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Vahagn Galstyan, IIIS, Trinity College Dublin
Michael Wycherley, IIIS, Trinity College Dublin



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Vahagn Galstyan

Michael Wycherley

IIIS, Trinity College Dublin

IIIS, Trinity College Dublin

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Abstract

The standard New Keynesian model allows the derivation of optimal monetary policy on the assumption that the economy is composed of a single sector. This paper analyses optimal policy on the basis that the economy comprises a number of different sectors. It shows that the composition of output matters, that policy should take into account the source of shocks as well as their aggregate magnitude, and that policy tools impacting individual sectors can be welfare improving. If sectoral policy is not adopted, then commitment in tax policy is important in similar ways and for similar reasons to commitment in monetary policy. With sectoral policy, commitment for tax and monetary policies ceases to be important.

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*Email: v.galstyan@tcd.ie, wycherlm@tcd.ie.

1. Introduction

"...monetary policy can target only the aggregate price level...it would be a mistake to target these relative prices."¹

Output and inflation are generally more volatile at a sectoral level than an aggregate level, as illustrated by Figure 1 for the UK and US, which also shows that they often do not move in tandem. The natural question emerges as to whether policy makers should be concerned with fluctuations in aggregate inflation and output gap only or the objective function should be modified to include movements in "relative inflation" and "relative output gap"?² Alternatively, does the sectoral composition of output matter for welfare?³

The standard literature on monetary policy, such as the classic texts by Woodford (2003) and Galí (2008), is generally focused on aggregate output and inflation, without being concerned about the composition of that output. As a consequence of this policy makers have been focused on stabilizing aggregate variables. In this paper, our aim is to show that the composition of output matters for welfare, and that welfare can be improved through policies that target individual sectors.

We use a two-sector version of the standard New Keynesian model to show that sectoral policy tools are useful in the presence of either sector specific shocks or aggregate shocks with effects that vary across sectors. We derive the representative households' welfare-based quadratic loss function, where the weight on each sectoral inflation rate is increasing in the degree of price rigidity in that sector.⁴ We highlight that, in a two sector model, welfare is affected not only by the volatility of aggregate inflation and output gap, as in a standard one sector model, but also by the volatility of the relative output gap, relative inflation and the covariance between relative and aggregate inflation. It follows that policy tools that affect these variables can generate an improvement in the household's welfare.

The optimal tax and monetary policy is derived and we show that, in the presence of either sector specific shocks or differential responses to common shocks, for example

¹King, M., 2003. The Governor's Speech at the East Midlands Development Agency/Bank of England Dinner.

²Movements in the real exchange rate to some extent reflect movements in the relative price of tradeables and non-tradeables. A straightforward extension of our model is that the sectors represent tradeables and non-tradeables, and so movements in the real exchange rate matter for welfare.

³Blanchard et al (2010) also highlights the importance of output composition.

⁴Aoki (2001) shows a similar result for an economy with a flexible price and a sticky price sector and Benigno (2004) for a currency union. Erceg and Levin (2006) show that the social welfare function involves sector-specific output gaps and inflation rates.

through differing price rigidities, welfare can be improved through sector specific tax policy. If the government can levy sector specific taxes on firms, taxes rise in sectors hit by a positive productivity shock to limit price dispersion within a sector and so ensure efficient allocation of resources within the sector. These taxes can also offset cost push shocks. For reasonable parameterisations optimal sectoral policy seeks to ensure efficiency within sectors rather than across sectors. Therefore government should stabilise sectoral output and prices relative to the steady state and not relative to the flexible price level, and so should respond countercyclically. We also show that adopting sectoral policy of this kind significantly reduces the welfare losses from the business cycle, and that the gain from adopting the correct sectoral policy is substantial relative to the gain from having no sectoral policy and moving from discretionary policy to policy under commitment. Finally, we show that under discretionary policy using aggregate tax and monetary policy to respond to productivity shocks is inferior to using monetary policy alone. However this is not true if there are sector specific taxes or if aggregate taxes are set under commitment. In fact, with sectoral policy, commitment for tax and monetary policies ceases to be important.

This study complements an already vast literature on monetary policy in a multi-sector setting. Carlstrom et al (2006) demonstrate that in a multi-sector model local determinacy is guaranteed as long as central banks respond aggressively to shocks, regardless of the price index they target. Erceg and Levin (2006) show that a rule that stabilizes a more narrow measure of final goods price inflation performs poorly in terms of social welfare. A rule that targets a weighted average of aggregate wage and price inflation, on the other hand achieves higher social welfare. Carvalho (2006) and Nakamura and Steinsson (2007) show that sectoral heterogeneity in price stickiness leads to larger and more persistent effects of monetary shocks.

These studies analyse the implications of different sectors for aggregate policy. Our paper shows that by utilizing sectoral tools, such as payroll taxes that can be varied at a sectoral level, policy makers can achieve a better allocation of resources and consequently higher welfare.

The rest of the paper is structured as follows, the model to be used is outlined in the second section, the third section examines the welfare implications of different policy tools, the fourth section quantifies the effects and the fifth concludes.

2. The model

This section outlines and solves the model that is used throughout the rest of the paper. The model is a two sector version of the standard New-Keynesian closed economy model. It is assumed that labour can move freely across both sectors and firms, so that all firms face the same wage rate.

2.1. Households

A representative household chooses a consumption bundle (C) and labour (N) to maximise the present discounted value of expected utility given by

$$\Psi_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\gamma N_s^{1+\varphi}}{1+\varphi} \right) \right\}, \quad (1)$$

with $0 < \beta < 1$ representing time preference factor, $\sigma > 0$ the inverse of the elasticity of intertemporal substitution, φ the inverse of Frisch elasticity and γ a relative weight assigned by the household to work effort. The household faces the standard transversality condition and the period budget constraint, $C_t + \frac{Q_t}{P_t} B_t \leq \frac{1}{P_t} B_{t-1} + \frac{W_t}{P_t} N_t + \Pi_t + T_t$, where B_t represents purchases of one-period bonds at price Q_t , W_t is the wage, P_t the price of the consumption bundle, Π_t profits and T_t lump sum payments by government. Household optimization then implies that the standard Euler equation and labour supply conditions hold

$$Q_t = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right\}, \quad (2)$$

$$\frac{\gamma N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \quad (3)$$

The consumption bundle is composed of goods from two different sectors that are aggregated according to

$$C_t \equiv \left(\omega^{\frac{1}{\eta}} C_{a,t}^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} C_{b,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

where $\eta > 0$, so that the sectors are complements, and $0 < \omega < 1$.⁵ This implies that the

⁵It is straightforward to extend the model from two to many sectors, but no additional insights are gained.

household's relative demand is inversely proportionate to relative sectoral prices

$$\frac{C_{a,t}}{C_{b,t}} = \frac{\omega}{1-\omega} \left(\frac{P_{a,t}}{P_{b,t}} \right)^{-\eta}. \quad (4)$$

The sectoral consumption bundle $C_{j,t}$ is, in turn, given by a CES aggregator over a continuum of goods indexed on the interval $i \in [0, 1]$ with a sector-specific elasticity of substitution $\theta_j > 1$. Defining $p_{j,t}(i)$ as the price of good i in sector j at time t , it follows that demand for individual goods is given by $c_{j,t}(i) = (p_{j,t}(i)/P_{j,t})^{-\theta_j} C_{j,t}$ while the sectoral price index is $P_{j,t} \equiv \left(\int_0^1 p_{j,t}^{1-\theta_j}(i) di \right)^{1/(1-\theta_j)}$ and the aggregate price index is a generalised weighted average of sectoral prices

$$P_t \equiv \left(\omega P_{a,t}^{1-\eta} + (1-\omega) P_{b,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}.$$

