PY5021 - 6 Field Mapping

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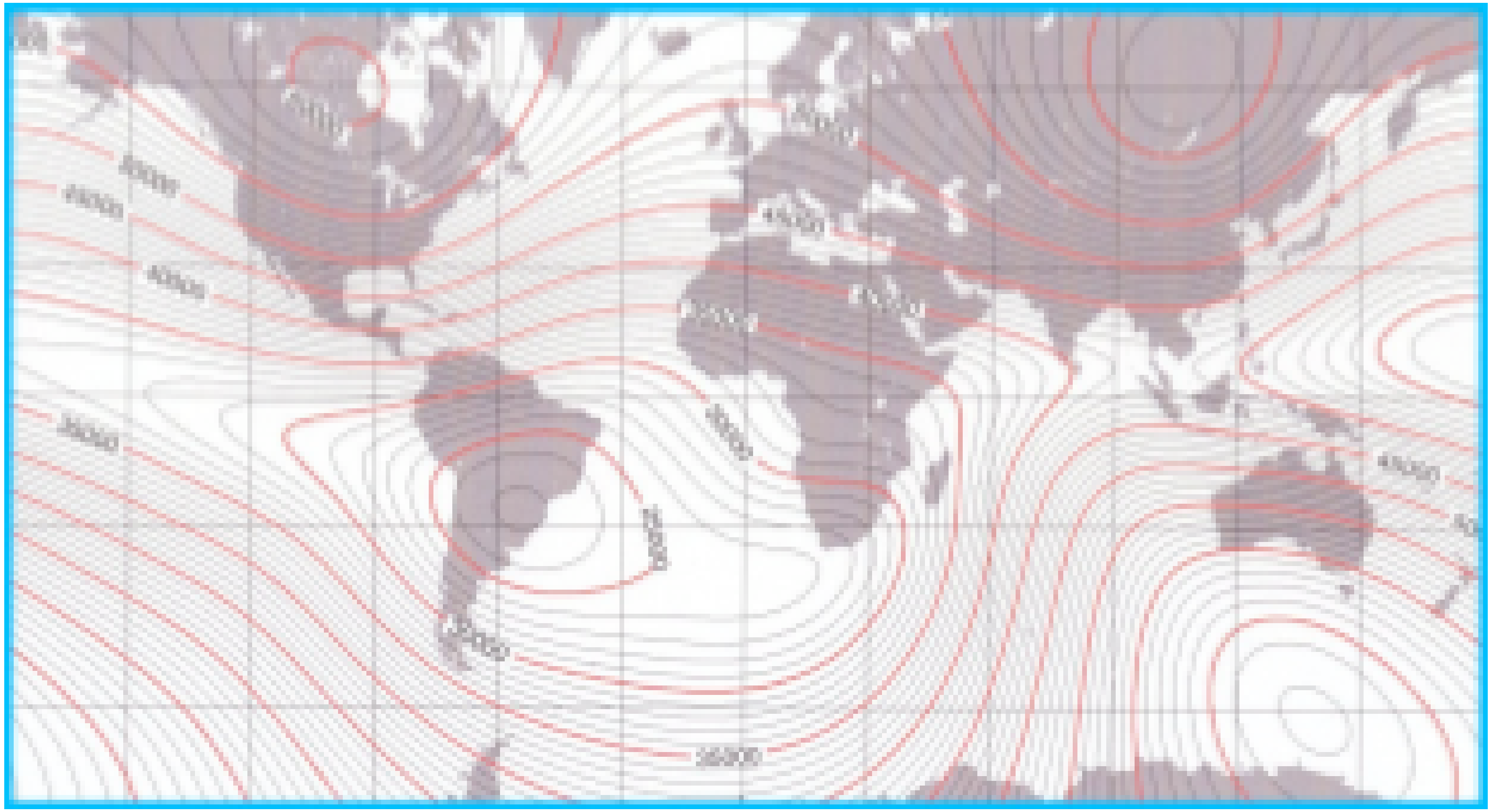
Ireland.

1. The Earth's field
2. Space
3. Prospection and detection

Comments and corrections please: jcoey@tcd.ie

www.tcd.ie/Physics/Magnetism

6.1 The Earth's field

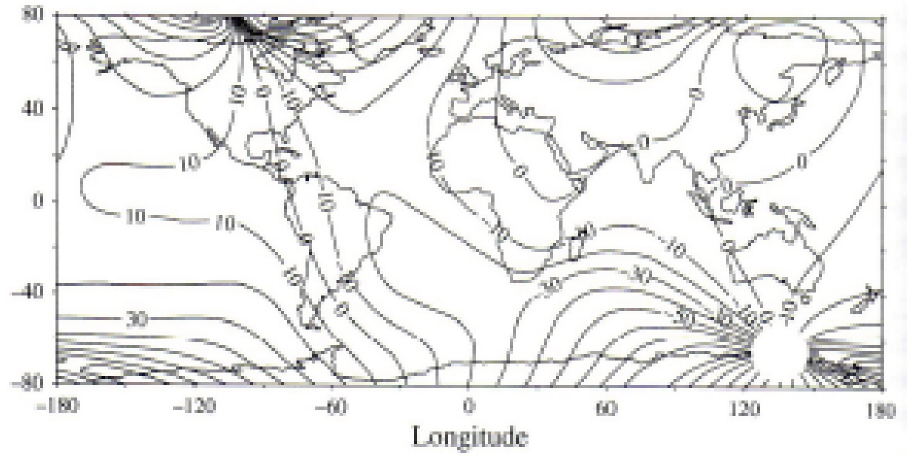


Magnitude of the Earth's field in nT

The Earth's field is best mapped from a satellite equipped with magnetometers.

MAGSAT and OERSTED have mapped the steady field and its short-time fluctuations in great detail.

D



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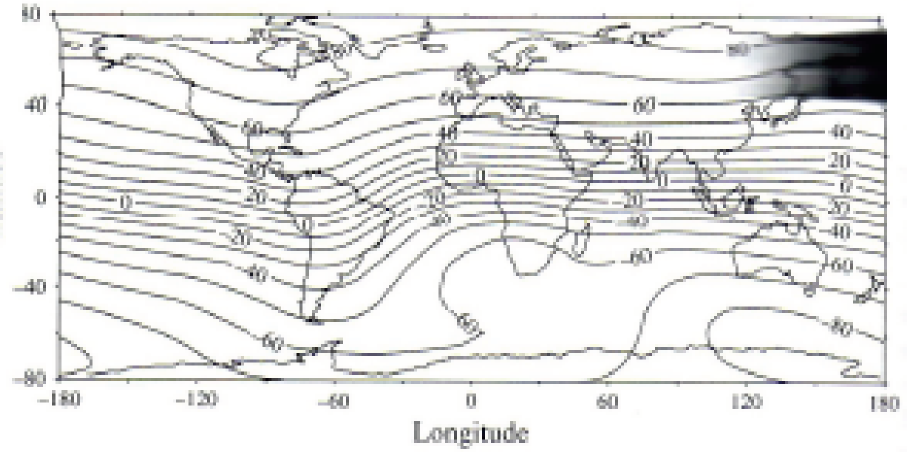
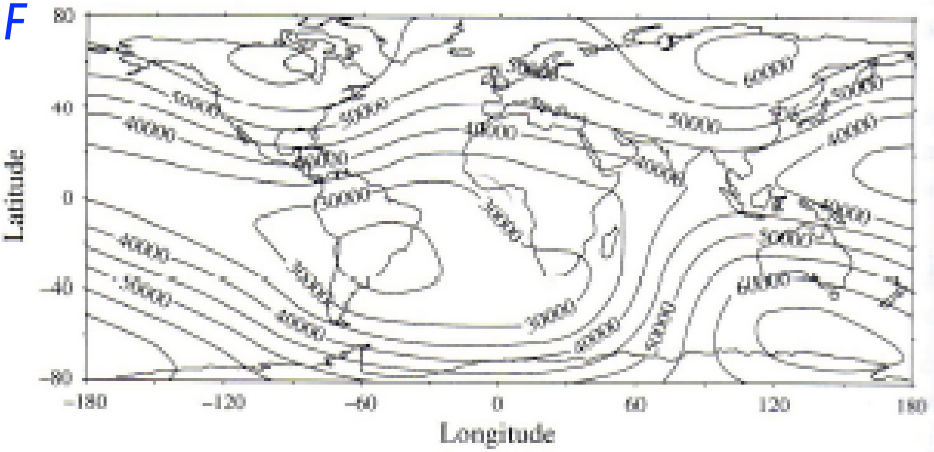
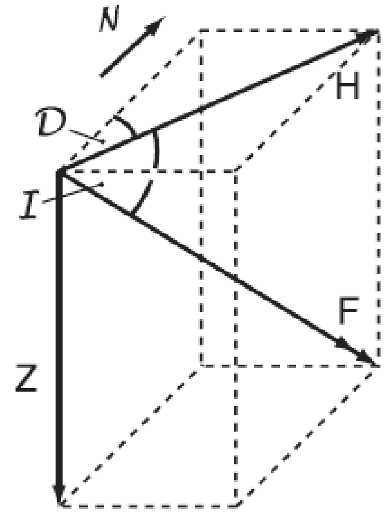


F





Horizontal and vertical components H, Z of the Earth's field, which has magnitude F, and direction defined by the declination (variation) D and inclination (dip) I.

Any three components define the field. Four offer a consistency check.

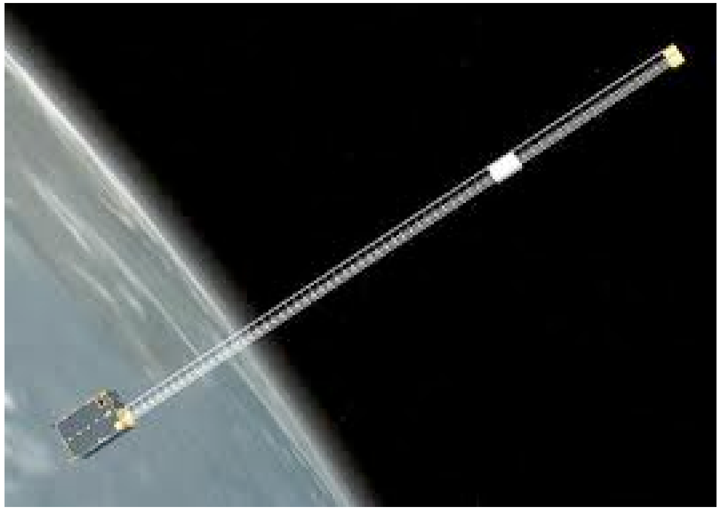
An Overhauser magnetometer and a fluxgate were placed at the end of an 8 m long boom on the Danish Oersted satellite, launched in 1999, into a sun synchronous low-earth orbit. The objective of Oersted was to map the Earth's field, and the associated high-energy charged particle environment. The drift of the Earth's magnetic poles appears to be accelerating, which may prefigure a reversal.

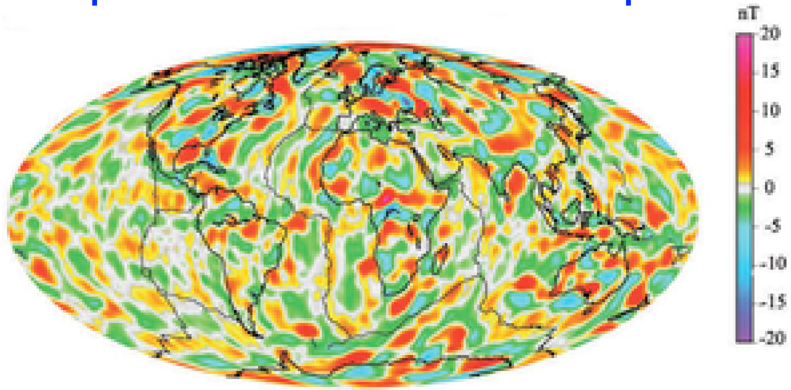
Overhauser magnetometer, magnitude with resolution < 1 nT

Three-axis fluxgate with resolution of 3 - 5 nT, orientational precision < 20"

6 particle detectors (electrons 0.03 - 1 MeV, protons 0.2 - 30 MeV, alphas 1 - 100 MeV)

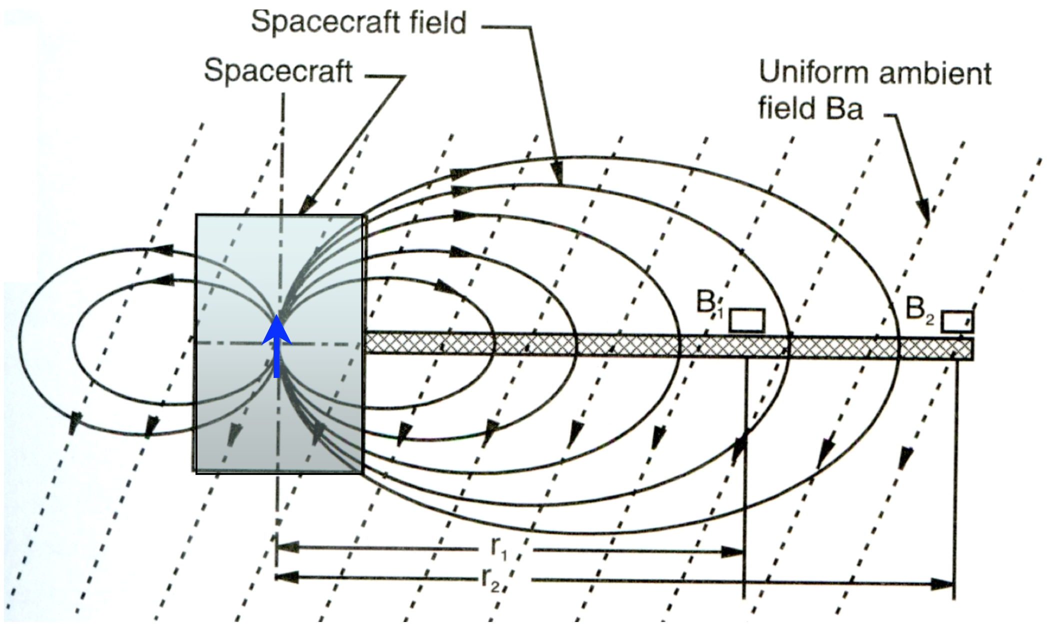
GPS to within 2 - 10 cm; 54 W average on-board power from GaAs solar panels.





Deviation of the measured Earth's field from a model

In order to separate the stray fields produced by the spacecraft $\boldsymbol{B}_{sc}(\boldsymbol{m},\boldsymbol{r})$ from the ambient field $\boldsymbol{B}_a$, the two magnetometers are deployed on the boom at different distances $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ from the spacecraft. $\boldsymbol{m}$ is the magnetic moment created by the electric currents on board.



Hence $$\boldsymbol{B}_1 = \boldsymbol{B}_a + \boldsymbol{B}_{sc}(\boldsymbol{m},\boldsymbol{r}_1)$$

$$\boldsymbol{B}_2 = \boldsymbol{B}_a + \boldsymbol{B}_{sc}(\boldsymbol{m},\boldsymbol{r}_2)$$

but $\boldsymbol{B}_{sc}(\boldsymbol{m},\boldsymbol{r}_1) = C\boldsymbol{B}_{sc}(\boldsymbol{m},\boldsymbol{r}_2)$, where $C = (r_2/ r_1)^3$

Hence $\boldsymbol{B}_1 - \boldsymbol{B}_a = C(\boldsymbol{B}_2 - \boldsymbol{B}_a)$

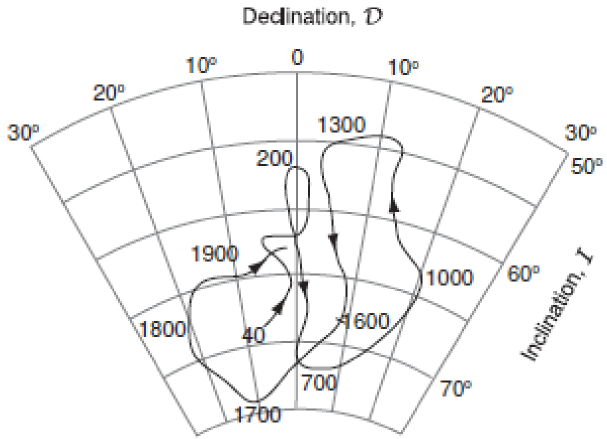
$$\boldsymbol{B}_a = (\boldsymbol{B}_1 - C\boldsymbol{B}_2)/(1 - C)$$

The Earth's field is varying on different timescales.

The internally-generated field (~99%) exhibits secular variation in both magnitude and direction, and also reversals.

