12**, 253-260.

Ait-Sahalia, Y. and M.W. Brandt (2001) Variable Selection for Portfolio Choice, *Journal of Finance* 56(4), 1297-1355.

Alles, A, L., and J.L Kling (1994) Regularities in the Variation of Skewness in Asset Returns, *Journal of Financial Research*, **17**, 427-438.

Ang, A. and J. Chen (2002) Asymmetric Correlations of Equity Portfolios, *Journal of Financial Economics,* **63,** 3, 443-494.

Barone-Adesi, G. (2003) Arbitrage Equilibrium with Skewed Asset Returns, *Journal of Financial and Quantitative Analysis* 20(3), 299-313.

Black, F. (1976) Studies in Stock Volatility Changes, Proceedings of the 1976 Meeting of the Business and Economics Statistics Section, *American Statistical Association,* 177-181.

Brown, B. Wm. Jr. and M. Hollander (1977) *Statistics: A Biomedical Introduction*, John Wiley & Sons, Inc.

Chen, J., H. Hong and J. Stein (2000) Forecasting Crashes: Trading Volume, Past Returns and Conditional Skewness in Stock Prices, *Journal of Financial Economics,* **61,** 345-381.

Chunhachinda, P., K. Dandepani, S. Hamid and A.J. Prakash (1997) Portfolio Selection and Skewness: Evidence from International Stock Markets, *Journal of Banking and Finance* 21(2), 143-167.

Conine, T. and M. Tamarkin (1981) On Diversification Given Asymmetry in Returns, *Journal of Finance* 36, 1143-1155.

Conover, W.J. (1999) *Practical Nonparametric Statistics (Third Edition),* John Wiley & Sons, Inc.

Cont, R. (2001) Empirical Properties of Asset Returns: Stylised Facts and Statistical Issues *Quantitative Finance*, **1**, 223-236.

Engle, R.F. and A.J. Patton (2001) What Good is a Volatility Model, *Quantitative Finance*, **1**, 237-245.

Fama, E.F. (1965) The Behaviour of Stock Market Prices, *Journal of Business,* **38,** 34–105.

Fogler, H. R. and R.C. Radcliffe (1974) A Note on Measurement of Skewness, *Journal of Financial and Quantitative Analysis*, **9** 485-489.

Friend, I. and R. Westerfield (1980) Co-skewness and Capital Asset Pricing, *Journal of Finance* 35, 897-919.

Gibbons, J. D. (1971) *Nonparametric Statistical Analysis*, McGraw-Hill.

Gray, J.B and D.W. French (1990), Empirical Comparisons of Distributional Models for Stock Index Returns, *Journal of Business Finance and Accounting*, 17, 451-459.

Harvey, C.R. and A. Siddique (1999) Autoregressive Conditional Skewness, *Journal of Financial and Quantitative Analysis,* **34,** 465 – 487.

Harvey, C.R. and A. Siddique (2000) Conditional Skewness in Asset Pricing Tests, *Journal of Finance* 55, 1263-1295.

Jondeau, E. and M. Rockinger (2003) Conditional Volatility, Skewness and Kurtosis: Existence, Persistence and Co-movements, *Journal of Economic Dynamics and Control* 27(10), 1699-1737.

Kon, S. (1984) Models of Stock Returns – A Comparison, *Journal of Finance*, **39**, 147–165.

Kraus, A. and R. Litzenberger (1976) Skewness Preference and the Valuation of Risk Assets, *Journal of Finance* 31, 1085-1100.

Kraus, A. and R. Litzenberger (1983) On the Distributional Conditions for a Consumption-oriented Three-Moment CAPM, *Journal of Finance* 38, 1381-1391.

Lai, T. (1991) Portfolio Selection with Skewness, *Review of Quantitative Finance and Accounting* 1, 293-305.

Lim, K-G. (1989) A New Test of the Three-Moment Capital Asset Pricing Model, *Journal of Financial and Quantitative Analysis* 24(2), 205-216.

Mandelbrot, B.B. (1963) The Variation of Certain Speculative Prices, *Journal of Business,* **36**, 4, 394-419.

Patton, A.J. (2004), On the Out-of Sample Importance of Skewness and Asymmetric Dependence for Asset Allocation, *Journal of Financial Econometrics*, 2, 1, 130-168.

Peiró, A. (1994) The Distribution of Stock Returns: International Evidence, *Applied Financial Economics,* **4**, 431-439.

Peiró, A. (1999) Skewness in Financial Returns, *Journal of Banking and Finance***,** **23**, 847-862.

Peiró, A. (2002) Skewness in Individual Stocks at Different Investment Horizons, *Quantitative Finance* **2,** 139-146.

Prakash, A.J., C.H. Chang and T. E. Pactwa (2003) Selecting a Portfolio with Skewness: Recent Evidence from US, European and Latin American Equity Markets, *Journal of Banking and Finance* 27(7), 1375-1390.

Simkowitz, M.A. and W.L. Beedles (1980), Asymmetric Stable Distributed Security Returns, *Journal of the American Statistical Association*, 75, 306-312.

Singleton, J. and J. Wingender (1986) Skewness Persistence in Common Stock Returns, *Journal of Financial and Quantitative Analysis* 13, 335-341.

Sun, Q. and Y. Yan (2003) Skewness Persistence with Optimal Portfolio Selection, *Journal of Banking and Finance* 27(6), 1111-1121.

Institute for International Integration Studies
The Sutherland Centre, Trinity College Dublin, Dublin 2, Ireland