2.2. Firms

In both sectors monopolistically competitive firms hire labour to produce a differentiated good, and to do so must pay wages (W) and a tax on payroll expenditure (τ) so that costs per unit of labour are given by $W(1 + \tau)$. In sector j , good i is produced according to $y_{j,t}(i) = A_{j,t} N_{j,t}(i)$, with $A_{j,t}$ representing an exogenous sector specific productivity parameter and $N_{j,t}(i)$ labour employed by the firm. Following the standard conventions of the New Keynesian model, we assume staggered price setting a la Calvo (1983) where any firm may reset its price with probability $(1 - \mu_j)$. Thus the problem of the firm is to choose the price $p_{j,t}^*(i)$ that maximizes the expected present discounted value of future profits subject to a sequence of demand constraints $y_{j,s}(i) = Y_{j,s} \left(p_{j,t}^*(i)/P_{j,s} \right)^{-\theta_j}$ for $s \in [t, \infty)$. Solution of the problem implies that prices are set according to

$$p_{j,t}^*(i) = \frac{\theta_j}{\theta_j - 1} \frac{E_t \left\{ \sum_{s=t}^{\infty} \mu_j^{s-t} Q_{t,s} MC_{j,s} y_{j,s}(i) \right\}}{E_t \left\{ \sum_{s=t}^{\infty} \mu_j^{s-t} Q_{t,s} y_{j,s}(i) \right\}}, \quad (5)$$

where $Q_{t,s} = \prod_{h=t}^s Q_h$ is the stochastic discount factor, $MC_{j,s} = W_s(1 + \tau_{j,s})/A_{j,s}$ is the marginal cost of production for the firm at time s , and $\tau_{j,s}$ is the payroll tax rate set by government in sector j .

2.3. Flexible price equilibrium

In the flexible-price equilibrium, all firms in a given sector set the same price $p_{j,t}^*(i) = \frac{\theta_j}{\theta_j-1} MC_{j,t} = \frac{\theta_j}{\theta_j-1} \frac{W_t(1+\tau_{j,t})}{A_{j,t}}$. In this case the optimal payroll tax is set to offset the monopolistic distortion, such that $\tau_{j,t}^f = -1/\theta_j$.⁶ Therefore it follows from the definition of $P_{j,t}$ that $W_t^f = P_{a,t}^f A_{a,t} = P_{b,t}^f A_{b,t}$ and relative sectoral prices are inversely proportional to sectoral productivity, with aggregate prices given by $P_t = W_t/A_t$, where $A_t \equiv \left(\omega A_{a,t}^{\eta-1} + (1-\omega) A_{b,t}^{\eta-1} \right)^{\frac{1}{\eta-1}}$.

The relative demand (equation 4) and market clearing ($C_{j,t} = Y_{j,t}$) conditions determine equilibrium relative labor supply. Combining the equilibrium relative labor supply with labor market clearing and the intratemporal labour supply condition (equation 3) allows us to solve for the flexible price level of aggregate and sectoral outputs

$$\begin{aligned} Y_t^f &= A_t N_t^f = \gamma^{-\frac{1}{\varphi+\sigma}} A_t^{\frac{\varphi+1}{\varphi+\sigma}}, \\ Y_{a,t}^f &= \left(\frac{A_{a,t}}{A_t} \right)^\eta \omega Y_t^f, \\ Y_{b,t}^f &= \left(\frac{A_{b,t}}{A_t} \right)^\eta (1-\omega) Y_t^f. \end{aligned}$$

2.4. Linearization

Defining $\tilde{x}_t \equiv \ln(x_t/x_t^f)$, $\hat{x}_t \equiv \ln(x_t/x)$ and $i_t = -\ln Q_t$ then to a first order approximation, household optimisation (equations 2 and 3), market clearing and relative demand (equation 4) imply

$$i_t = \sigma \left(E_t \left\{ \tilde{Y}_{t+1} \right\} - \tilde{Y}_t \right) + \sigma \frac{1+\varphi}{\varphi+\sigma} E_t \left\{ \hat{A}_{t+1} - \hat{A}_t \right\} + E_t \left\{ \pi_{t+1} \right\} - \ln \beta, \quad (6)$$

$$\hat{N}_t = -\frac{\sigma}{\varphi} \hat{C}_t + \frac{1}{\varphi} \left(\hat{W}_t - \hat{P}_t \right), \quad (7)$$

$$\tilde{Y}_{a,t} - \tilde{Y}_{a,t-1} = \tilde{Y}_{b,t} - \tilde{Y}_{b,t-1} - \eta \left(\pi_{a,t} - \pi_{b,t} + \hat{A}_{a,t} - \hat{A}_{a,t-1} - \hat{A}_{b,t} + \hat{A}_{b,t-1} \right). \quad (8)$$

It is possible to show that, defining inflation in sector $j = a, b$ at time t as $\pi_{j,t}$ and $\bar{\tau}_{j,t} \equiv (\tau_{j,t} + 1/\theta_j) / (1 - 1/\theta_j)$, the optimal price setting equation (5) implies equation

⁶Superscript f implies the variable represents the flexible price value of the variable.

(9), a sectoral version of the standard New Keynesian Phillips curve (NKPC).⁷⁸

$$\pi_{j,t} = \beta E_t (\pi_{j,t+1}) + \frac{(1 - \beta\mu_j)(1 - \mu_j)}{\mu_j} \left(\left(\varphi + \sigma - \frac{1}{\eta} \right) \tilde{Y}_t + \frac{1}{\eta} \tilde{Y}_{j,t} + \bar{\tau}_{j,t} \right) + u_{j,t}, \quad (9)$$

where

$$\tilde{Y}_t = \omega \tilde{Y}_{a,t} + (1 - \omega) \tilde{Y}_{b,t}, \quad (10)$$

$$\pi_t = \omega \pi_{a,t} + (1 - \omega) \pi_{b,t}. \quad (11)$$

The NKPC includes an exogenous term, $u_{j,t}$, which is a sector specific version of the standard cost push shock. This could arise, for example, through a shock to θ_j , the elasticity of substitution within a sector.

2.5. Welfare

The private sector equilibrium for given interest and tax rates is described by equations (6), (8), (9), (10) and (11). Government chooses monetary and tax policy to maximise welfare subject to these constraints. Net revenue from payroll taxes and subsidies is spent on or financed by lump sum transfers from households. The payroll taxes have two purposes, firstly, with flexible prices they are used to offset market power to ensure an efficient level of output ($\tau_j = -1/\theta_j$), and secondly they can be varied in response to shocks that hit the economy to offset price stickiness. It is possible to show that, to a second order approximation around the flexible price level, welfare (equation 1) is given by:⁹

$$\frac{\Psi_t}{C^{1-\sigma}} = -E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{\psi_s}{C^{1-\sigma}} \right\} + O(\|a\|^3), \quad (12)$$

$$-\frac{2\psi_s}{C^{1-\sigma}} = \frac{\omega}{\eta} \tilde{Y}_{a,s}^2 + \frac{1-\omega}{\eta} \tilde{Y}_{b,s}^2 + \left(\sigma + \varphi - \frac{1}{\eta} \right) \tilde{Y}_s^2 + \chi_a \omega \pi_{a,s}^2 + \chi_b (1 - \omega) \pi_{b,s}^2. \quad (13)$$

where $\chi_j = \theta_j \mu_j / [(1 - \mu_j)(1 - \beta\mu_j)]$.