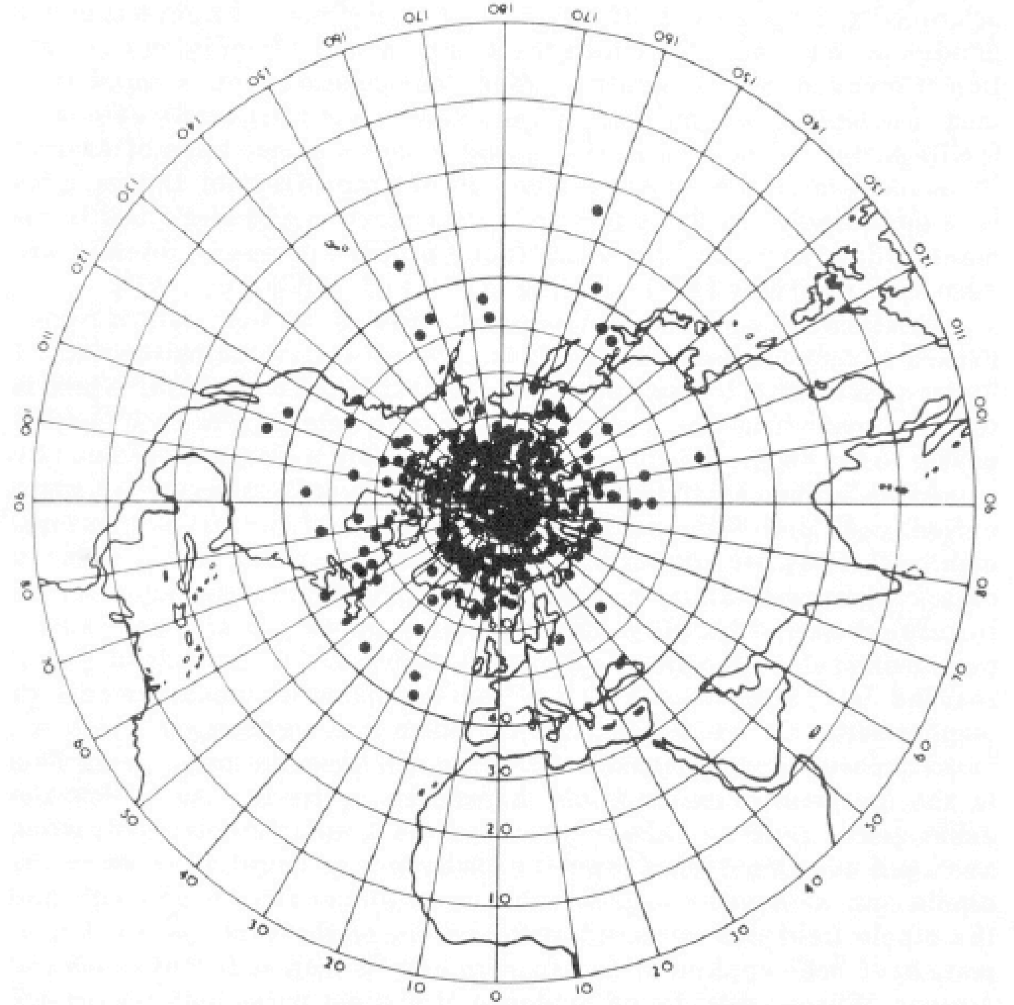
The externally-generated field due to flux of charged particles varies on a timescale of minutes or hours.

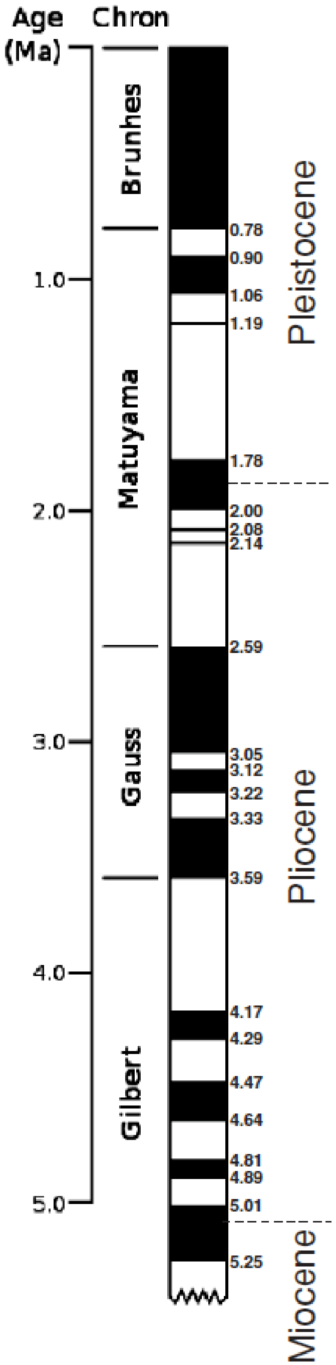
Edmond Halley led three research voyages from 1698 - 1701 to map the Earth's magnetic field.



The scalar variation of the Earth's field deduced by combining observations in Paris (>1600) with measurements of the remanence of baked clay (<1600).

Position of the Earth's magnetic pole deduced from measurements of recently formed ingeneous rocks. Half of the points have the present polarity, while the other half are reversed. On average the magnetic field is that of a geocentric axial dipole.

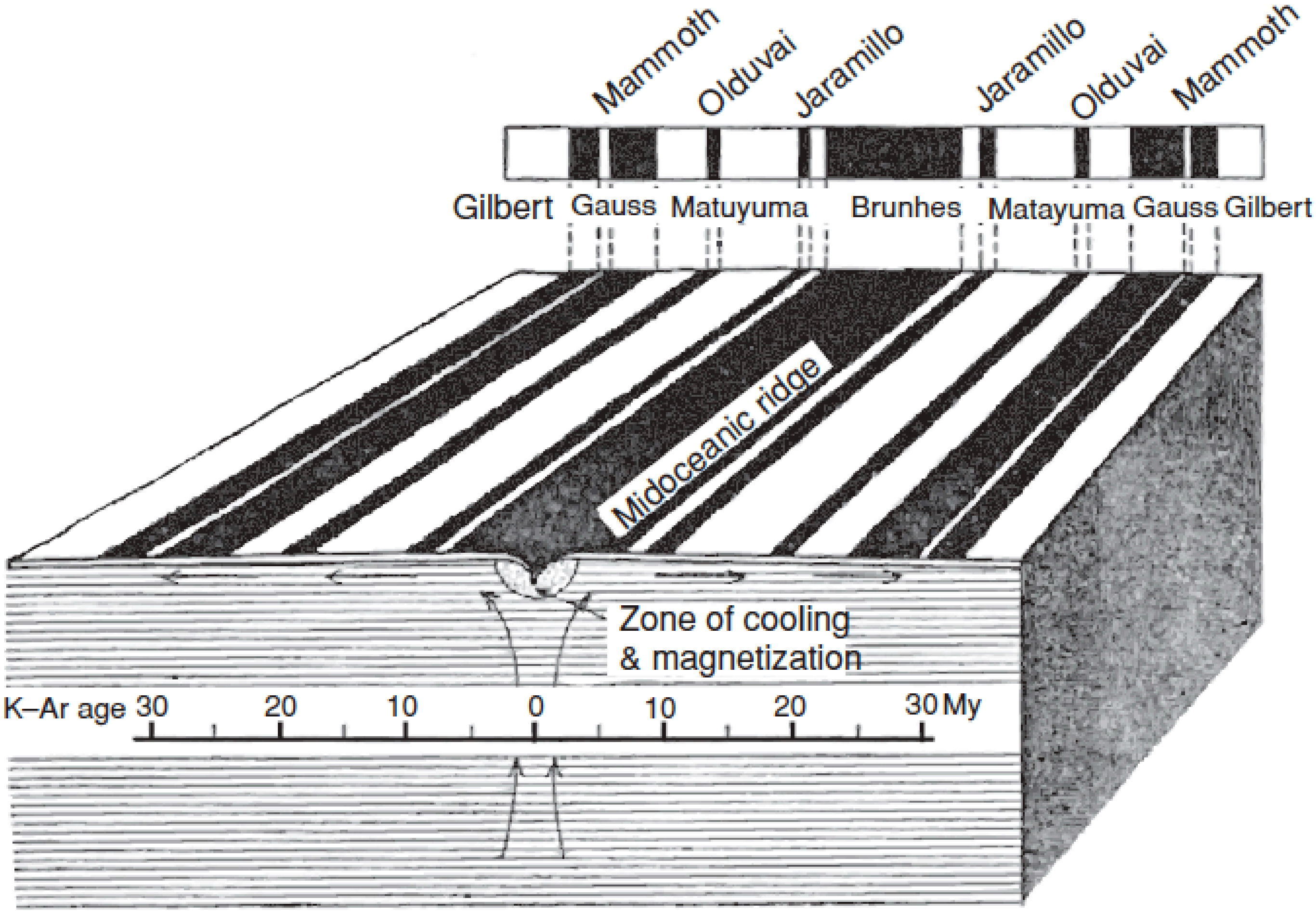




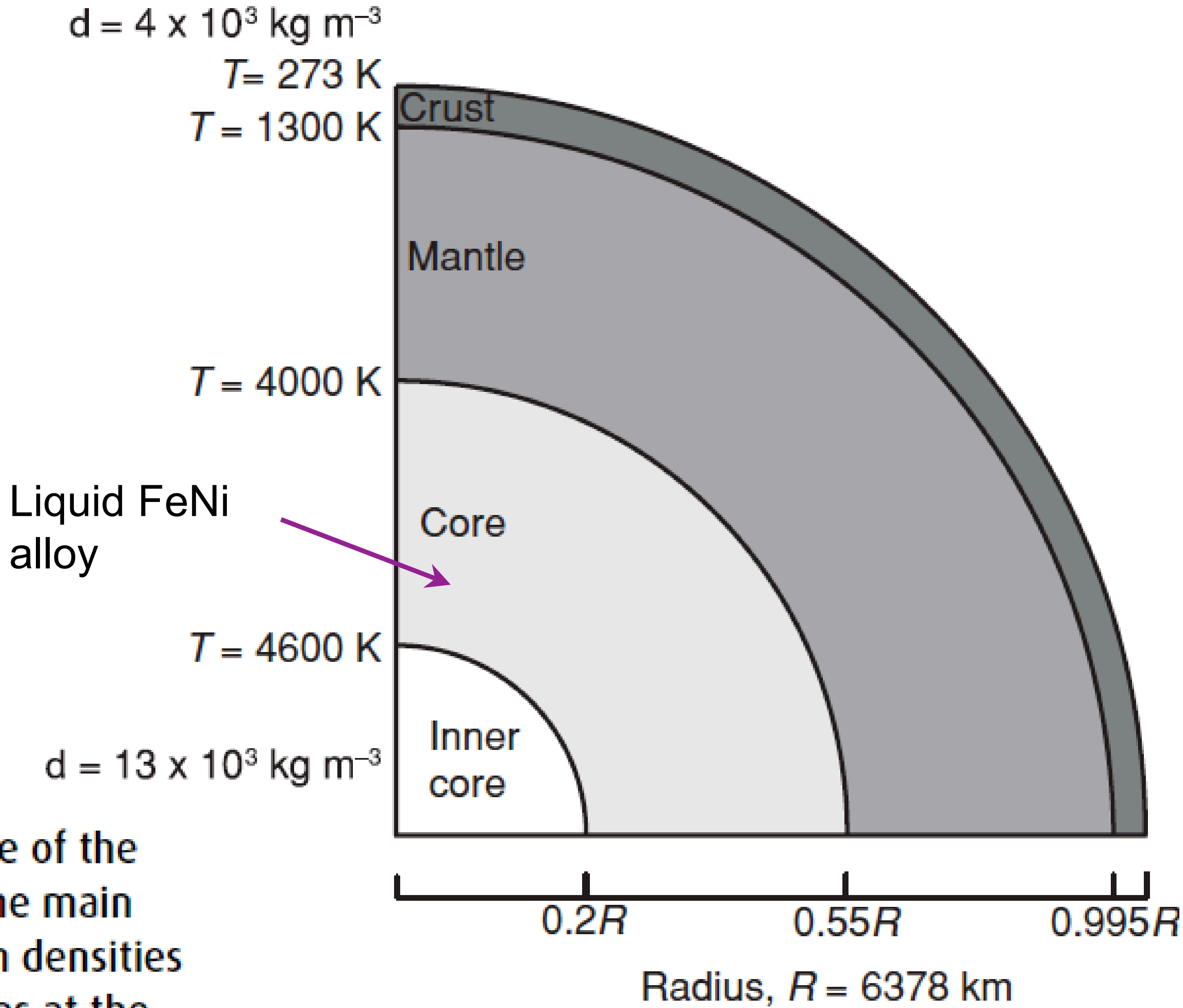
There is an almost random sequence of reversals of the Earth’s field.

The last one was 700 ka ago.

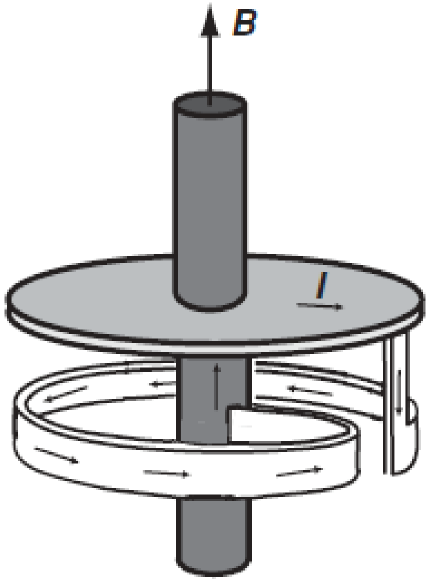
Schematic representation of plates separating at a mid-ocean ridge. The pattern of magnetization of sea-floor basalts measured across the North Atlantic led to the ideas of seafloor spreading and global plate tectonics.



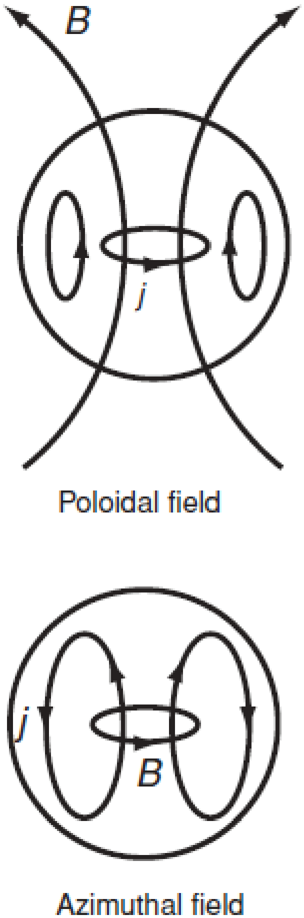
Apparent polar wander paths which are used to reconstruct the past positions of plates on the globe. Data from rocks in Europe (open circles) and North America (solid circles) can be made to coincide by closing up the Atlantic ocean.



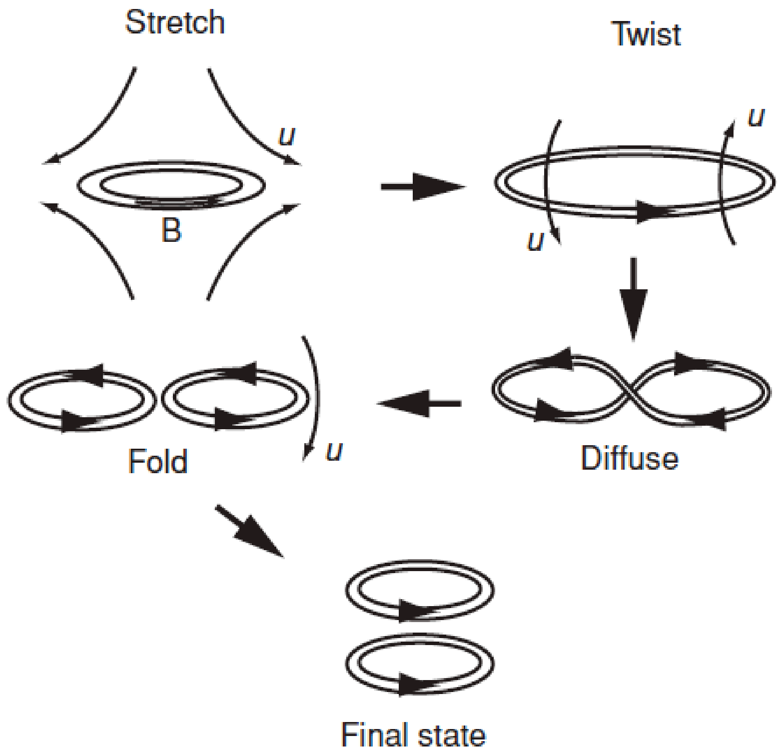
Internal structure of the Earth. Radii of the main structures, mean densities and temperatures at the centre and surface are given.



A mechanical model of a self-exciting dynamo.

