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#09-1258: Measuring Network Reliability Considering Paradoxes: The Multiple Network Demon Approach

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Introduction

Road networks are now required to be highly reliable to ensure that users can experience smooth travel under both normal and abnormal traffic conditions. This requirement has led many researchers to develop various reliability measures or indicators to assess the network reliability, including *encountered reliability* and *travel time reliability*. These two reliabilities can be studied through the game theoretic approach.

The Game Theoretic Approach

- Traffic assignment is described as a fictitious zero-sum game being played between:
  - **Users (travellers)**
    - choose paths to minimise their trip costs
    - non-cooperative themselves
    - risk-averse and highly pessimistic about the state of the road network
    - cannot anticipate the decisions of demons and other users
  - **Demons (evil entities)**
    - choose links to damage to increase the travel costs of users
    - cannot anticipate decisions of users
- Known as 'Risk-Averse Traffic Assignment'
- 1 OD-specific demon per OD pair
  - not free to 'roam' networks by damaging links
  - restricts number of demons = number of OD pairs
  - number of links damaged = Number of OD pairs
- Link Capacity Reduction Assumption:
  - Only accounted for 2 operational states: congested and un-congested
  - Always 50% reduction for any number of demons

The Multiple Network Demon Approach

Recently we proposed a more general game theoretic formulation via the nonlinear complementarity problem (NCP) approach by relaxing the assumptions on the number/type of demons and the form of capacity reduction:

- Extended the existing game theoretic formulation to allow demons to be free to select any links to damage and to impose no restriction on the number of demons or number of links to be damaged.
- However, the formulation is a path based one that requires path enumeration. This is not attractive from the computational point of view.

Another assumption of the game is that the users selfishly and non-cooperatively seek to maximize their own utilities by seeking the expected minimum cost routes without regard to the overall effect this may have on other network users. This assumption in the traditional user equilibrium problem can give rise to the seemingly counter-intuitive result known as "Braess' Paradox". This paradox describes how adding a new link to a transportation network might not improve the operation of the system in the sense of reducing the total system travel time in the network. However, to date no study has considered the existence of analogues of Braess' paradox in risk-averse user equilibrium assignment or the resulting implications for travel time and encountered reliability measurements.

Objectives

- Propose a *link-based* NCP multi-demon formulation which:
  - Avoids path enumeration and is proved to be equivalent to the path-based one and proved to have at least one solution
  - Reformulated into a variational inequality problem and an unconstrained minimization problem that can be solved by many existing solution algorithms
- Assess the impact of this extension on the travel time and encountered reliabilities
  - Challenge the effect of the assumption of 50% capacity reduction, especially when there are multiple demons involved, and when more than one demon selects a particular link to damage.
- Demonstrate the existence of two analogues of Braess' paradox in risk-averse user equilibrium assignment:
  - Stochastic Braess' paradox and reliability-paradox, that if one adds a road to a network then all the travelers may be worse off in terms of expected network travel time/cost and travel time reliabilities respectively

Risk-Averse Assignment with Multiple Network Demons: The Link Based NCP Formulation

The proposed problem is formulated through the link-based approach that adopts link flows as decision variables. The number of variables required in this approach is therefore much fewer because there are considerably fewer links than routes in large networks.

The Demon Problem

The problem for multiple network demons can be written as the following nonlinear complementarity conditions:

$$p_{l_m}^m \left\{ \pi^m - \theta_{l_m}^m \right\} = 0, m = 1,\ldots,M, l_m = 1,\ldots,n$$

$$\pi^m - \theta_{l_m}^m \ge 0, m = 1,\ldots,M, l_m = 1,\ldots,n$$

$$p_{l_m}^m \ge 0, m = 1,\ldots,M, l_m = 1,\ldots,n$$

Probability of link lm selected by spoiler m

where:

Expected total network cost when spoiler m selects link lm (payoff to spoiler m)

$$\theta_{l_m}^m = \sum_{l_1=1}^{n} \cdots \sum_{l_q=1, q\neq m}^{n} \cdots \sum_{l_M=1}^{n} \left\{ \left(\prod_{i=1, i\neq m}^{M} p_{l_i}^i \right) \left[\sum_a t_{a,l_1 \ldots, l_m, \ldots, l_M}(\mathbf{v}) v_a \right] \right\}$$

$\pi^m = \max_{l_m} \theta_{l_m}^m$ is the maximum expected payoff to demon m

The User Problem

The first-order condition of deterministic traffic assignment can be expressed as the following link-based non-linear complementarity conditions:

$$v_a^{rs} \varphi_a^r = 0, \forall r,s,a,$$

$$\varphi_a^r \ge 0, \forall r,a,$$

$$v_a^{rs} \ge 0, \forall r,s,a,$$

v_a^{rs} denotes the flow on link a departed from origin r to destination s

where: φ_a^r is the difference between the expected travel time on link a (directly connecting x and y) and the minimal expected travel time from x to y for flows departing from r

$$\varphi_a^r = \sum_k t_{ak}(\mathbf{v}) q_k - \left[\phi^{ry} - \phi^{rx} \right], \forall r, a = (x,y)$$

Expected travel time on link a

Minimal expected travel time from the entry node of a to the exit node of a

Probability of Scenario k $\quad q_k = \prod_{m=1}^{M} p_{l_m}^m \quad$ Probability of Link lm selected by spoiler m $\quad k = (l_1, \ldots, l_m, \ldots, l_M)$

The disaggregated link flow defines the aggregated link flows v_a:

$$v_a = \sum_{rs} v_a^{rs}, \forall a,$$

and must satisfy the flow conservation condition at each node:

$$\sum_{a\in O(z)} v_a^{rs} - \sum_{b\in I(z)} v_b^{rs} - Q_z^{rs} = 0, \ \forall r,s,z$$

$$\text{where } Q_z^{rs} = \begin{cases} -N^{rs}, \text{if } z = s \\ 0, \text{ otherwise} \end{cases}$$

N^{rs} is the travel demand of OD pair rs

Link-Based NCP Formulation

This problem can be reformulated as a link-based nonlinear complementarity problem (NCP): to find **y** such that:

$$\mathbf{y} \ge \mathbf{0}, \ \mathbf{H}(\mathbf{y}) \ge \mathbf{0}, \text{ and } \mathbf{y}^{\mathrm{T}} \cdot \mathbf{H}(\mathbf{y}) = 0,$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \hat{\phi}^{rsz}, \forall rs, z/r \\ \pi^m, \forall m \end{bmatrix}, \mathbf{H}(\mathbf{y}) = \begin{bmatrix} \mathbf{F}(\mathbf{x}) \\ \sum_{a\in O(z)} v_a^{rs} - \sum_{b\in I(z)} v_b^{rs} - Q_z^{rs}, \forall rs, z/r \\ \sum_{l_m} p_{l_m}^m - 1, \forall m \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} v_a^{rs}, \forall rs, a \\ p_{l_m}^m, \forall m, l_m \end{bmatrix},$$

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \psi_a^{rs}, \forall rs, a \\ \pi^m - \theta_{l_m}^m, \forall m, l_m \end{bmatrix}.$$

$$\hat{\phi}^{rsz} = \phi^{rz}, \forall r,s,z, \text{ and}$$

$$\psi_a^{rs} = \varphi_a^r, \forall r,s,a.$$

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Solution Method

The NCP is transformed to an unconstrained minimization program via a gap function:

$$\min_{\mathbf{y}} G(\mathbf{y}) = \frac{1}{2}\sum_i \left[\sqrt{\left(y_i^2 + H_i(\mathbf{y})^2\right)} - \left(y_i + H_i(\mathbf{y})\right)\right]^2$$

This minimization program can be solved by a number of existing optimization algorithms. In this paper, this program is solved by the Generalized Reduced Gradient (GRG) Algorithm.