Households are concerned about inflation since, with sticky prices, changes in the price level imply an inefficient use of labour. This follows as an approximation of output

⁷Available on request from the authors

⁸From now on when we refer to tax rate we mean $\bar{\tau}$.

⁹Also available on request from the authors.

in sector j is given by

$$\tilde{Y}_{j,t} = \tilde{N}_{j,t} - \frac{\theta_j}{2} \frac{\mu_j}{1 - \mu_j} \sum_{s=0}^t \mu_j^{t-s} \pi_{j,s}^2. \quad (14)$$

Therefore, the weight in the welfare function of inflation in a sector depends upon both the sector's weight in aggregate output and the degree to which the price change causes inefficiency, i.e. the degree of price rigidity in the sector. For this reason the weight on each sectorial inflation term is higher the greater the price rigidity in that sector.¹⁰

Alternatively, the welfare function can be rewritten in aggregate and relative terms as

$$\begin{aligned} -\frac{2\psi_s}{C^{1-\sigma}} = & (\sigma + \varphi) \tilde{Y}_s^2 + (\omega\chi_a + (1 - \omega)\chi_b) \pi_s^2 \\ & + \frac{\omega(1 - \omega)}{\eta} \tilde{Y}_{a-b,s}^2 + \omega(1 - \omega)((1 - \omega)\chi_a + \omega\chi_b) \pi_{a-b,s}^2 \\ & + 2\omega(1 - \omega)(\chi_a - \chi_b) \pi_s \pi_{a-b,s}, \end{aligned}$$

where $x_{a-b,s} = x_{a,s} - x_{b,s}$. Thus, as in the standard New-Keynesian model, consumers' welfare depends on the volatility of aggregate inflation and aggregate output. But in a two sector model, welfare is also affected by the volatility of the relative output gap, relative inflation and the covariance between relative and aggregate inflation. It is now clear that both relative prices and relative quantities matter for welfare and that, since they are linked through the relative demand condition (equation 8), they are jointly determined. The extent to which households are concerned about the covariance term depends on the degree to which the relationship between output and inflation through the NKPC differs across sectors (given by χ_j). The more similar the sectors, the more indifferent households are about the covariance between aggregate and relative inflation. This is because the more similar the sectors, the better the weight in aggregate output (given by ω) represents the welfare weight placed on sectorial inflation and the less likely governments are to trade-off higher aggregate inflation for lower inflation in a sector with greater price stickiness.

For a symmetric case ($\mu_j = \mu$ and $\theta_j = \theta$) the covariance between relative and aggregate inflation becomes irrelevant. In this case it can be shown that relative inflation is determined solely by past and future expected productivities, taxes and cost push shocks

$$\pi_{a-b,t} = \sum_{i=0}^{\infty} \mu^i (\xi_{t-i} - \xi_{t-1-i}),$$

¹⁰As mentioned in the introduction, Aoki (2001) shows a similar result for an economy with a flexible price and a sticky price sector and Benigno (2004) for a currency union.

where

$$\xi_t \equiv \sum_{i=0}^{\infty} (\mu\beta)^i E_t \left\{ \mu [u_{a,t+i} - u_{b,t+i}] - (1-\mu)(1-\mu\beta) \left[(\hat{A}_{a,t+i} - \hat{A}_{b,t+i}) - (\bar{\tau}_{a,t+i} - \bar{\tau}_{b,t+i}) \right] \right\}.$$

Therefore, in the symmetric case, the variance of relative inflation and the relative output gap is determined independently of monetary policy. Since relative inflation and the relative output gap matter for welfare, it follows that policy tools that can affect these variables can generate an improvement in the household's welfare.

More generally, in the standard New Keynesian model governments have two objectives, zero aggregate inflation and zero aggregate output gap, output matters because it affects both consumption and labour, while prices matter because they affect the efficiency with which labour is used, and so the relationship between labour and output. In this framework governments have two additional objectives, stabilizing the relative output gap and relative prices. In a two sector model inefficiencies arise through both the inefficient allocation across sectors and through the inefficient allocation of resources within sectors. The next section shows that there are welfare gains from sectoral policy tools (payroll taxes) that enable governments to target relative prices and output, and under which conditions these welfare gains are likely to be large.

3. Optimal policy

Governments have two types of policy tools, a nominal interest rate that affects demand through the IS curve and payroll taxes or subsidies that affect firms' costs and so their price setting and output decisions. Sectoral policy tools imply that the sectoral NKPCs of equation (9) do not bind, so there is no need for commitment. Without sectoral tools (either $\bar{\tau}_t = \bar{\tau}_{b,t} = \bar{\tau}_{a,t}$ or $\bar{\tau}_{b,t} = \bar{\tau}_{a,t} = 0$) results are qualitatively similar under either discretion or commitment to a rule. The key difference is that there is a welfare gain in going from policy under discretion to policy under commitment. Under discretion, using aggregate tax and monetary policy to respond to productivity shocks is inferior to using monetary policy alone. This is because, under discretion, aggregate taxes are set without taking into account their effect on inflation expectations, and so, as with monetary policy, there is a gain to commitment. With sectoral policy there is no difference between policy under discretion and policy with commitment. Simulations reveal that the gains from commitment are small compared to the welfare gains from adopting sectoral policy (see Table 1). Therefore, we assume that governments are unable to commit to a policy rule and so seek to maximise welfare as given by equation (13) subject to the

sectoral NKPCs of equation (9) and the relative demand constraint given by equation (8).

3.1. Common tax policy

If government is unable to vary payroll taxes across sectors ($\bar{\tau}_t = \bar{\tau}_{b,t} = \bar{\tau}_{a,t}$) then optimal policy is to set payroll taxes and nominal interest rates such that

$$0 = \omega \tilde{Y}_{a,t} + (1 - \omega) \tilde{Y}_{b,t}, \quad (15)$$

$$\sum_{j=a,b} \varpi_j \left(\tilde{Y}_{j,t} + \theta_j \pi_{j,t} \right) \left(1 + \frac{\theta_j}{\chi_j} \right) = \beta E_t \sum_{j=a,b} \varpi_j \theta_j \pi_{j,t+1}, \quad (16)$$

where $\omega_a = \omega$ and $\omega_b = 1 - \omega$. If $\mu_a = \mu_b$ equation (16) becomes $(1 - \varpi) \theta_b \pi_{b,t} + \varpi \theta_b \pi_{b,t} = 0$. Therefore, the government ensures that aggregate output is always at the flexible price level. If the sectors are symmetric then stabilising the aggregate output gap also stabilises aggregate prices, but not otherwise. The intertemporal nature of the optimal policy comes from the relative demand condition (8). This implies that relative sectoral inflation depends on the change in the relative output gap compared to the previous period, and so when deciding on the inflation-output trade-off this period the effects on next period's relative demand will be taken into account. This is different from the intertemporal nature of the NKPC as the latter is forward looking rather than backward looking and so depends on expectations rather than actual realizations. The optimal equilibrium without sectoral taxes is defined by equations (6, 8, 9, 10, 11, 16 and 15).