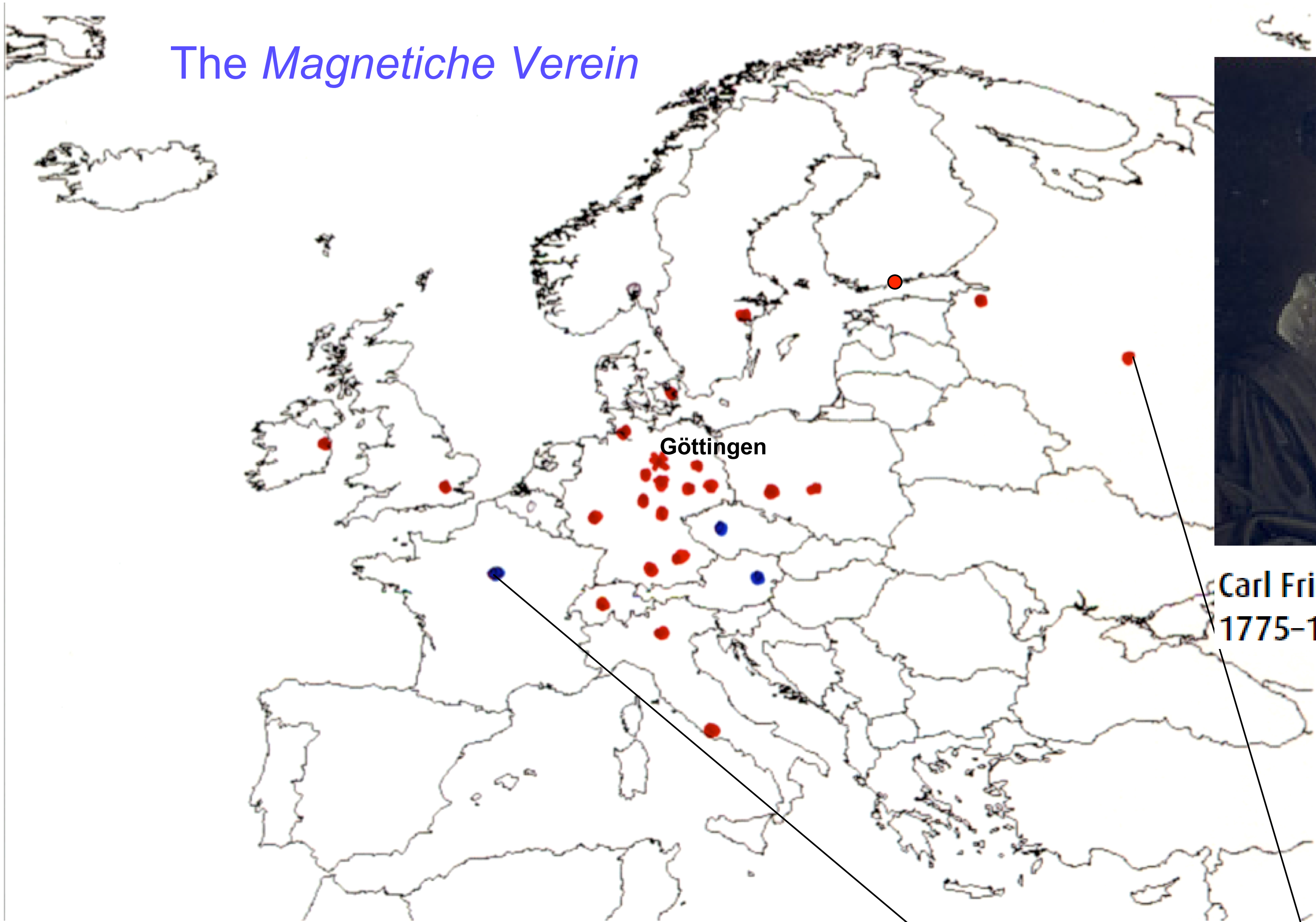


Azimuthal currents create poloidal fields, and vice versa.



Magnetic field is intensified in a fluid core by a process of stretching and twisting flux lines. *u* is the fluid velocity.

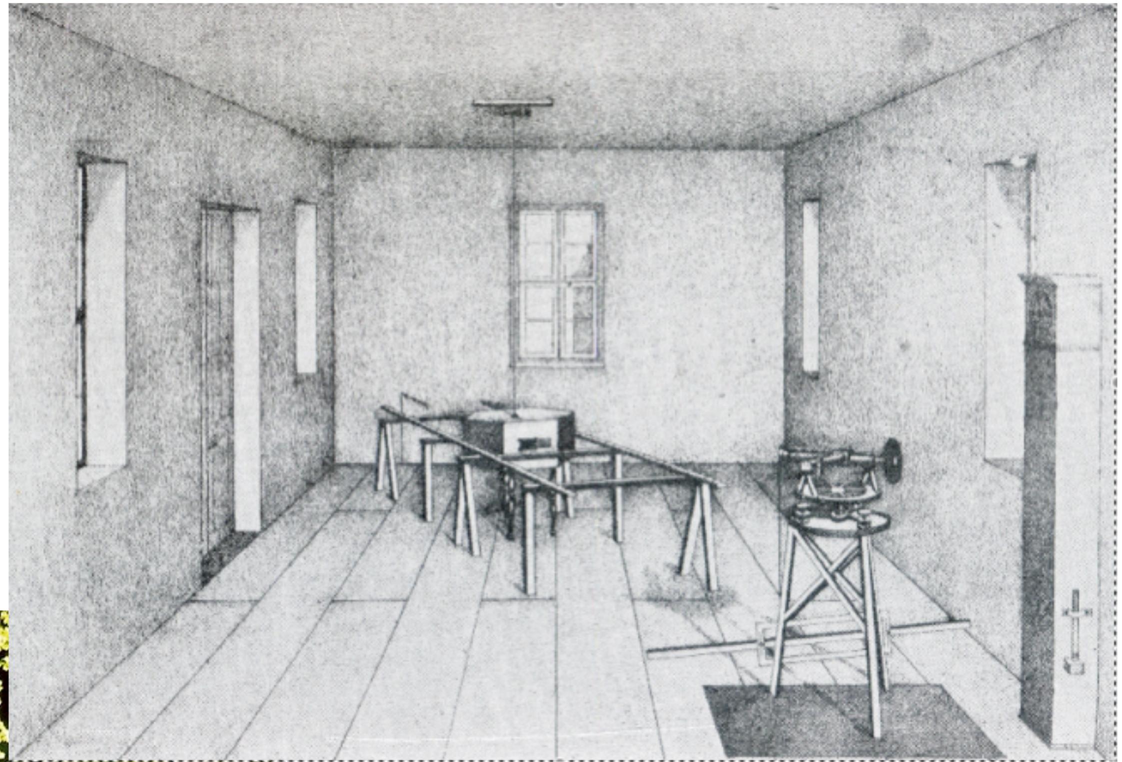
The *Magnetiche Verein*



Carl Friedrich Gauss, 1775–1855.

Network of Magnetic observatories ≈ 1836

In 1830 simultaneous measurements were made of the fluctuations of the Earth’s magnetic field during a 24 hour period in Paris and in Kazan



Gauss’s magnetic observatory
in Göttingen 1830.

The magnetical observatory established at Trinity College, Dublin, in 1835.

Carl Friedrich Gauss, 1775–1855.

$$\nabla^2 \varphi_m = 0$$

external internal

$$\varphi_m = \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left[A_l^m r^l + B_l^m r^{-(l+1)} \right] Y_l^m(\theta, \phi)$$

About 99% of the Earth’s magnetic field has an *internal* origin. It changes slowly

About 1% has an external origin. If fluctuates rapidly, on a daily basis

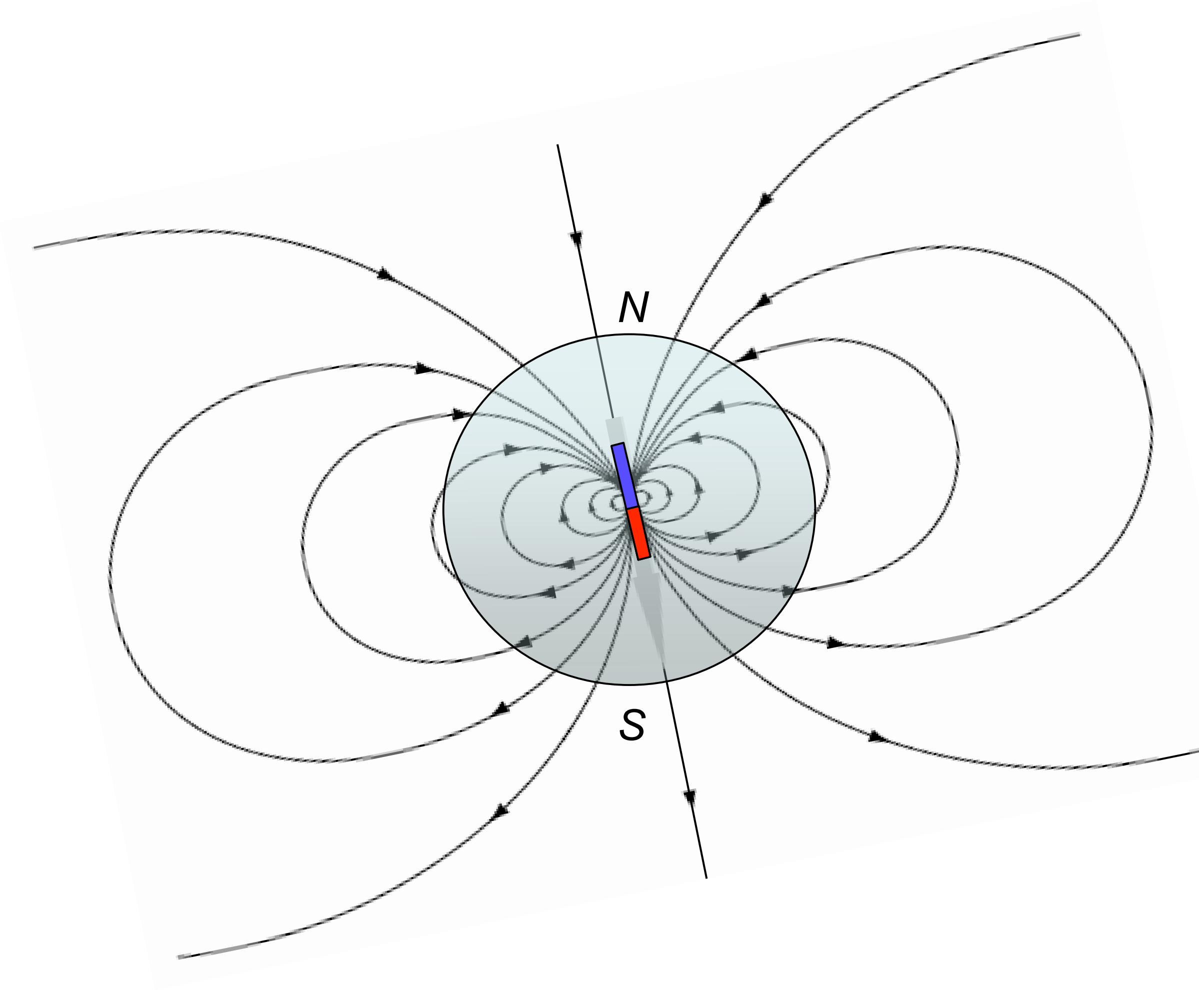
Spherical harmonic coefficients of the Earth's magnetic field (1985) in nanoteslas

Coefficient	Degree (m)	Order (n) 1	2	3	4
	4				169
g_n^m	3			835	−426
	2		1691	1244	363
	1	−1903	2045	−2208	780
g_n^o	0	−29877	−2073	1300	937
	1	5497	−2191	−312	233
	2		−309	284	−250
h_n^m	3			−296	68
	4				−298

Potental due to internal sources: q - colatitude, f - longitude

$$\varphi_{mi} = \frac{a}{\mu_0} \sum_{l=1}^{\infty} \sum_{m-0}^{l} \left(\frac{a}{r}\right)^{l+1} P_e^m(\cos\theta)\left[g_e^m \cos\phi + h_l^m \sin\phi\right]$$

Potental j in amperes, g_l^m and h_l^m in nT.



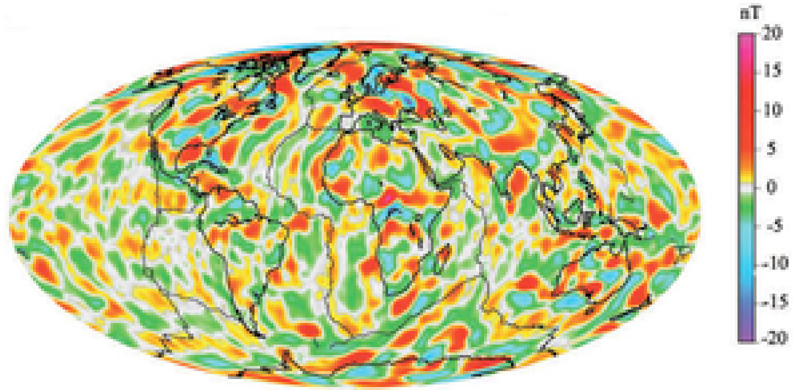
90% of the field is accounted for by a dipole of magnitude $(4a^3/m_0)[g_1^{02} + g_1^{12} + h_1^{12}]^{1/2}$

m = 7.9 10^{22} A m^2

$q = \tan^{-1}[g_1^{0}/(g_1^{12} + h_1^{12})^{1/2}]$

$q = 15°$

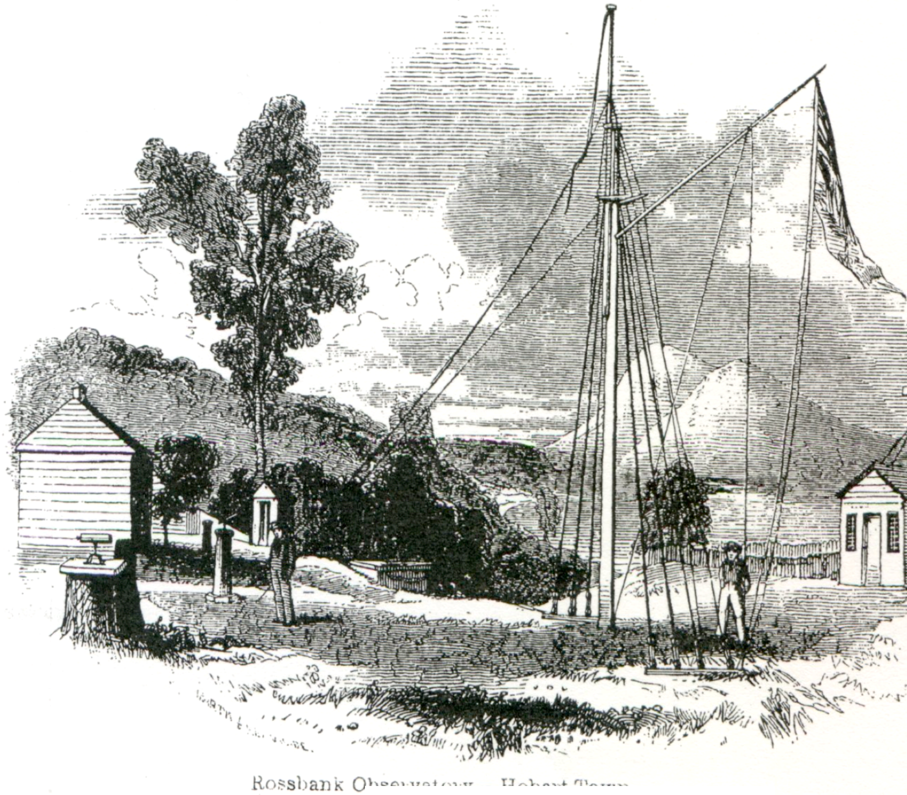
The first ~ eight harmonics represent the field produced in the Earth's core. Higher order terms represent the field produced by magnetized rocks in the first 30 km of earth's crust. (where $T > T_C$)



The humblest student of astronomy, or of any other physical science if he is to profit at all by his study must in some degree go over for himself, in his own mind, if not in part with the aid of his own observation and experiment, that process of induction which leads from familiar facts to obvious laws, then to the observation of facts that are more remote and to the discovery of laws of higher orders. And even if this study *be a personal act, much more must that* discovery *have been individual. Individual energy, individual patience, individual genius have all been needed to tear fold after fold away which hung before the shrine of nature; to penetrate gloom after gloom into those Delphic depths, and force the reluctant Sibyl to utter her oracular responses.*