Numerical Studies

The simple test network and parameters used to generate results is the classical Braess network (see below). Since this network has only one OD pair, the demon(s) here can be considered as OD-specific demon(s), and the results here can also be used to explain the effect of introducing more than one demon per OD pair. The following parameters are also adopted here:

Normal link capacities:

$c_{1,k}= c_{2,k}=c_{3,k}=c_{4,k}=c_{5,k}=1$ if a link is not selected for damage by any demon

Damaged link capacities:

Case 1: 50% Capacity Reduction:

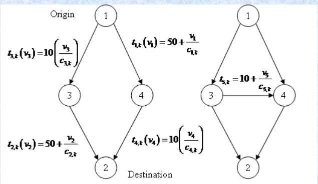
$c_{1,k}= c_{2,k}=c_{3,k}=c_{4,k}=c_{5,k}=0.5$ if a link is selected for damage by one or more demons

Case 2: Nonlinear Capacity Reduction:

$c_{1,k}= c_{2,k}=c_{3,k}=c_{4,k}=c_{5,k}=0.5$ if a link is selected for damage by one demon;

$c_{1,k}= c_{2,k}=c_{3,k}=c_{4,k}=c_{5,k}=0.25$ if a link is selected for damage by two demons simultaneously;

$c_{1,k}= c_{2,k}=c_{3,k}=c_{4,k}=c_{5,k}=0.125$ if a link is selected for damage by three demons simultaneously.



(a) Braess' network before improvement (b) Braess' network after improvement

The Effect of Multiple Network Specific Demons and Multiple States of Link Damage to Reliability Measurement

In this example we consider the effect of:

- (i) introducing multiple network-specific demons into the game, and
- (ii) relaxing the capacity reduction assumption, on the network reliability measures defined earlier.

Travel Time Reliability The travel time reliabilities are calculated for the case of one, two and three demons under the two capacity reduction assumptions: 50% capacity reduction (case 1) and nonlinear capacity reduction (case 2). For the case of 50% reduction, the impact of the multiple demons is clear – the greater the number of demons the greater is the drop in path travel time reliability since the network capacity is lower on average. In more congested conditions the additional demons have a far greater impact since there are many more users traveling on degraded paths. For the case of nonlinear capacity reduction, the path travel time reliabilities decrease even more quickly though of course this is to be expected since the demons can combine to inflict even higher expected network costs on the users. In fact it seems that the capacity reduction assumption may be even more important than the number of demons to assume due to the additive effect of the demons in significantly lowering the travel time reliabilities. It also seems more appropriate for the capacity to degrade in this way with respect to the increasing number of demons if we consider multiple capacity states in reality and real–life events which can combine to lower the capacity of a road. The overall trend of the network travel time reliabilities shown for both cases can also be explained in a similar way to the path travel time reliabilities.

Encountered Reliability The results show that when the number of demons is fixed, the path encountered reliability remains the same with increasing levels of travel demand. This is because the network is symmetric, leading to the outcome that both paths have the same flow and path selection probabilities for damage. For asymmetric networks, the path encountered reliability changes with demand levels. Moreover, the path encountered reliability decreases with the increasing number of demons present in the network. This result is expected in any network since more demons allow more links to be degraded in the network at the same time which means that the network capacity is lower on average. As a result, the users then have a higher probability in encountering a link failure in their journey and consequently the encountered reliability is lower for multiple demons than one demon. In fact in the special case where the network is symmetric like here, any assumption on capacity reduction will produce the same set of results, since by definition the encountered reliability is the probability of a trip-maker encountering failure of *any* magnitude on their path or journey.

Stochastic Braess' Paradox and Reliability Paradox in Risk-Averse User Equilibrium Traffic Assignment

We adopt the improved Braess network (i.e., after the fifth link has been added) and for simplicity we consider the capacity reduction to be 50% regardless of the numbers of demons present. By comparing the Expected Total System Travel Times (ETSTT) with those obtained from the 'un-improved' Braess network (i.e., four links network) in our first example, we can demonstrate the occurrence of stochastic Braess' paradox. Furthermore, by comparing the network travel time reliabilities between the two cases we demonstrate the existence of a paradoxical phenomenon in reliability measurement. The results are summarized in the table above in the section dealing with stochastic Braess' paradox. It is worth mentioning here that failure to appreciate the effects of stochastic Braess' paradox would result in a misleading expectation that the network costs would be lower after the new link is added.