The welfare loss comes about despite achieving the flexible price level of aggregate output because government is unable to ensure that output is produced efficiently. Price rigidities imply that output at the firm level is different from the flexible price level for some firms and so firms produce too much or too little compared to the flexible price level. Thus sticky prices imply that an inefficient amount of labour is used to achieve the flexible price level of output, and welfare is lower than with flexible prices.

With symmetry across sectors, optimal policy is given by

$$i_t = \sigma \frac{1 + \varphi}{\varphi + \sigma} \left(E_t \left\{ \hat{A}_{t+1} \right\} - \hat{A}_t \right) - \ln \beta, \quad (17)$$

$$\bar{\tau}_t = -\mu_b \frac{\omega u_{a,t} + (1 - \omega) u_{b,t}}{(1 - \beta \mu_b)(1 - \mu_b)}. \quad (18)$$

So with symmetric sectors, taxes can be chosen to offset any aggregate cost push shock (where $u_{a,t} = u_{b,t}$) leaving no effect on aggregate or sectoral prices and output. Aggregate productivity shocks (where $\hat{A}_{b,t} = \hat{A}_{a,t}$) lead, with symmetric sectors, to changes in

the nominal interest rate to ensure that output is at the flexible price level. The mechanism at work is that changes to the nominal interest rate affect current consumption. This affects the labour supply decision, in turn inducing changes in output. With a positive aggregate productivity shock, under flexible prices, prices fall to increase consumption and hours worked, while with price rigidities, government lowers interest rates to increase consumption and hours worked. Relative prices are constant in the event of an aggregate productivity shock because sectors are affected equally while aggregate prices are kept constant because higher demand pushes up wages, offsetting the downward pressure on prices from higher productivity.

Symmetry across sectors implies that both sectors respond in similar ways to a change in aggregate policy, and so aggregate policy is powerless to address sectoral shocks that offset each other (i.e. $\omega u_{a,t} + (1 - \omega)u_{b,t} = 0$ or $\hat{A}_t = 0$). Sector specific shocks that do not offset each other lead aggregate policy to stabilise the aggregate output gap with respect to the flexible price level and to stabilise aggregate prices. This would occur at the expense of being unable to affect relative prices and output, which have both been shown to matter for welfare, meaning a policy tool that could target relative prices could be welfare improving.

Without symmetry across sectors, aggregate policy tools are able to target relative prices because the sectors will respond in different ways to a given policy. Governments still seek to stabilise the aggregate output gap (equation 15). However, they now care about the source of shocks, not just the aggregate effect, with the optimal trade-off given by equation (16). As might be expected it places more weight on stabilisation in sectors with more weight in the welfare function, with a higher degree of price stickiness (higher μ) or a larger elasticity of substitution (θ). The objective is no longer to stabilise aggregate prices because extra weight above that in output is now placed on stabilising sectors with a higher degree of price stickiness or elasticity of substitution. Therefore, with asymmetric sectors, nominal interest and aggregate tax rates both respond to sector specific productivity and cost push shocks. This contrasts with the symmetric case where nominal interest rates respond only to aggregate productivity shocks and aggregate tax rates respond only to aggregate cost push shocks.

3.2. Sector specific tax policy

If government is able to vary payroll taxes across sectors then optimal policy is to set payroll taxes and nominal interest rates such that equation (15) as well as¹¹

$$\tilde{Y}_{a,t} = \chi_a (\pi_{a,t} - \beta E_t \{\pi_{a,t+1}\}), \quad (19)$$

$$\chi_a \varpi \pi_{a,t} + \chi_b (1 - \varpi) \pi_{b,t} = 0. \quad (20)$$

Once again the government ensures that output is always at the flexible price level and, if the sectors are symmetric, then stabilising the aggregate output gap also stabilises aggregate prices, but not otherwise. As with aggregate taxes, the relative demand condition (equation 8) implies an intertemporal element to the optimal trade off. Aside from the intertemporal element, sectoral policy is very similar to aggregate policy in a one sector model (see Gali 2008 for example). The optimal equilibrium with sectoral taxes is defined by equations (6, 9, 8, 10, 11, 15, 19 and 20).

The difference with sectoral policy is that government is now able to trade off inflation and output at a sectoral level rather than at some composite aggregate level. Thus both sectoral cost push shocks and aggregate cost push shocks with asymmetric sectors can now be offset by sectoral tax changes without them affecting sectoral prices or output. This is in contrast to the case without sectoral policy where cost push shocks could only be offset if the sectors were symmetric and were hit by the same shock. If only one sector is hit by a productivity shock then through the relative demand condition, equation (8), there are implications for output and prices in the other sector. Thus sectoral taxes respond to cost push shocks in their sectors and productivity shocks anywhere in the economy. Nominal interest rates continue to respond to changes in aggregate productivity and prices. As with aggregate taxes, if the sectors are symmetric stabilising the aggregate output gap also stabilises aggregate prices, but not otherwise.

Productivity shocks cause two types of inefficiency. Firstly, some firms within a sector are unable to adjust their prices and so resources are inefficiently employed within a sector (equation 14). At the same time price rigidities make it unlikely that relative sectoral prices adjust appropriately, and so resources are then deployed suboptimally across sectors (equation 8). Sectoral policy tools enable government to target sectoral prices, and they can do so either to stabilise prices within a sector at the steady state level or to target relative sectoral prices at the flexible price level. The former implies that resources are allocated efficiently within the sector but that output in the sector deviates from the flex-

¹¹These equations hold under both commitment and discretionary policy, so that with sectoral policy commitment ceases to be important,

ible price level (equation 8 implies that, if prices are constant, then productivity shocks lead to output gaps). Targeting relative prices implies that resources are allocated efficiently across sectors, but the required change in prices leads to the inefficient use of labour within a sector (equation 14).

To achieve the flexible price level of relative sectoral prices the firms that can adjust their prices should be induced to do so by more than they would under flexible prices to offset the effect of firms that are unable to adjust their prices. Relative sectoral prices under flexible prices are given by $P_{a,t}^f A_{a,t} = P_{b,t}^f A_{b,t}$ so that a positive relative productivity shock to sector a would lead to lower relative sectoral prices in that sector. Thus achieving the flexible price level of relative prices implies tax cuts in sectors with high productivity, while achieving stable sectoral prices implies tax rises in sectors with high productivity in order to limit price changes. Therefore government cannot simultaneously use sectoral taxes to achieve both the efficient allocation within sectors and the efficient allocation across sectors. Which it prefers to pursue determines whether taxes in a given sector rise or fall in response to a positive productivity shock in that sector.

It is possible to use equations (15 and 19) in the NKPCs (equation 9) to show that

$$\bar{\tau}_{j,t} = -\frac{\theta_j - \eta}{\eta\theta_j} \tilde{Y}_{j,t} - \frac{\chi_j}{\theta_j} u_{j,t}. \quad (21)$$

Therefore, assuming no cost push shock, if output in a given sector is below the flexible price level then, for plausible parameterisations, taxes in that sector are above the steady state level. This implies that, in the event of productivity shocks alone, sectoral policy tends to stabilise prices within a sector, ensuring the efficient allocation of resources within a sector rather than across sectors. Thus, with nominal rigidities, the impact of sectoral productivity booms is optimally reduced by sectoral policy below what it would be both without sectoral policy and without price rigidities. The lower the degree of substitution across sectors relative to within sectors ($\eta - \theta$), the more likely government is to focus on efficiency within a sector since there is less to be gained from reallocating labour across sectors. Equation (21) also implies that sectoral taxes, unlike aggregate taxes, respond to both cost push and productivity shocks regardless of whether the sectors are symmetric.