William Rowan Hamilton $\mathcal{H}$

The Magnetic Crusade

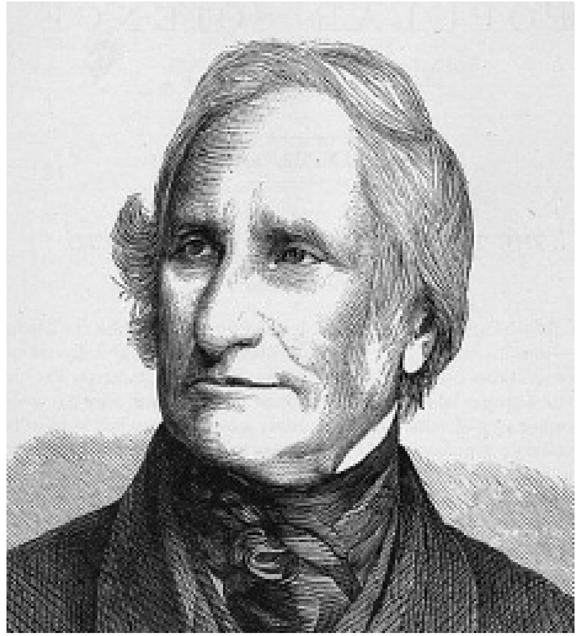


Lt Joseph Kay

1841

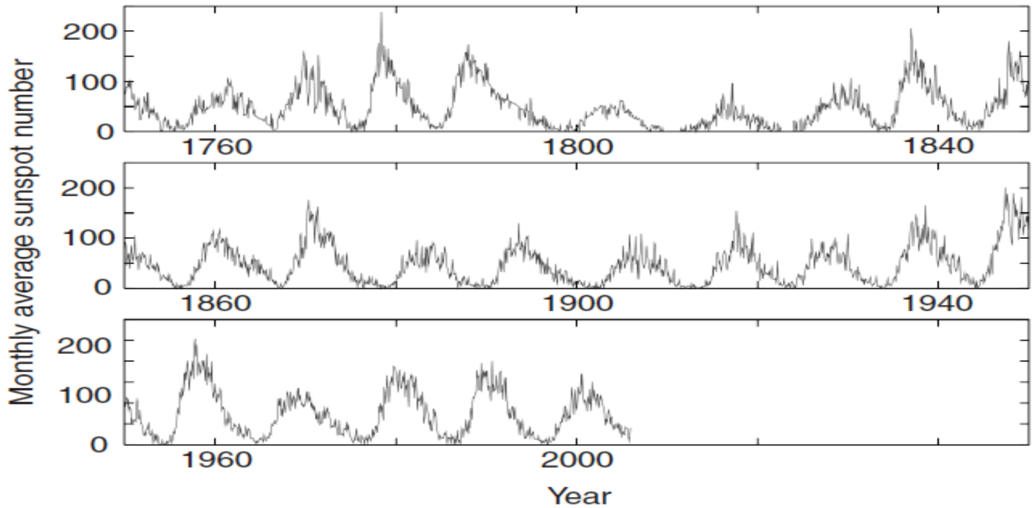
1847

1854



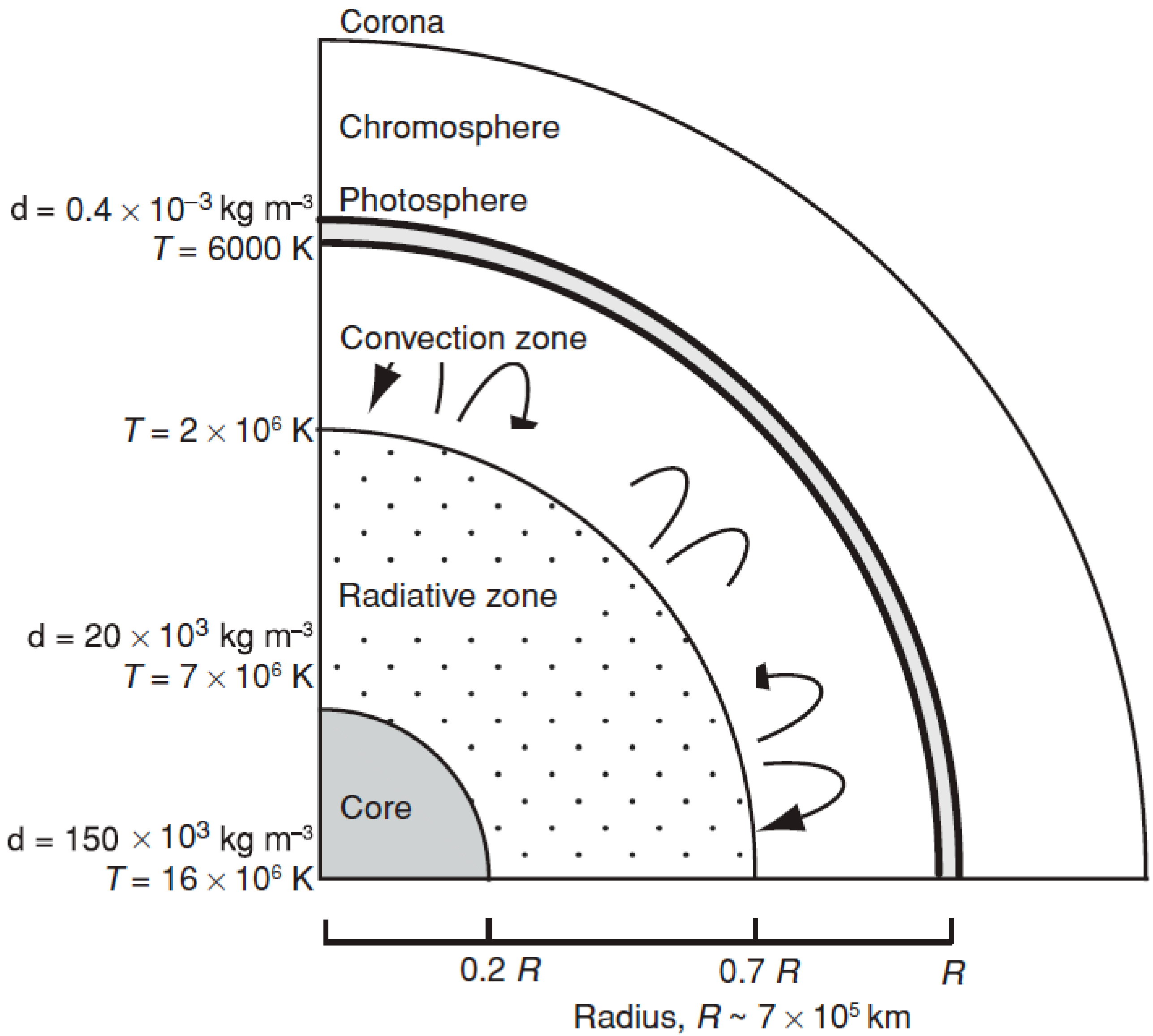
Edward Sabine, 1788–1883.

The 11-year sunspot cycle from 1760–2000.



The data from the observatories showed short-term fluctuation that exactly reflect the 11-year sunspot cycle!

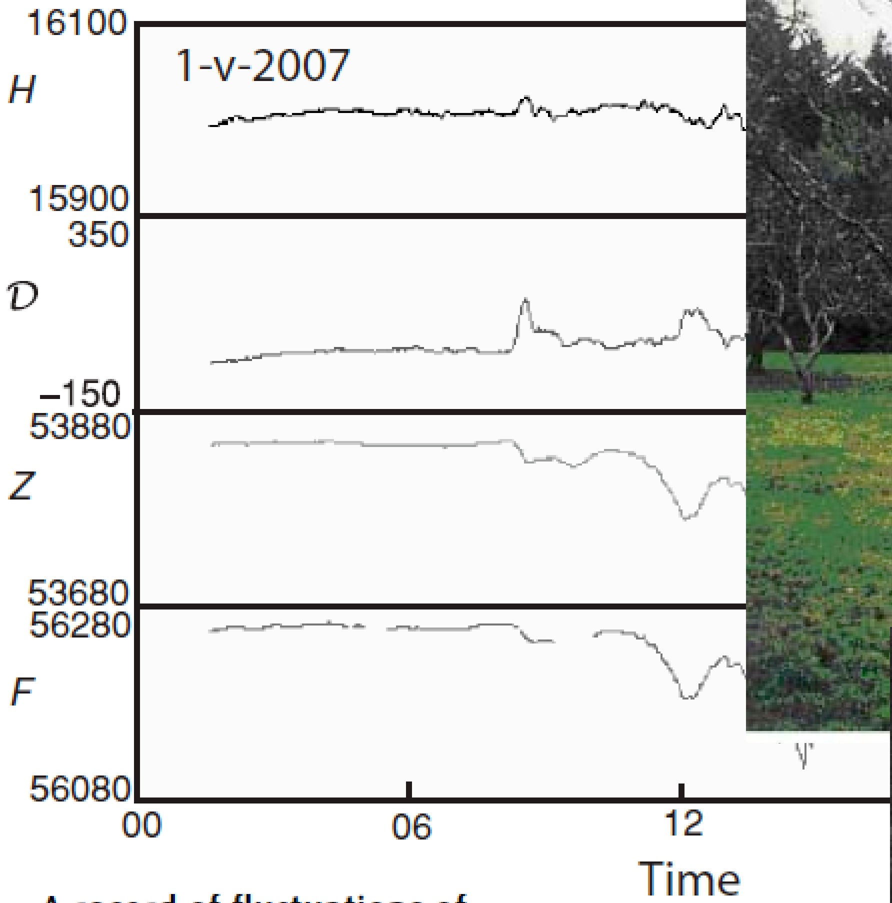
The internal structure of the Sun. Radii of the main structures, mean densities and temperatures at the centre and surface are given.



A flux tube which gas been pushed out through the surface of the Sun, forming two sunspots.

$$\mathcal{R}_m = vl\sigma\mu$$

INTERMAGNET - A world-wide network



A record of fluctuations of the Earth's magnetic field, taken at Sitka in Alaska on 1 May 2007. A magnetic observatory has existed on this site since 1842. Units of H, Z and F are nanoteslas, the units of $\mathcal{D}$ are degrees.

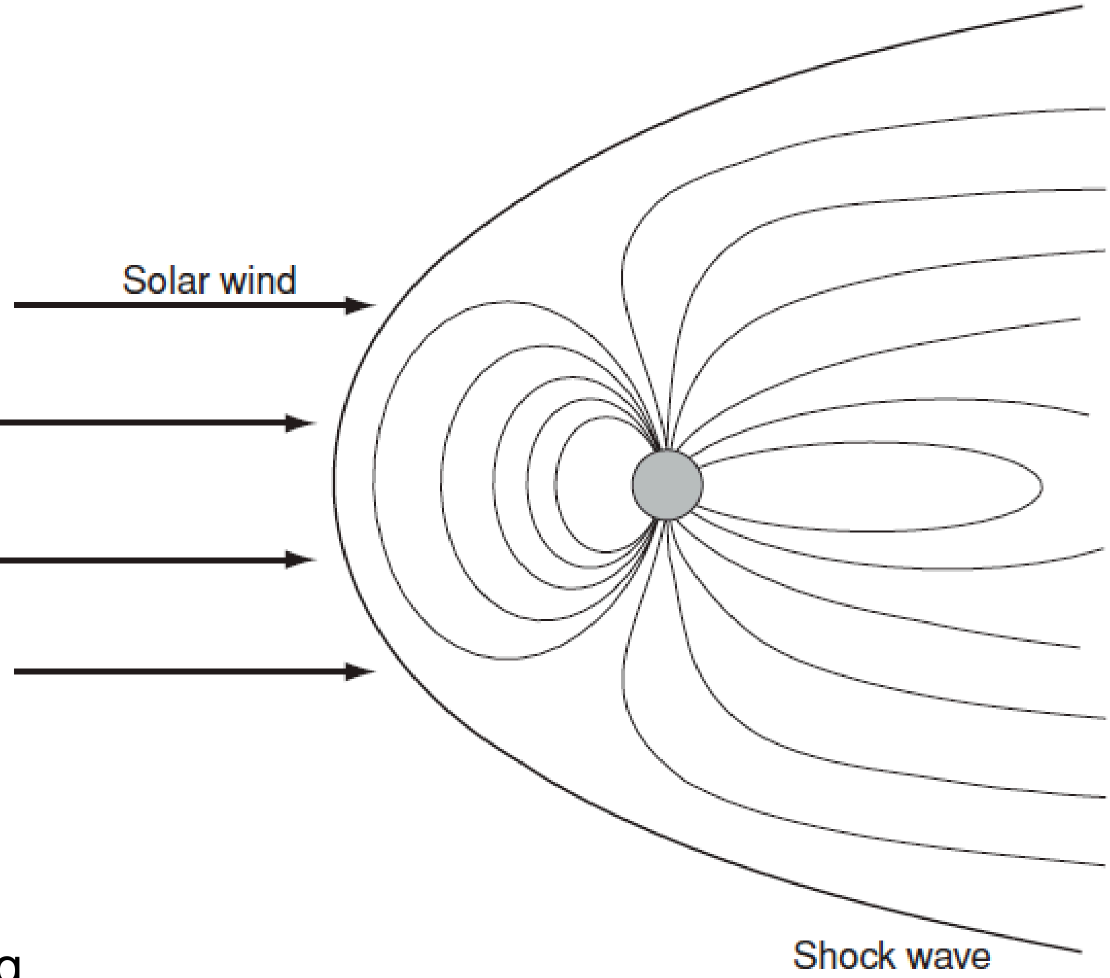
Compass

Measurement of the Earth's field for direction finding is conveniently done with integrated sensors to detect each of the three components of the Earth's field.

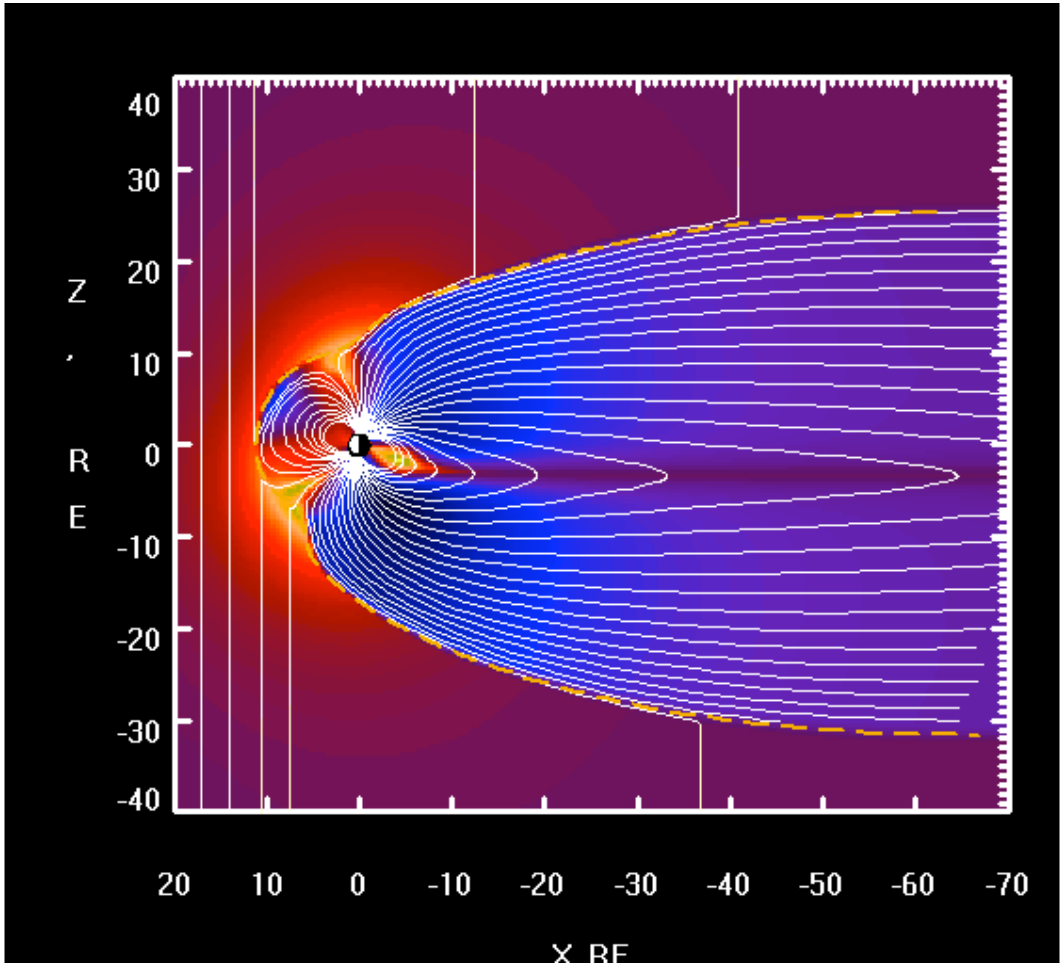
These may be Hall sensors, which can be integrated on silicon, or GMI sensors. It is awkward to measure three orthogonal components in a thin-film structure, but two are easily done.

6.2 Space

The solar wind, which is deflected by the Earth's magnetic field.

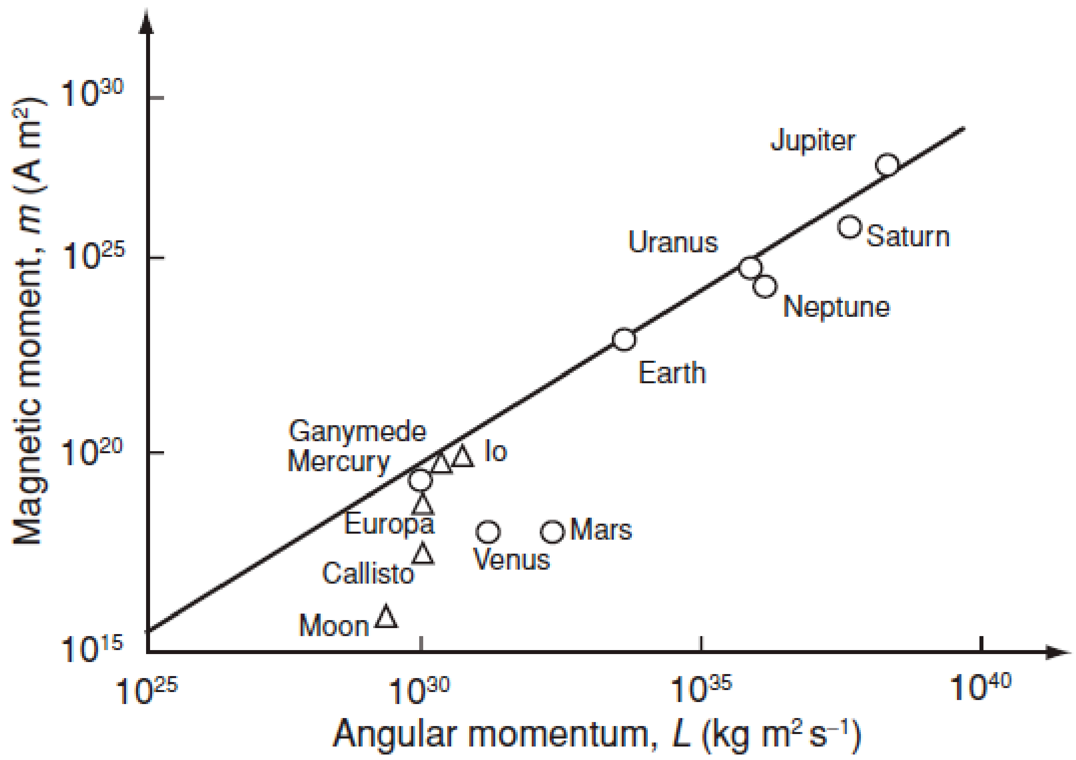


Space weather forecasting can be critical.