Table 1: The effect of multiple network specific demons, multiple states of link damage and Braess' paradox on network reliability measurement

Travel Time Reliabilities for a Range of Demand Levels under Different Levels of Capacity Uncertainty for Braess' Four-Links Network

Case 1: 50% Capacity Reduction

Demand	Path Travel Time Reliability: 1 Demon	Path: 2 Demons	Path: 3 Demons	Network Travel Time Reliability: 1 Demon	Network: 2 Demons	Network: 3 Demons
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	0.5	0.25	0.125	1	0.5	0.25
4	0.5	0.25	0.125	1	0.5	0.25
5	0.5	0.25	0.125	1	0.5	0.25
6	0.5	0.25	0.125	1	0.5	0.25
7	0.5	0.25	0.125	1	0.5	0.25
8	0.5	0.25	0.125	0	0	0
9	0.5	0.25	0.125	0	0	0
10	0.5	0.25	0.125	0	0	0

Case 2: Nonlinear Capacity Reduction

Demand	Path Travel Time Reliability: 1 Demon	Path: 2 Demons	Path: 3 Demons	Network Travel Time Reliability: 1 Demon	Network: 2 Demons	Network: 3 Demons
1	1	0.75	0.50	1	1	0.750
2	1	0.75	0.50	1	0.50	0
3	0.50	0.250	0.125	1	0.50	0
4	0.50	0.250	0.125	1	0.50	0
5	0.50	0.250	0.125	1	0.50	0
6	0.50	0.250	0.125	1	0.50	0
7	0.50	0.250	0.125	1	0.50	0
8	0.50	0.250	0.125	0	0	0
9	0.50	0.250	0.125	0	0	0
10	0.50	0.250	0.125	0	0	0

Braess' Network Before and After the Network is Improved when the Demand Level is 3 Users

Number of Demons Present	Before/After Improvement	Travel Time Reliabilities: Path 1-3-2	Path 2-4-2	Path 1-3-4-2	Network	ETSTT
1	Before	0.50	0.50	-------	1.00	222.00
	After	0.50	0.50	0.00	0.00	266.31
2	Before	0.25	0.25	-------	0.50	233.25
	After	1.00	1.00	0.50	0.50	269.99
3	Before	0.125	0.125	-------	0.25	238.88
	After	1.00	1.00	0.25	0.25	271.54

The table of results also shows the effect of ignoring stochastic Braess' paradox to the travel time reliability measures that we defined earlier. In fact, this gives rise to another type of paradox, namely the reliability paradox. For all cases of the number of demons present, the reliabilities of paths one and two are equal due to the fact that these two paths are symmetric. When link (3,4) is added to the network an additional path is created (path 1-3-4-2) and the reliability of this path varies depending on how many demons we assume in our model. The network travel time reliability in this case is also zero which strongly suggests that the new link should not have been added at all since not alone is the new path highly unreliable but the network as whole becomes highly unreliable.

Conclusions

- Developed a link-based formulation of the risk-averse traffic assignment via the NCP approach that allows for the presence of multiple network-specific demons and therefore allows us to consider the effect of greater network uncertainty in terms of capacity degradation.
- This model can avoid path enumeration which is important particularly when dealing with multiple network demons which becomes even more significant when applying the model to larger real-life networks.
- Demonstrated the effect of assuming multiple demons and relaxing the capacity assumption to network reliability measurement through numerical studies. The results show just how important it is to accurately capture the link capacity degradations particularly when we assume the presence of more than one demon. Relaxing this assumption also allows for more operational states to be modeled.
- Demonstrated the existence of stochastic Braess' paradox and reliability paradox in risk-averse assignment, and provided some further insights into the implications for network design if these paradoxes are ignored.

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