It follows that sectoral policy is welfare improving when either there are sector specific shocks or there are asymmetries between sectors that mean the sectors respond differently to either common shocks or to common policies. The question is whether the welfare gain is significant, and this is the subject of the next section where the model is calibrated and the welfare implications are quantified.

4. Quantifying the effect

This section calibrates the model to study significance of welfare gains from sectoral policy and the variation in sectoral taxes required to achieve them. It is assumed that sectoral productivities and cost push shocks follow a zero mean first order autoregressive process with sector specific autoregressive coefficients ($\rho_{A_a} \leq \rho_{A_b}$ and $\rho_{u_a} \leq \rho_{u_b}$). The economy is assumed to start in steady state and then be hit by one of four sets of disturbances: (i) cost push shock in sector a ($\varepsilon_{u_a,t} = 0.01$); (ii) opposing sectoral cost push shocks ($\varepsilon_{u_a,t} = 0.01$ and $\varepsilon_{u_b,t} = -0.01$); (iii) productivity shock to sector a ($\varepsilon_{A_a,t} = 0.05$); (iv) opposing sectoral productivity shocks ($\varepsilon_{A_a,t} = 0.05$ and $\varepsilon_{A_b,t} = -0.05$). In all of the cases $\varepsilon_{u_j,t}$, and $\varepsilon_{A_j,t}$ are exogenous i.i.d. shocks, and unless specified $\varepsilon_{u_j,t}$, and $\varepsilon_{A_j,t}$ are zero.

4.1. Symmetric sectors

We take what might be thought of as a fairly standard baseline calibration where $\beta = 0.99$, which implies a steady state annual real return on financial assets of about four percent. We also assume $\sigma = 1$ giving an intertemporal elasticity of substitution of 1, and $\varphi = 1$ (a unitary Frisch elasticity of labour supply), values commonly found in the business cycle literature. In addition we assume $\mu_b = \mu_a = 0.66$, which implies an average price duration of three quarters, a value consistent with the empirical evidence. The elasticity of substitution within a sector is set to give a mark up of 20%, so that $\theta_a = \theta_b = 6$, and the elasticity of substitution across sectors, η , is arbitrarily set at 0.5. It is assumed that the sectors carry equal weight and so $\omega = 0.5$, and shocks have equal persistence $\rho_{u_j} = \rho_{A_j} = 0.9$.

This implies that the sectors are symmetric, so that both the aggregate output gap relative to the flexible price level and prices relative to the steady state are stabilised. Aggregate shocks are then offset by the aggregate policy tools without the need for sectoral policy tools and it is only sector specific shocks that generate trade-offs. Equations (17) and (18) show that, without sectoral taxes, symmetry across sectors implies that offsetting shocks lead to no policy changes, while aggregate taxes respond to sector specific cost push shocks and nominal interest rates respond to productivity shocks. Sectoral taxes can entirely offset sectoral cost push shocks, and so, with sectoral policy, cost push shocks have no welfare implications.¹² Therefore, with sectoral policy tools, equations (9), (15), (6) and (21) imply that nominal interest rates change only in response to changes in

¹²Not surprisingly if these cost push shocks come from shocks to the elasticity of substitution within a sector.

aggregate productivity ($\hat{A}_t$), while sectoral taxes change in response to both changes in relative sectoral productivity and sector specific cost push shocks.

Figure 2 illustrates the response of the economy to a cost push shock to sector a alone, both with and without sectoral policy. In both policy environments the aggregate output gap and aggregate prices are constant. With sectoral policy, taxes in sector a are reduced and taxes in sector b remain constant. Therefore, with sectoral policy there are no implications for sectoral output, prices or aggregate welfare from the shock. However, without sectoral policy, government's best response is to reduce the common tax rate to approximately the average of the tax rates with sectoral policy. The lack of sectoral policy implies that the cost push shock to sector a has implications for relative prices and the distribution of output and therefore for welfare. Thus sectoral policy is welfare improving in the event of sector specific shocks with symmetric sectors.

The case of offsetting cost push shocks is illustrated in Figure 3. Again, symmetry implies that aggregate prices and aggregate output are kept constant, while sectoral policy enables both sectoral taxes to completely offset the effect of the shocks. Without sectoral policy neither nominal interest rates nor the aggregate payroll tax respond and again relative sectoral prices and output deviate from their flexible price levels and there is a welfare loss from the shock, so again, welfare is improved by sectoral policy.

A more interesting case occurs when the economy is hit by productivity shocks. In the case of a productivity shock to sector a alone, aggregate prices and the aggregate output gap are unchanged, while equation (6) implies that nominal interest rates respond similarly with and without sectoral policy. Equation (18) shows that, without sectoral policy, payroll taxes remain constant. Therefore, without sectoral policy, in sector a relative prices fall and relative output rises, but neither changes by as much as with flexible prices. With sectoral policy, equation (21) with the given parameterisation implies that taxes rise in sector a and fall in sector b . Thus, the effect of higher productivity on prices and output is offset by sectoral policy and both prices and output remain closer to their steady state level. Optimal sectoral policy then inhibits the economy moving away from the steady state level of sectoral output and prices and so increases the divergence from the flexible price equilibrium. These results also hold for offsetting productivity shocks with symmetric sectors, although in this case nominal interest rates are constant, so only sectoral policy tools are used (if they are available).

4.2. Asymmetric sectors

To see the effect of asymmetries between sectors the same exercise is undertaken with differing price rigidities at the sectoral level. It is now assumed that sector b has lower

price flexibility, with an average price duration of five quarters, so that $\mu_b = 0.8$, all the other parameters are left unchanged from the baseline. The effect of asymmetric sectors is that nominal interest rates and a common tax rate can affect relative prices. Therefore, both nominal interest rates and tax rates tend to change in response to both sector specific and aggregate shocks, unlike with symmetric sectors, where nominal interest rates respond only to aggregate productivity shocks.

With asymmetric sectors, policy does not ensure that aggregate prices are stable, as the weighting of sectoral inflation in the welfare function depends partly on the weight of the sector in output and partly on the degree of price stickiness of the sector. Therefore, as Figure 4 shows, without sectoral policy nominal interest rates respond to a cost push shock to sector a . The cost push shock can only be entirely offset by sectoral taxes, so without these taxes government has to decide how much to stabilise sector a at the expense of destabilising sector b . The trade-off depends on the weight of the two sectors in output and the relative price rigidity of the two sectors. The shock will thus have aggregate effects if there are no sectoral taxes, and in this case the nominal interest rate responds to these aggregate effects to stabilise aggregate output at the flexible price level. With sectoral taxes cost push shocks are completely offset with no effects on either sectoral or aggregate inflation rates or output gaps. Asymmetries imply that, without sectoral policy, both nominal interest rates and the aggregate tax rate should respond to a sector specific cost push shock rather than just the aggregate tax rate as occurs with symmetry.

As might be expected from equation (9), with asymmetric sectors, opposing cost push shocks are no longer offsetting and so there is a role for both nominal interest rates and aggregate tax rates, unlike with symmetric sectors. Differing price rigidities implies that a common tax rate has different effects in each sector, and so the optimal rate is not zero, even if the shock is sector specific. Therefore, while sectoral policy remains valuable in the event of opposing cost push shocks and asymmetric sectors, if sectoral policy is not available then aggregate policy is also now valuable, because the asymmetries mean it can affect relative prices.