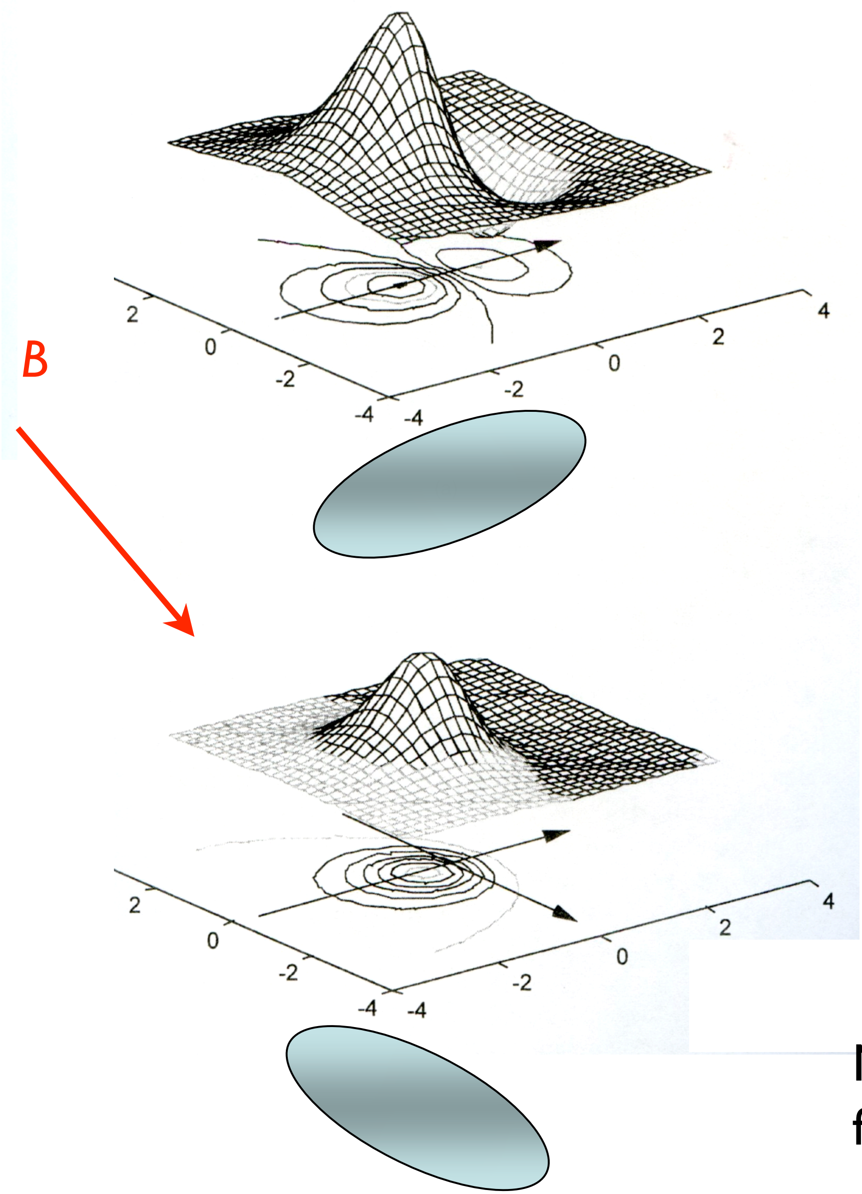


40
30
20
10
0
-10
-20
-30
-40
Z, RE
20
10
0
-10
-20
-30
-40
-50
-60
-70
X RE

Magnetic moments of planets and moons in the solar system, plotted against their angular momentum. (After P Rochette.)



6.3 Prospection and detection



Magnetic surveys have been used since 1640 is sweden to detect buried iron ore.

Magnetic signature of a buried ferrous object, aligned N-S or E-W

Magnetic surveys along a grid of closely-spaced points are aften made with a proton magnetometer , which measures B_t. Loss often with a fluxgate which measures $B_{z,}$ or a three- axis fluxgate that measures $\boldsymbol{B}_t$

The height and spacing of the grid must be comparable to the depth and spacing of the buried objects

Airborne surveys should fly as low as possible, usually < 200 m.

It is most valuable to map ΔB_z Both ΔB_t and ΔB_h (i.e. ΔB_{xy}) can be inferred from dense readings of ΔB_z, but not vice versa (unless both ΔB_x and ΔB_y are known).

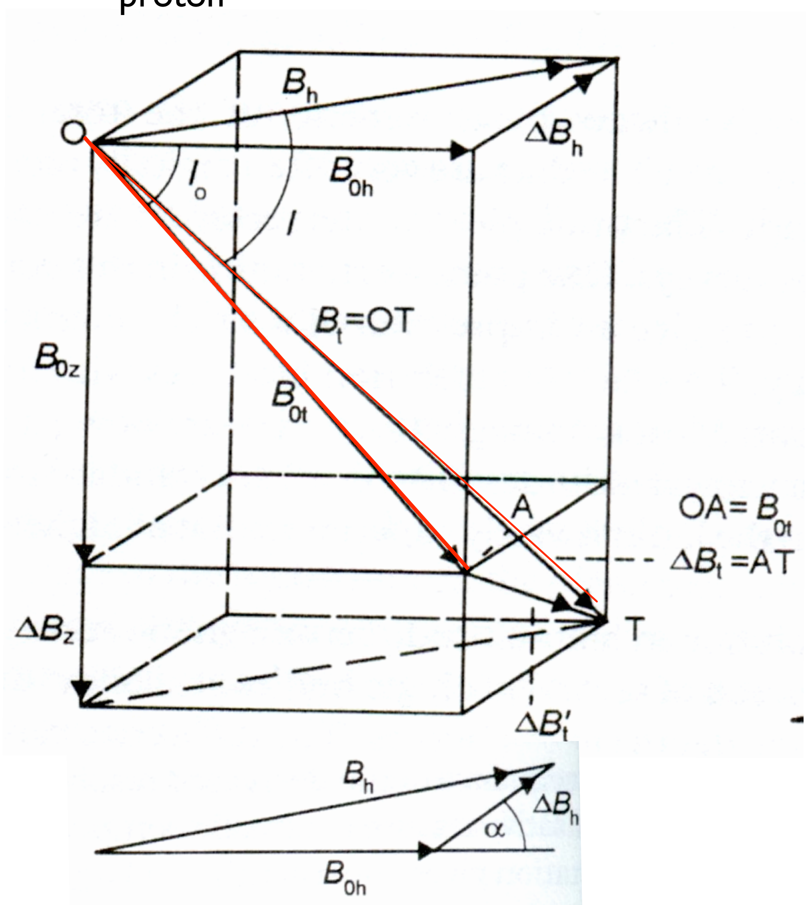
The total change in ΔB_z across buried objects such as spheres and thin dykes always exceeds that of ΔB_h at all latitudes.

$$\Delta B_t = |\boldsymbol{B}_t - \boldsymbol{B}_{t0}| = (\Delta B_z^2 + \Delta B_h^2)^{1/2}$$

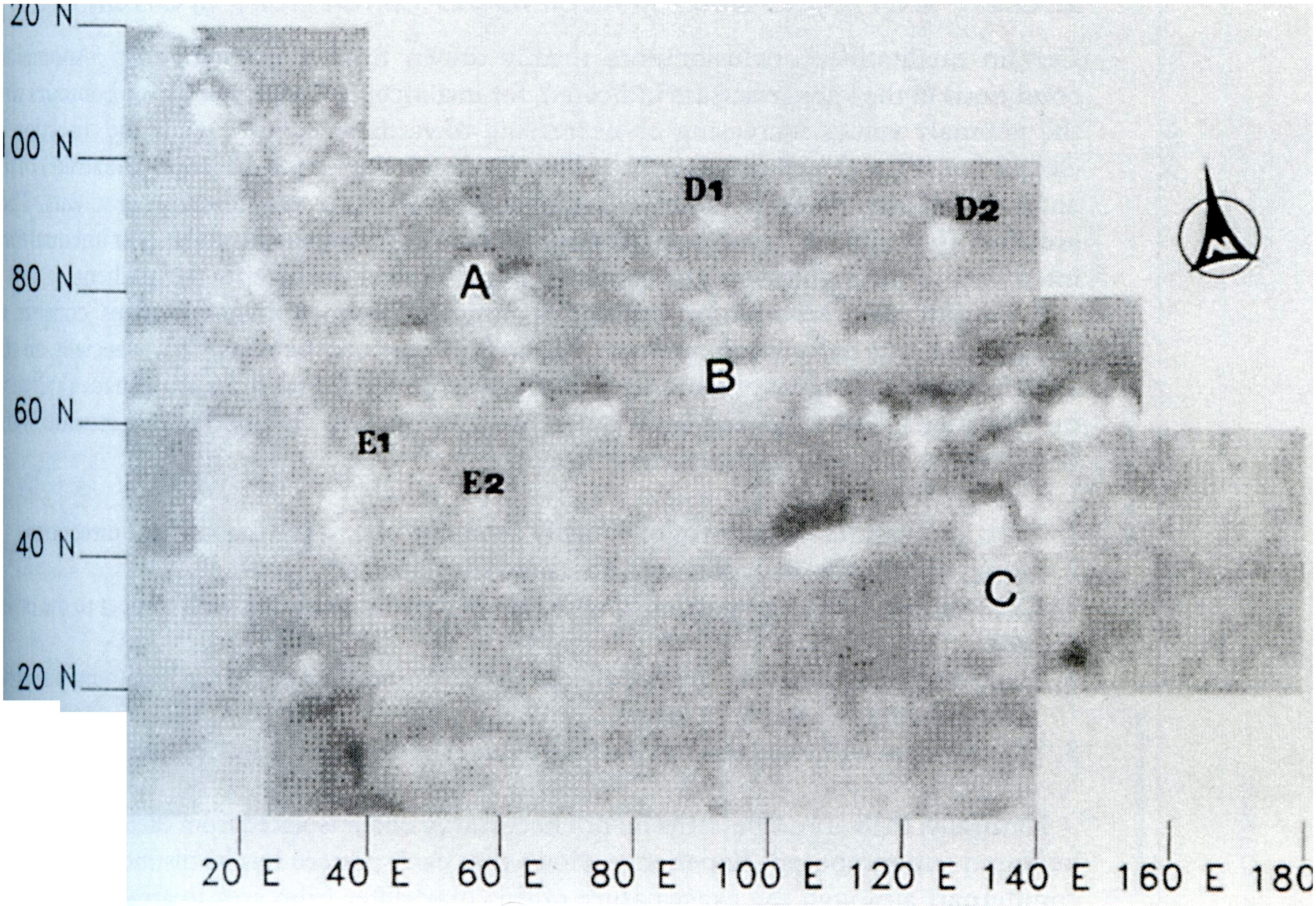
fluxgate

$$|\Delta B_t| = |\boldsymbol{B}_t| - |\boldsymbol{B}_{t0}| = (B_z^2 + B_h^2)^{1/2} - (B_{zo}^2 + B_{ho}^2)^{1/2}$$

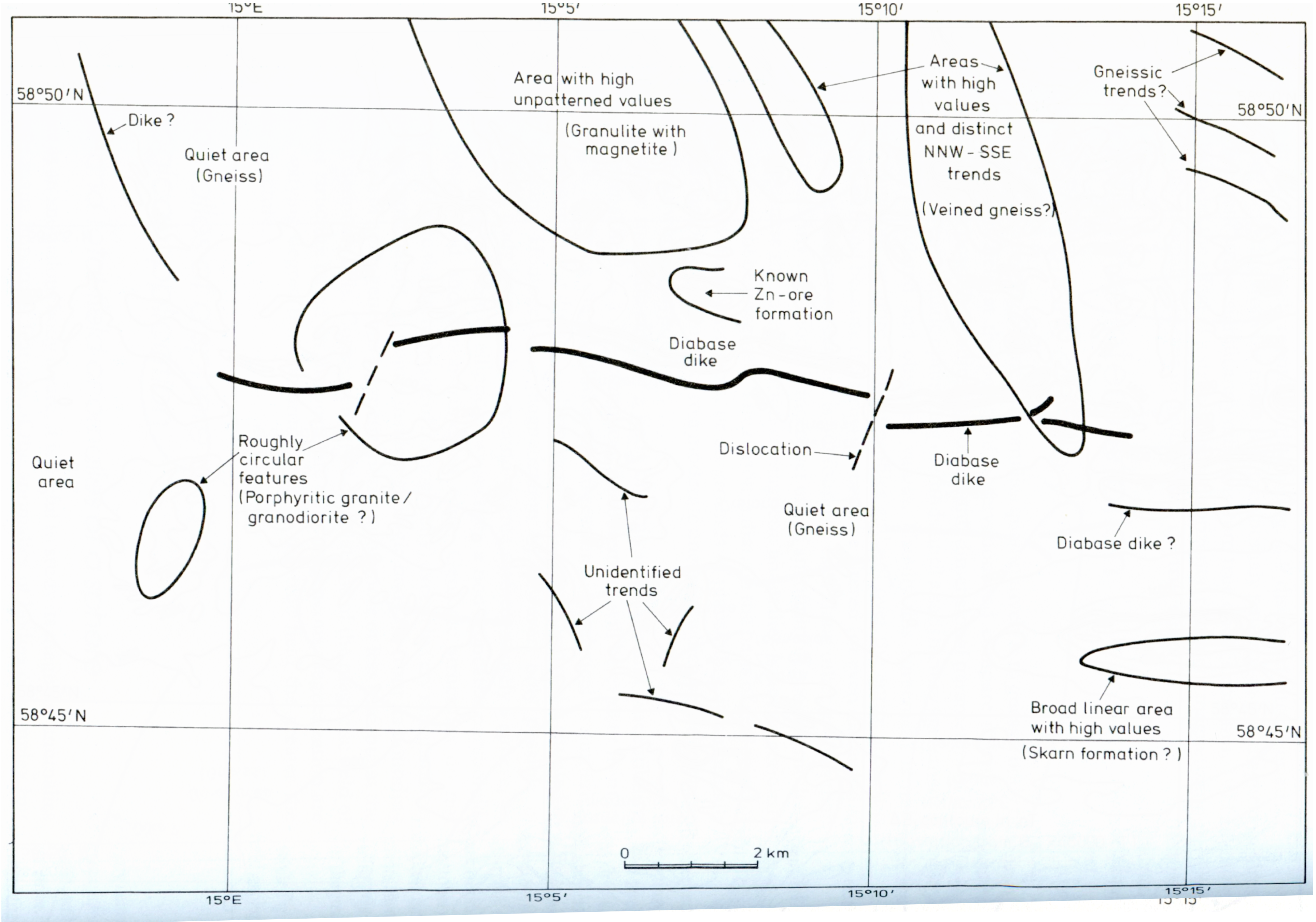
proton



$$\Delta B_t = \Delta B_h \cos\alpha \; \cos I \quad + \Delta Bz \sin Is$$

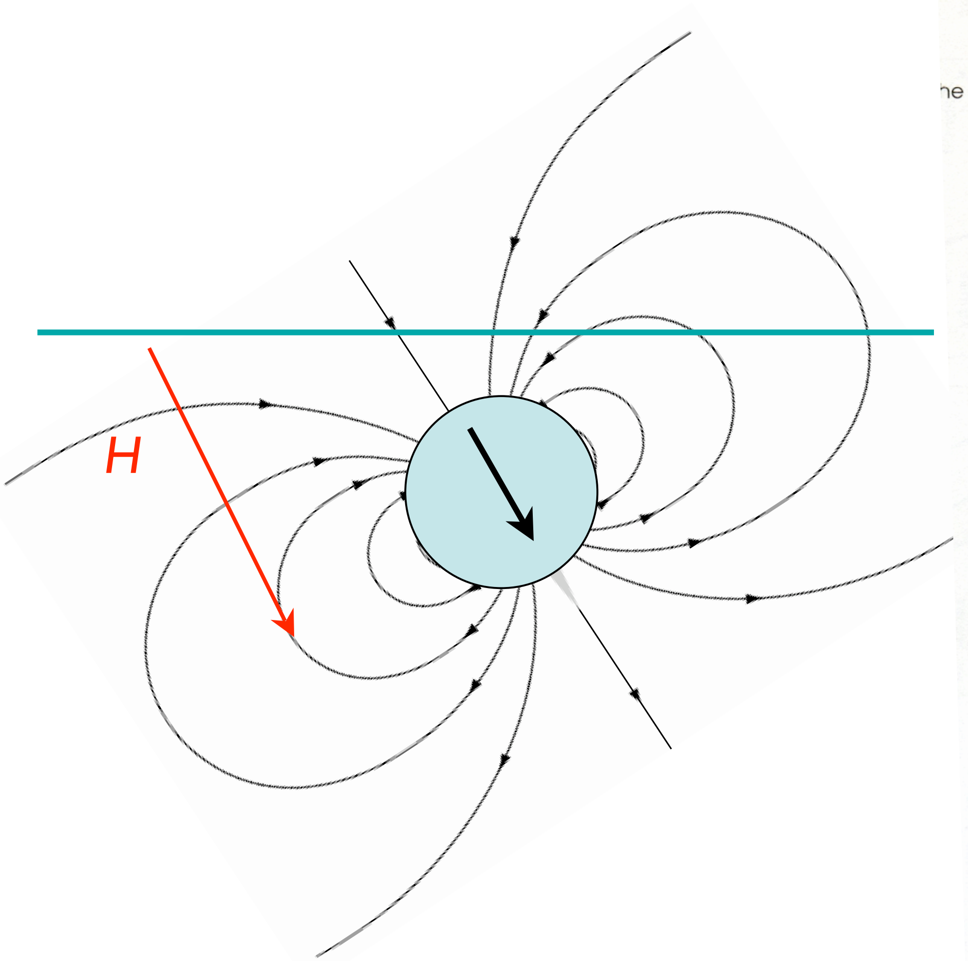


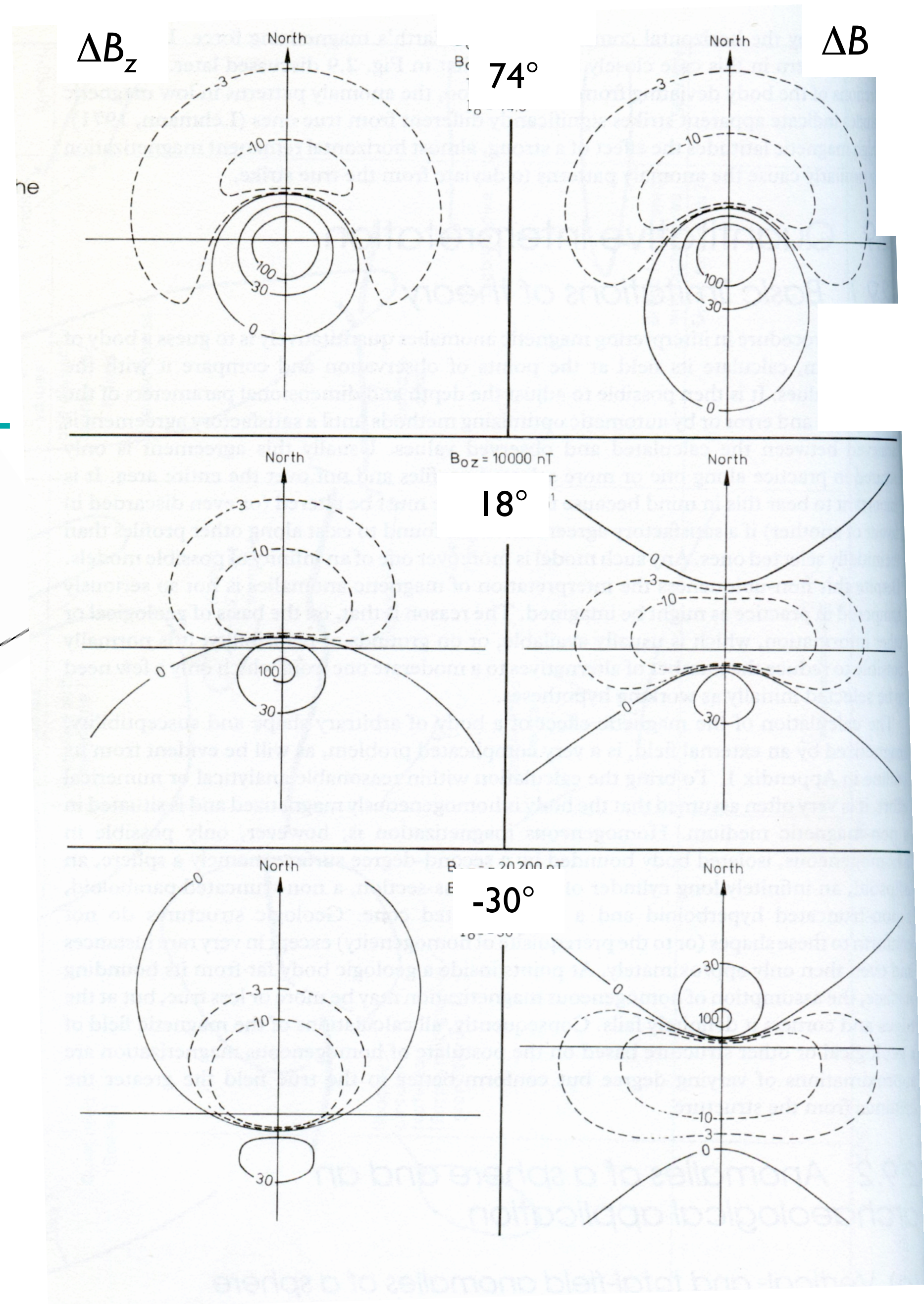
Grey-tone magnetic map of an archaeological site in Greece
Values range from − 150 nT (darkest) to + 280 nT (lightest). Coordinates in metres.

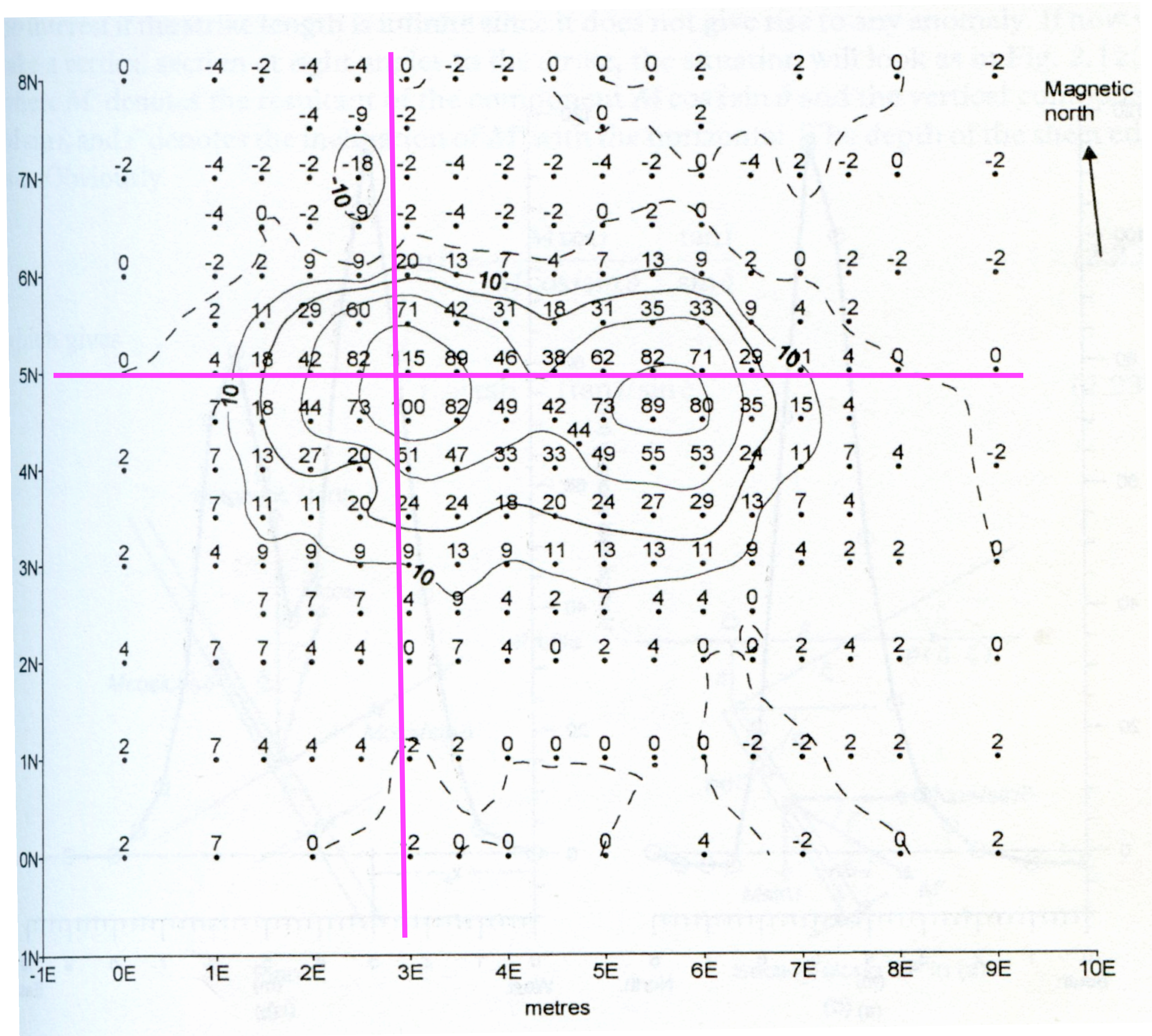


Airborne magnetic survey

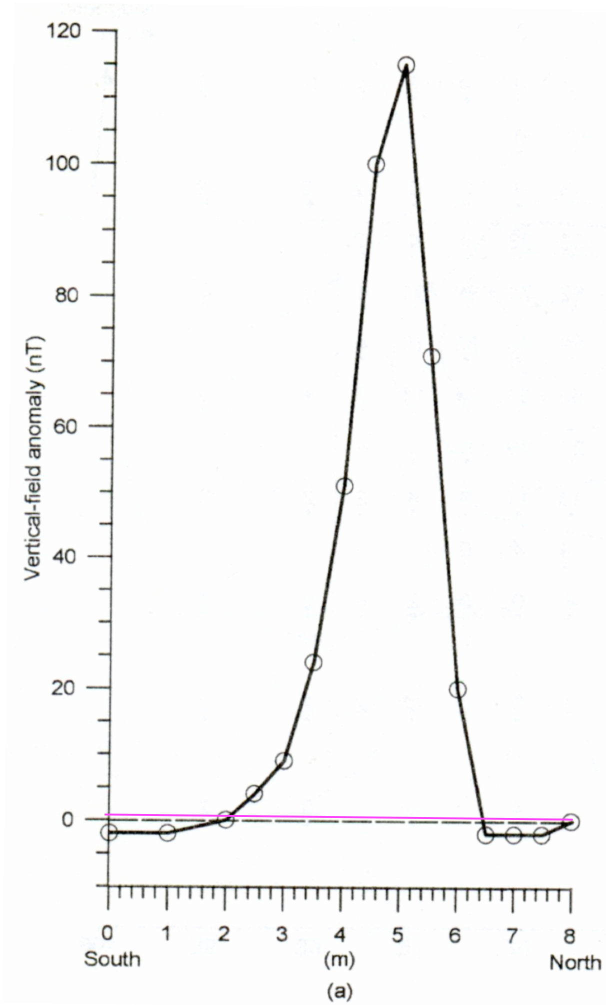
Magnetic anomaly due to a sphere



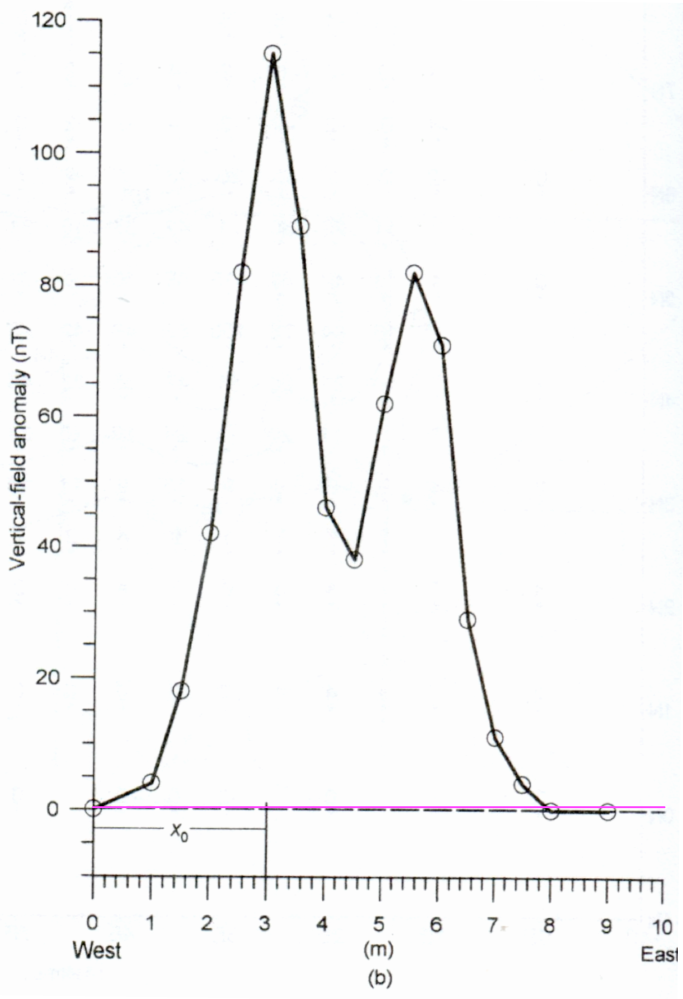




Danish archaeological site with slag from prehistoric ironworking



3E scan



5N scan

6.3.1 Potential calculations.

In magnetostatics, there are no electric currents ($\nabla \times \boldsymbol{H} = 0$) and no time-dependence. [$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \partial \boldsymbol{D}/\partial t$]

The ***H***-field can be derived from a scalar potential φ_m [$\nabla \times \nabla f(\boldsymbol{r}) = 0$]

$$\boldsymbol{H} = -\nabla \varphi_m$$

$$\nabla . \boldsymbol{B} = 0 \qquad \text{(no sources or sinks of } \boldsymbol{B}\text{)}$$

but $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$

$$\nabla . \boldsymbol{B} = \mu_0(\nabla . \boldsymbol{H} + \nabla . \boldsymbol{M})$$

$$\nabla . \boldsymbol{H} = -\nabla . \boldsymbol{M}$$

$\nabla^2 \varphi_m = \nabla . \boldsymbol{M}$ This is Poisson's equation. Volume charge density ρ_m

Volume charge density $\rho_m = \delta q_m / \delta V$

$$\boxed{\nabla^2 \varphi_m = -\rho_m}$$

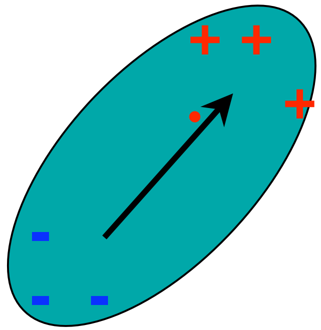
Units of magnetic charge q_m are Am

The magnetic potential of a point charge $\varphi_m = q_m / 4\pi r$. Units of φ_m are A

Resulting field $\boldsymbol{H} = -\nabla \varphi_m = q_m / 4\pi r^2$

Note that magnetic charges are just a convenient way to calculate the ***H***-field.

They have no physical reality. But there *is* a nice analogy with electric charge.



If a body is uniformly magnetized, $\nabla.\boldsymbol{M} = 0$ in the bulk. The only contribution arises from the surfaces.