Both aggregate and sectoral tax rates change in response to sector specific productivity shocks, unlike with symmetric sectors, due to their ability to affect relative prices. Sectoral policy tends to react similarly as in the case of symmetric sectors. Differences in optimal policy with and without sectoral taxes imply differences in aggregate inflation, and so nominal interest rates are adjusted differently with and without sectoral policy, unlike with symmetric sectors. This is shown in Figure 5. Varying the calibration shows that the larger the asymmetries across sectors, the larger the deviations in nominal inter-

est rates between the two policy regimes.

4.3. Welfare

The previous sections have shown that sectoral policy is useful either if there are sector specific shocks, or if there are asymmetric responses to common shocks. The question remains as to whether this is quantitatively important, and this is something that can be readily demonstrated for our calibrated model. Table 1 provides the present discounted value of utility ($U_t/C^{1-\sigma}$) with optimal policy, for monetary policy only ($\bar{\tau}_{a,t} = \bar{\tau}_{b,t} = 0$), without sectoral policy ($\bar{\tau}_{a,t} = \bar{\tau}_{b,t}$) and with sectoral policy and does so with and without symmetric sectors.

As we have shown, with symmetric sectors, aggregate taxes do not respond to productivity shocks or offsetting cost push shocks. We have also demonstrated that sectoral cost push shocks can be entirely offset by sectoral policy both with and without symmetric sectors. The table shows that the welfare loss from sectoral shocks is significantly greater with asymmetric sectors unless government employs sectoral policy. The final row reports the effect on welfare of the shocks with asymmetric sectors and no sectoral policy under commitment.¹³ It shows that the welfare gain from being able to commit to a monetary policy is significantly smaller than the welfare gain from adopting sectoral policy.¹⁴ Interestingly, welfare without sectoral policy would be improved by constraining the tax rate not to respond to productivity shocks. This is because, under discretion we do not take into account the effect of taxes on expected inflation leading to a gain from commitment which is particularly important when productivity shocks hit an economy with asymmetric sectors.

Therefore, a significant part of the welfare cost of nominal rigidities can be eliminated by the correct use of sectoral policy and this is particularly true when either the sectors are asymmetric and the economy is susceptible to cost push shocks, or when sectors are vulnerable to different productivity shocks.¹⁵

¹³With symmetric sectors policy under commitment and discretion is equivalent, namely to stabilise aggregate inflation and output.

¹⁴Simulations show that there is a welfare gain in going from sectoral policy under discretion to sectoral policy under commitment, but that this is small compared to the welfare gain from adopting sectoral policy.

¹⁵Changing our calibration so that $\omega = 0.3$ implies that the welfare gain from adopting sectoral policy is lower, but it remains large relative to the gains from commitment.

5. Conclusion

We have shown that, to a second order approximation, welfare depends on square deviations of all the sectoral inflations and sectoral output gaps. The weight on each sectoral inflation quadratic term is higher the greater the price rigidity in that sector. In addition, the relationship between sectoral inflation and the sectoral output gap depends on the degree of price rigidity through the NKPC. These imply that the monetary policy response to shocks should depend on the origins of the shock if there are sectoral differences.

Secondly, we showed that welfare can be improved by not just letting monetary policy respond to the origins of shocks, but also by utilising tools, such as payroll taxes, that can be varied at a sectoral level. For instance, a temporary sector specific productivity improvement requires higher taxes in the sector in order to reduce inefficiencies from price stickiness within the sector. These taxes would tend to reduce the rise in output in the sector. Quantitatively, sectoral policy can significantly reduce the welfare loss from price rigidities.

Finally, if sectoral policy is not adopted, then commitment in tax policy is important in similar ways and for similar reasons to commitment in monetary policy. With sectoral policy commitment for tax and monetary policies ceases to be important.

There are a number of potential extensions to the basic framework used here. The first is to examine optimal policy under commitment. The second would be to extend the model to incorporate fundamentally different sectors such as tradeables and non-tradeables, consumption and investment, or durables and non-durables. The first of these would be particularly interesting in a currency union, where other trade-offs also exist (see Galstyan and Wycherley 2012).

References

- Aoki, K., 2001. Optimal monetary policy responses to relative-price changes. *Journal of Monetary Economics*, 48(1), pp.55-80.
- Benigno, P., 2004. Optimal monetary policy in a currency area. *Journal of International Economics*, 63(2), pp.293-320.
- Bils, M. & Klenow, P.J., 2004. Some Evidence on the Importance of Sticky Prices. *Journal of Political Economy*, 112(5), pp.947-985.
- Blanchard, O., Dell'Ariccia, G. & Mauro, P., 2010. Rethinking Macroeconomic Policy. *Journal of Money, Credit and Banking*, 42(s1), pp.199-215.
- Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3), pp.383-398.
- Carlstrom, C.T., Fuerst, T.S. & Gihoni, F., 2006. Does it matter (for equilibrium determinacy) what price index the central bank targets? *Journal of Economic Theory*, 128(1), pp.214-231.
- Carvalho, C., 2005. Heterogeneity in Price Setting and the Real Effects of Monetary Shocks, EconWPA.
- Dhyne, E. et al., 2005. Price setting in the euro area: some stylized facts from individual consumer price data, European Central Bank.
- Erceg, C. & Levin, A., 2006. Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics*, 53(7), pp.1341 - 1359.
- Galstyan, V. & Wycherley, M. 2012. Optimal policy tools in a Monetary Union. Unpublished manuscript.
- Gali, Jordi, 2008. *Monetary Policy, Inflation and the Business Cycle*. Princeton University Press.
- Nakamura, E. & Steinsson, J., 2010. Monetary Non-Neutrality in a Multisector Menu Cost Model. *The Quarterly Journal of Economics*, 125(3), pp.961-1013.
- Woodford, Michael, 2003. *Interest and Prices*. Princeton University Press.

Table 1: $U_t/C^{1-\sigma}$ for optimal policy with and without sectoral variation in payroll taxes

		Shock i	Shock ii	Shock iii	Shock iv
symmetric sectors	MP only	-0.352	-0.883	-0.145	-0.581
	No SP	-0.221	-0.883	-0.145	-0.581
	SP	0	0	-0.055	-0.219
asymmetric sectors	MP only	-0.348	-5.996	-0.158	-0.631
	No SP	-0.245	-4.810	-0.162	-0.649
	SP	0	0	-0.059	-0.238
No SP under commitment		-0.223	-4.384	-0.154	-0.616

Shock i: cost push shock to sector a only $\varepsilon_{u_a,t} = 0.01$

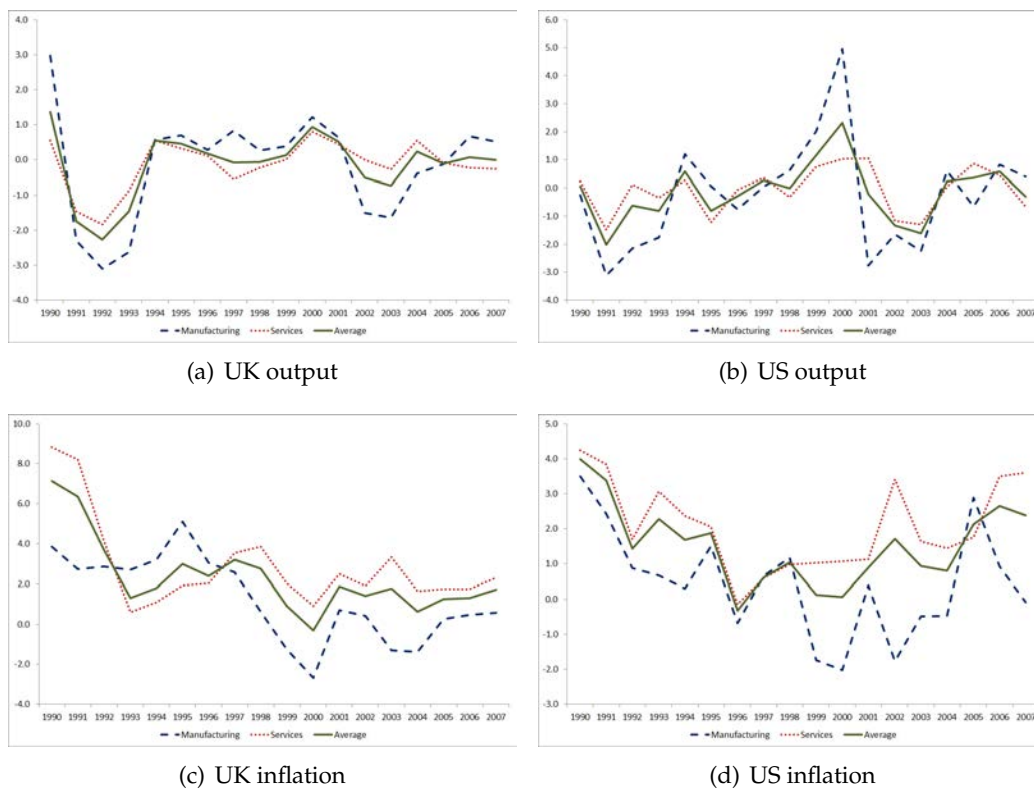
Shock ii: offsetting cost push shocks $\varepsilon_{u_a,t} = 0.01$ and $\varepsilon_{u_b,t} = -0.01$

Shock iii: productivity shock to sector a only $\varepsilon_{A_a,t} = 0.05$

Shock iv: offsetting productivity shocks $\varepsilon_{A_a,t} = 0.05$ and $\varepsilon_{A_b,t} = -0.05$

Note: With asymmetric sectors and an inability to commit to a policy rule, the policy response to productivity shocks with aggregate taxes produces lower welfare than if taxes were constrained to zero. This is because government does not take into account the effect that exploiting the ability to affect relative prices has on expectations of future inflation.

Figure 1: Inflation and output gaps



Note: Authors' calculations based on EU KLEMS. Services are composed of wholesale and retail trade, hotels and restaurants, transport and storage and communication, education, health and social work. Output gap is measured as the log deviation of actual output from Hodrick-Prescott trend. Average variables are constructed by weighting sectors with weights averaged over 1990-2007.