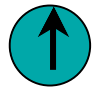
$\int_V \nabla.\boldsymbol{M}\ d^3r = \int_S \boldsymbol{M}.\mathbf{e}_n\ d^2r$ (divergence theorem)

The surface charge density is $\sigma_m = \boldsymbol{M}.\mathbf{e}_n$

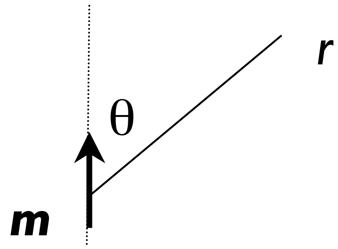
In general the *H*-field can be calculated from the magnetization via the potential

$$\varphi_m = -(1/4\pi) \int_V (\nabla.\boldsymbol{M})/r\ d^3r$$

Point dipole

The potential of a point dipole $-q_m \longrightarrow +q_m$ or  $\boldsymbol{m} = \boldsymbol{M}\delta V$

is $$\boxed{\varphi_m = (m/4\pi r^2)\cos\theta}$$

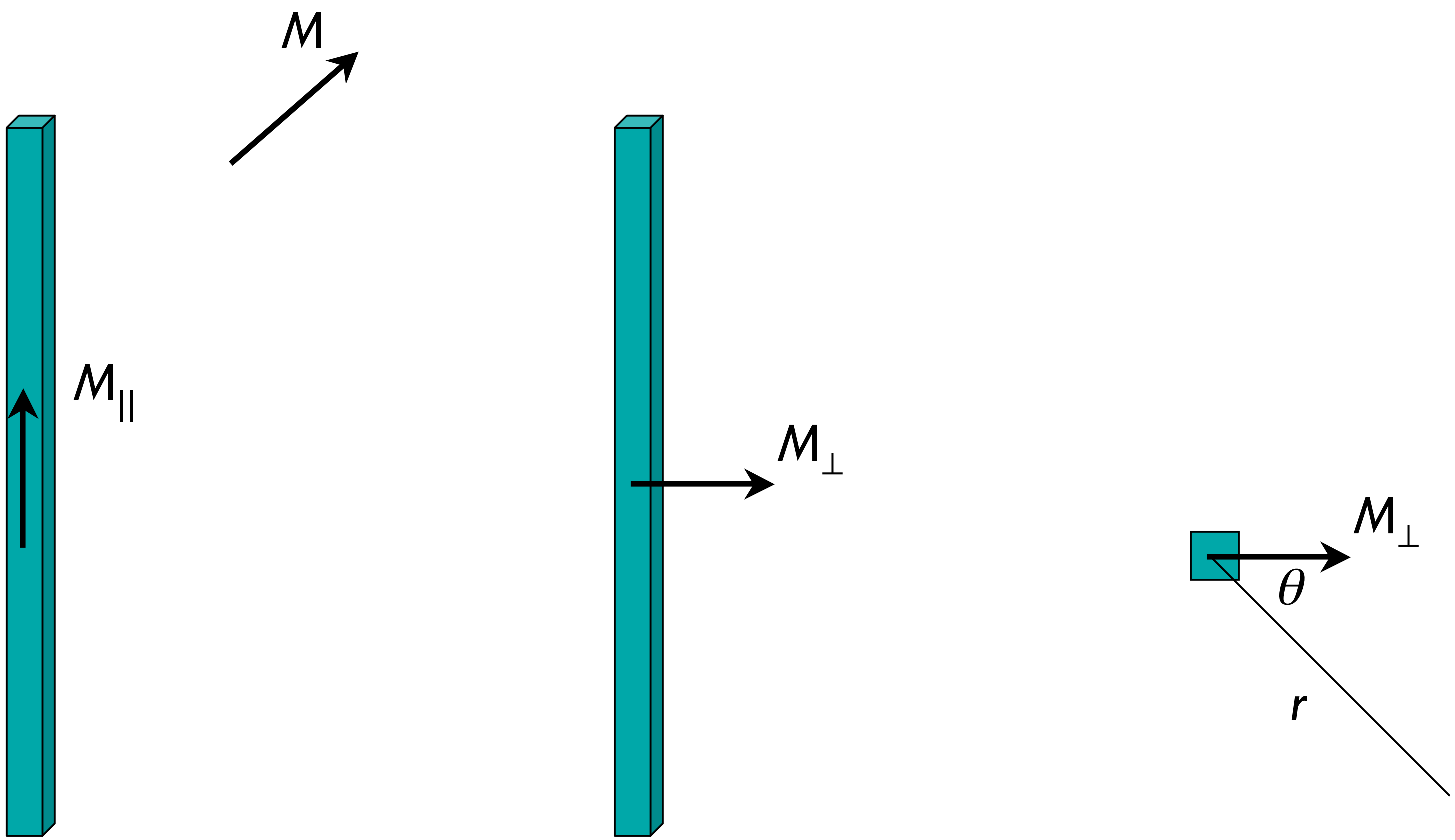


$$H_r = -\partial\varphi_m/\partial r = (m/4\pi r^3)\,2\cos\theta$$
$$H_\theta = -(1/r)\partial\varphi_m/\partial\theta = (m/4\pi r^3)\sin\theta$$

or in terms of components || and $\perp$ to $\boldsymbol{m}$

$$H_{||} = (m/4\pi r^3)\,(3\cos^2\theta - 1)$$
$$H_\perp = (m/4\pi r^3)\,(3\cos\theta\sin\theta)$$

Line dipole



$H_r = (\lambda/4\pi r^2)\cos\theta$
$H_\theta = (\lambda/4\pi r^2)\sin\theta$

Transverse magnetic moment λ A m

The longitudinal component produces no stray field

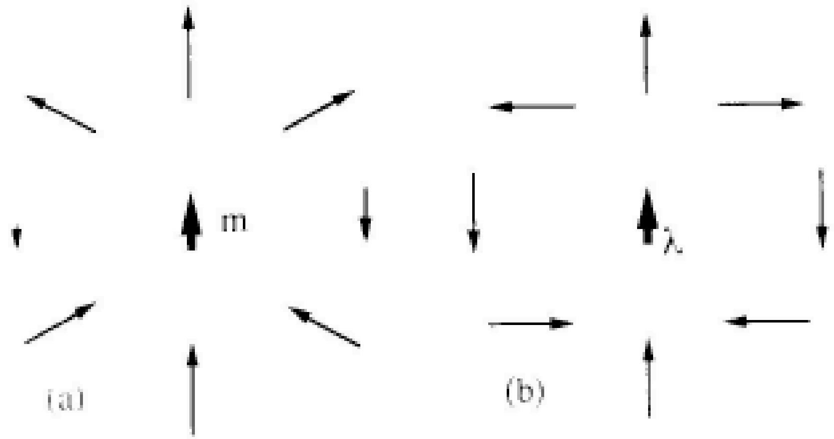
The magnetic field produced by a point dipole of moment m Am^2 is quite inhomogeneous In polar coordinates, it is

$H_r = 2m \cos\theta/4\pi r^3$, $\quad$ $H_\theta = m \sin\theta/4\pi r^3$, $\quad$ $H_\phi = 0$

The field due to an extended line dipole of length L and dipole moment λ Am per unit length is significantly different:

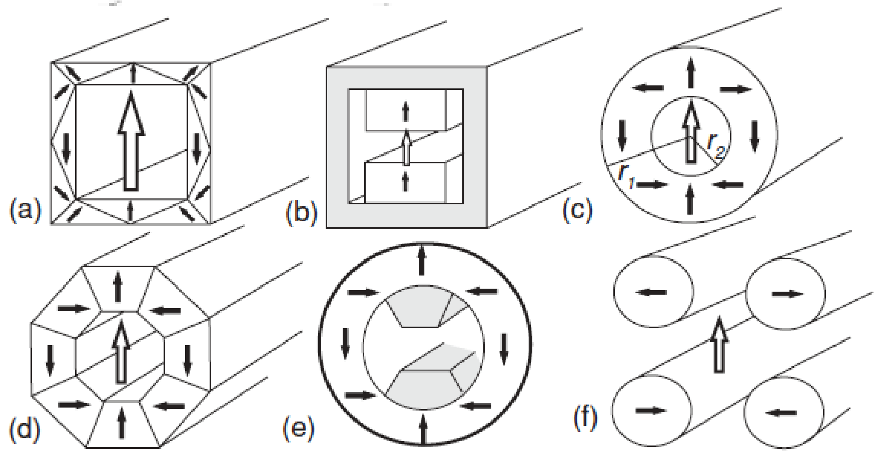
$H_r = \lambda \cos\theta,/4\pi r^2$, $\quad$ $H_\theta = \lambda \sin\theta/4\pi r^2$, $\quad$ $H_z = 0$

The magnitude of **H**, $\sqrt{(H_r^{\ 2} + H_\theta^{\ 2} + H_\phi^{\ 2})}$, is now independent of θ and its direction makes an angle 2θ with the orientation of the magnet.



Comparison of the magnetic field produced by a) a point dipole ***m*** and b) a line dipole λ.

Magnetic circuits made of long cylindrical segments may be used to generate uniform fields. An open cylinder or a design with flat cuboid magnets and a soft iron return path is used to for nuclear magnetic resonance (NMR). Permanent magnet flux sources supply fields of order 0.3 T with homogeneity of 1 part in 10^5 in a whole-body scanner:

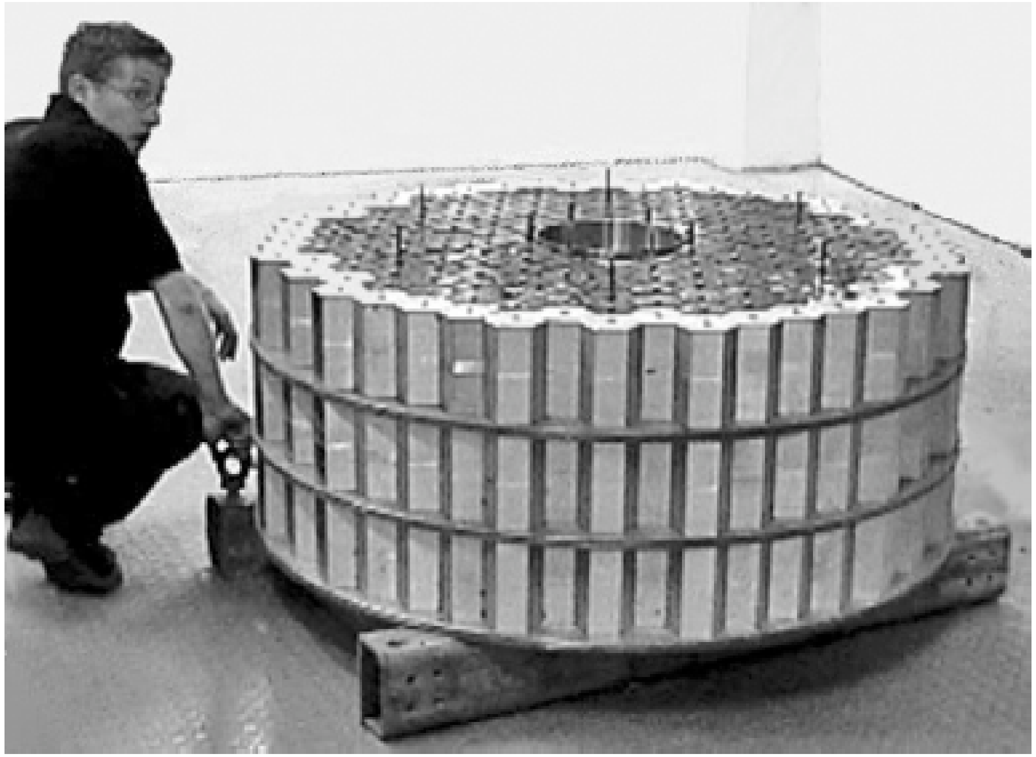


Designs for magnetic cylinders which produce a uniform transverse field.

Figure (c) shows a design where the direction of magnetization of any segment at angular position ϑ in the cylinder is at 2ϑ from the vertical axis. According to the equations for the line dipole, all segments now contribute to create a uniform field across the airgap in a vertical direction. Unlike the structure of Fig (a), the radii r_1 and r_2 can take any values without creating a stray field outside the cylinder. This ingeneous device is known as a *Halbach cylinder*, The field in the airgap is

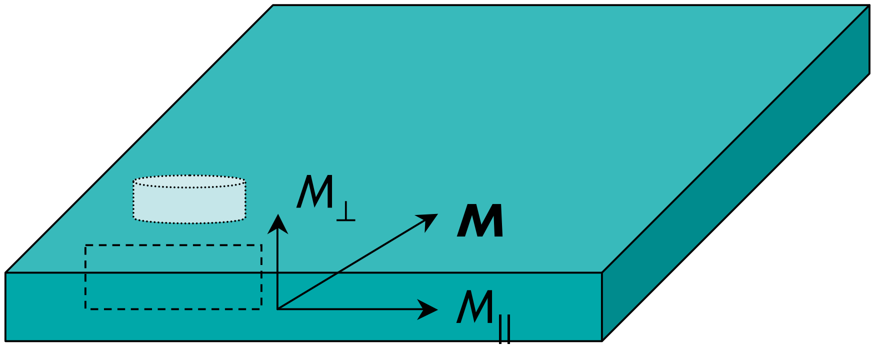
$$\boxed{B_0 = B_r \ln(r_2/r_1)}$$

In practice it is convenient to assemble the device from n trapezoidal segments, as illustrated in fig. (d) for n = 8.



A large Halbach cylinder, manufactured by Magnetic Solutions Ltd.

Uniformly-magnetized infinite sheet



Inside the film $H^i_{||} = H_d = -\mathcal{N}_{||}M_{ii} = 0$;

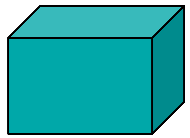
Integrating $\boldsymbol{H}\bullet\mathrm{d}\boldsymbol{l}$ around the dashed path $H^i = H^o$. Hence $H^o_{||} = 0$.

Inside the film, $B^i_\perp = \mu_0(H^i_\perp + M^i_\perp)$; but $H_\perp = -\mathcal{N}_\perp M^i_\perp$ where $\mathcal{N}_\perp = 1$, hence $B^i_\perp = 0$

From Gauss's law, the perpendicular component of B is continuous at the interface; $B^o_\perp = 0$

Since $\boldsymbol{B} = \mu_0\boldsymbol{H}$, both $\boldsymbol{B}^o$ and $\boldsymbol{H}^o$ are zero!

A magnet must be block-shaped to produce a stray field.



A convenient way to consider the *H*-field created by magnetized material is as originating from magnetic charge.

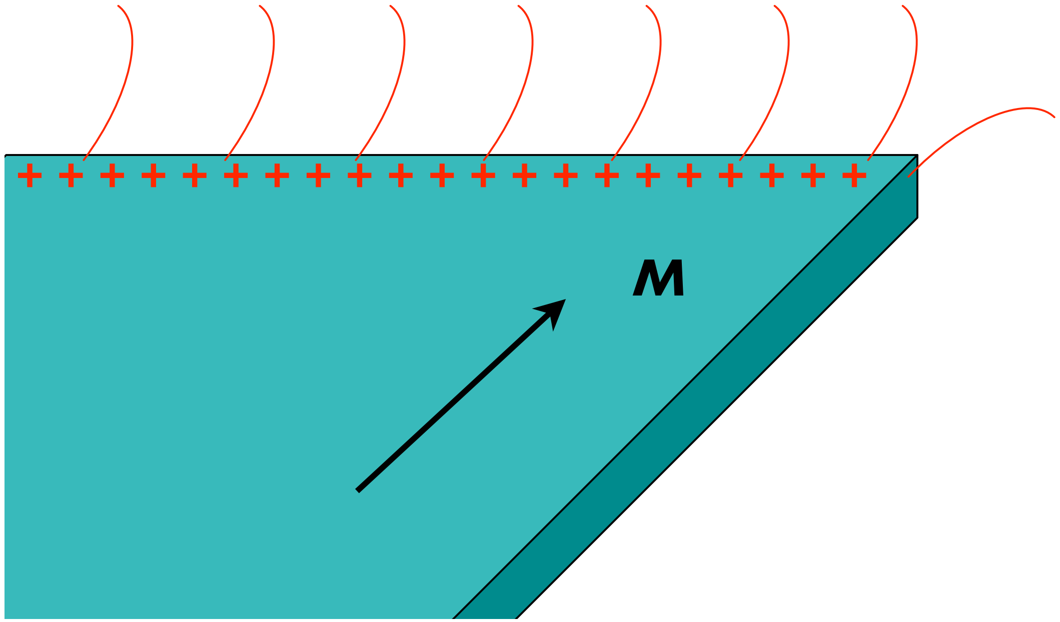
In the bulk, the charge density is $\rho_m = -\nabla.\boldsymbol{M}$. There is no bulk charge density when ***M*** is uniform.