Figure 2: Response to cost push shock in sector a with symmetric sectors

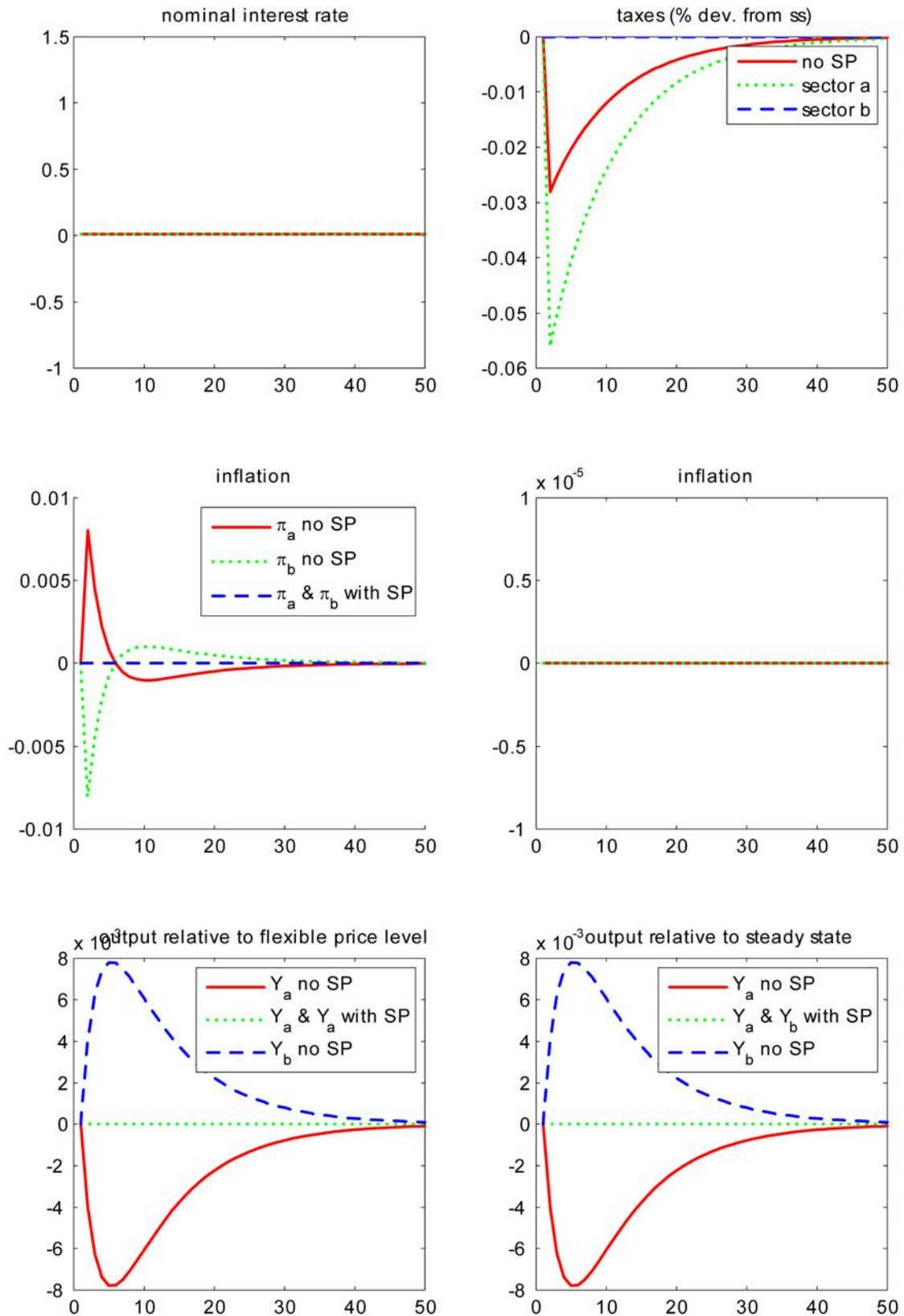


Figure 3: Response to offsetting cost push shocks with symmetric sectors

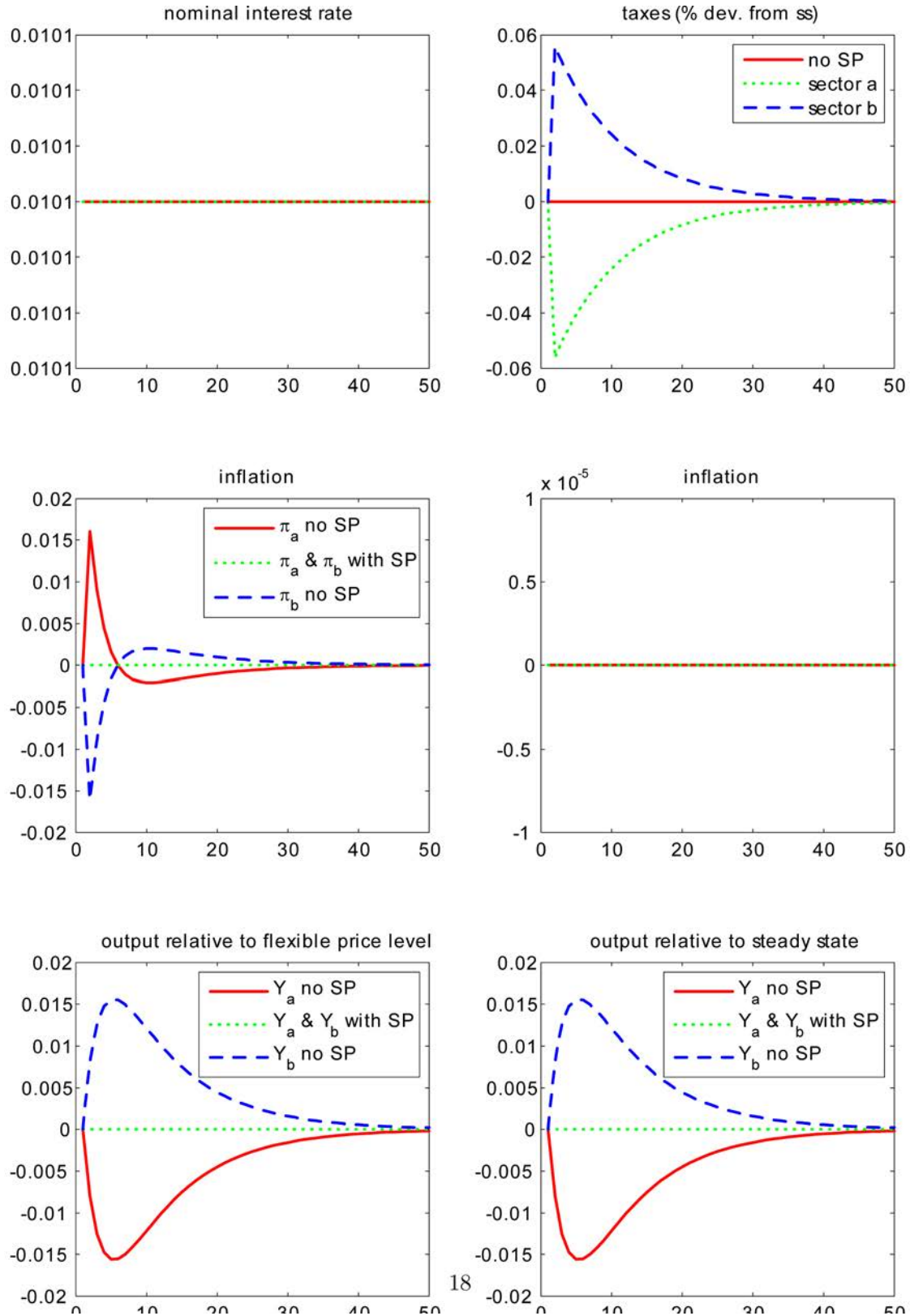


Figure 4: Response to cost push shock in sector a with asymmetric sectors

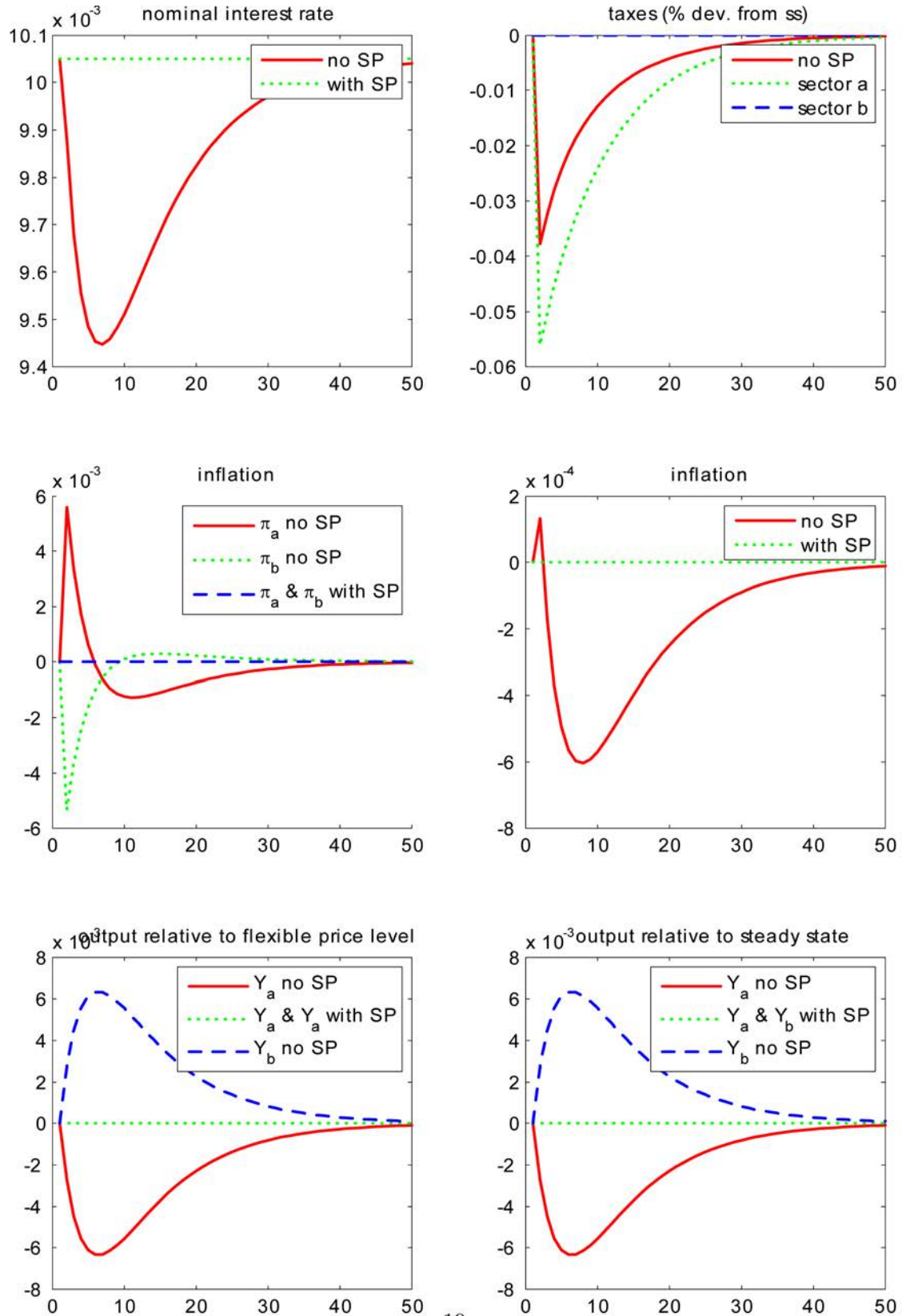


Figure 5: Response to productivity shock in sector a with asymmetric sectors