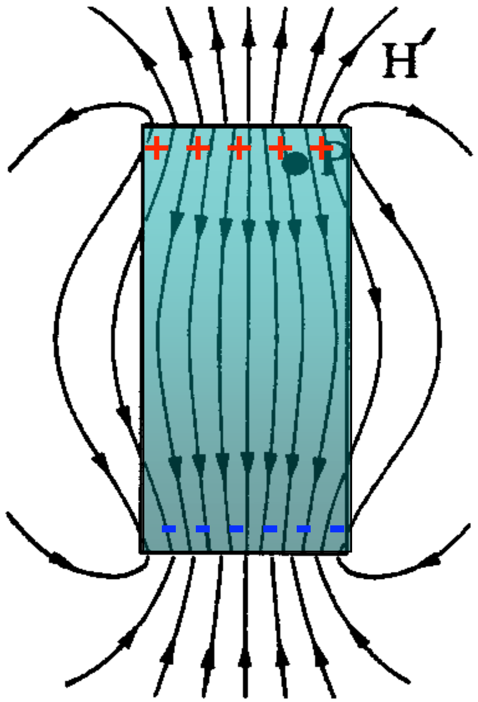
At the surface, the charge density $\sigma_m = \boldsymbol{M}.\mathbf{e}_n$

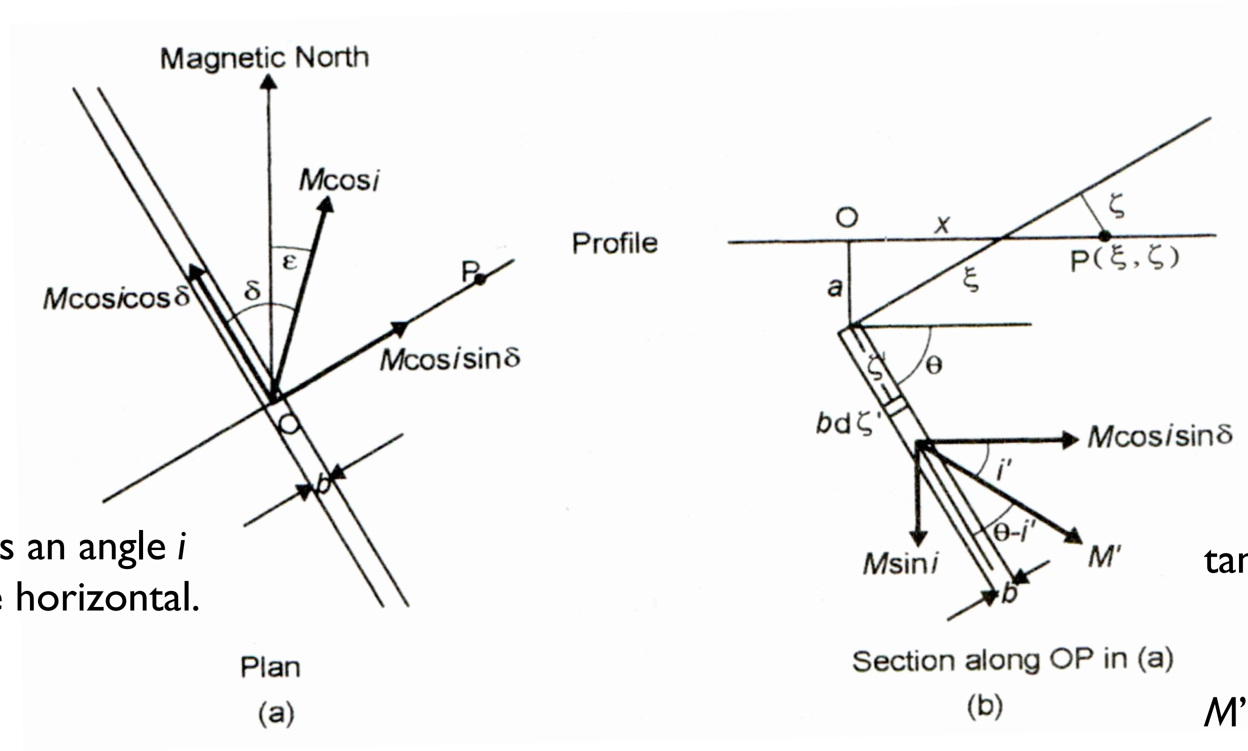
Units of magnetic charge q_m are Am

The magnetic potential $\varphi_m = q_m/4\pi r$

Resulting field $\boldsymbol{H} = -\nabla\varphi_m = q_m/4\pi r^2$







M makes an angle i with the horizontal.

$\tan i' = M\sin i / M\cos i \sin\delta$

$= \tan i/\sin\delta$

$M'^2 = M^2(\cos^2 i \sin^2\delta + \sin^2 i)$

$M' = M(1 - \cos^2 i \cos^2\delta)^{1/2}$

$$M'_{||} = M'\cos(\theta - i') \qquad M'_{\perp} = M'\sin(\theta - i')$$

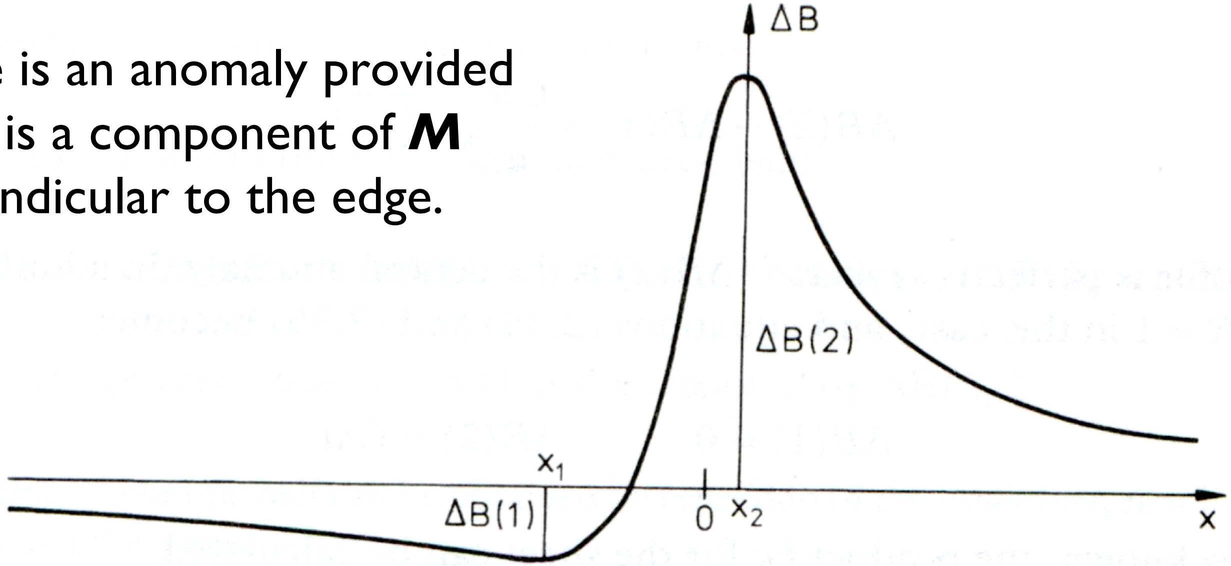
Integrating over long strips $\Delta B_h = -(\mu_0/4\pi)2b(xM'_{||} + aM'_{\perp})/(a^2 + x^2)$

$$\Delta B_z = -(\mu_0/4\pi)2b(aM'_{||} + xM'_{\perp})/(a^2 + x^2)$$

$$\Delta B_t = -(\mu_0/4\pi)2b[(a^2 + x^2)(M'^2_{||} + M'^2_{\perp}) + 4axM'_{||}M'_{\perp}]/(a^2 + x^2)$$

Magnetic anomaly due to a thin sheet

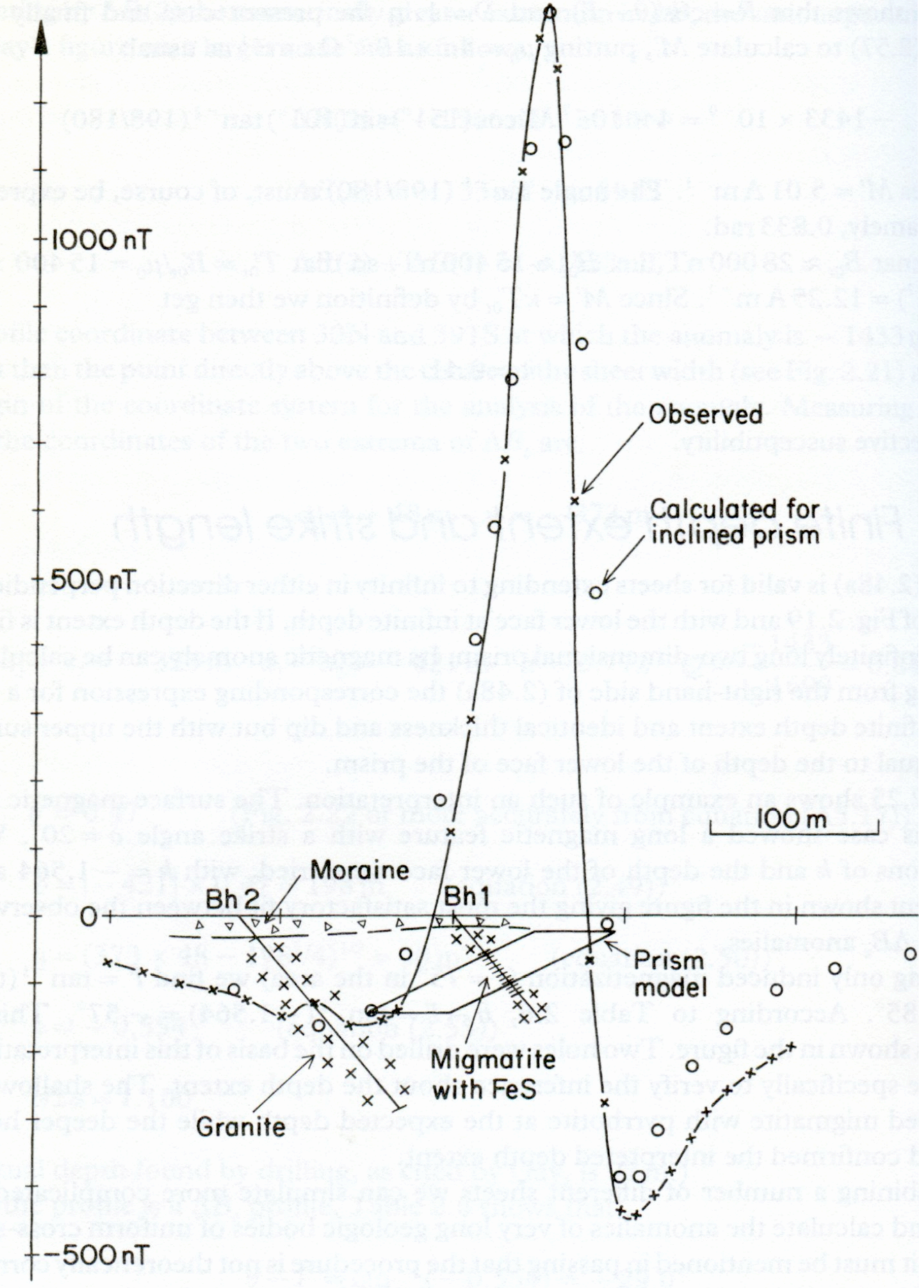
There is an anomaly provided there is a component of ***M*** perpendicular to the edge.



The form of the anomaly is similar whether ΔB_h, ΔB_z, ΔB_t is measured.

It is of the form $\Delta B = C(fa - fgx)/(a^2 + x^2)$

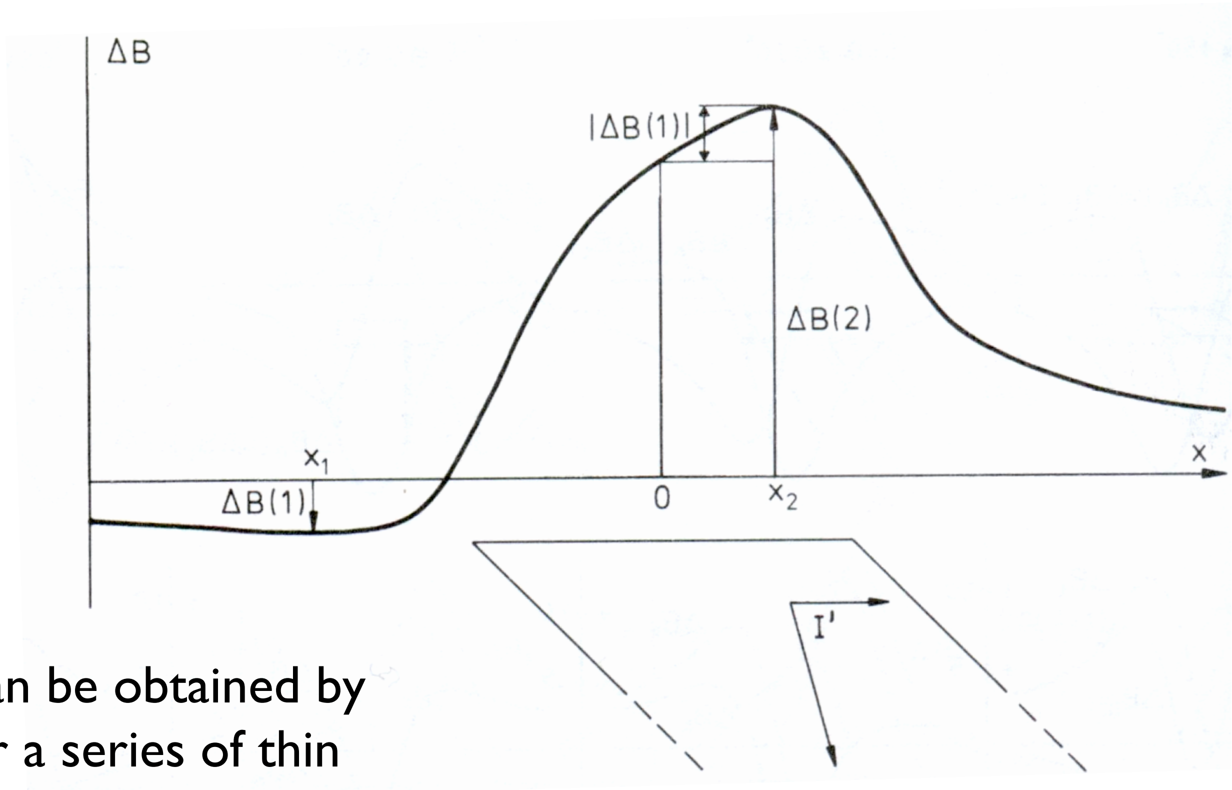
Here $C \propto M$; f, g are functions of the angles θ and i



1000 nT
500 nT
0
−500 nT
Observed
Calculated for inclined prism
100 m
Moraine
Bh 2
Bh 1
Prism model
Migmatite with FeS
Granite

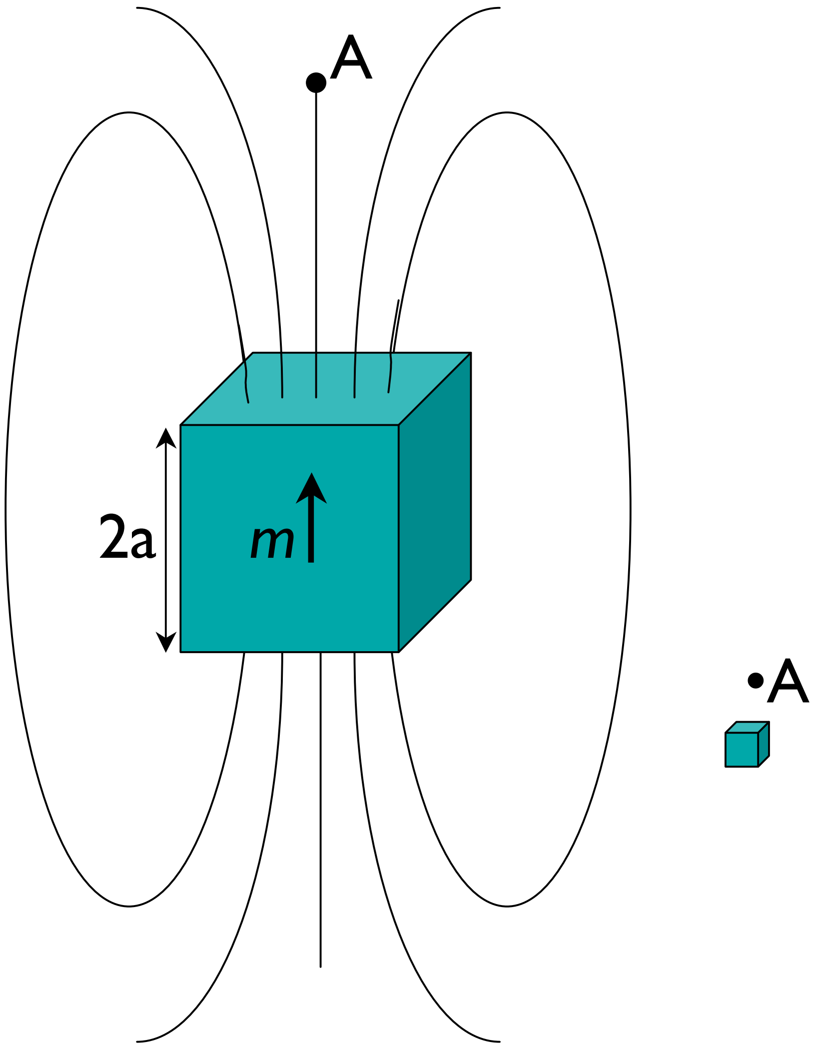
Magnetic anomaly due to a thick sheet

Again there is an anomaly provided there is a component of ***M*** perpendicular to the edge.



The anomaly can be obtained by integrating over a series of thin sheets.

Scaling



$\boldsymbol{H} = (m/4\pi r^3)[2\cos\theta\,\mathbf{e}_r + \sin\theta\,\mathbf{e}_\theta]$
$H_A = 2Ma^3/4\pi r^3$;

If $a = 0.1$m, $r = 4a$, $M = 1$ MAm^{-1}
$H_A = 2M/16\pi = 40$ kAm^{-1} (50 mT)

Magnet-generated fields are limited by M. *Scale-independent*

Mapping the field due to a permanent magnet.

A Hall probe was scanned 0.5 mm above the top surface of a ferrite magnet magnetized along Oz, and the 'ash tray' profile of B_z determined.

Sensor	Principle	Detects	Frequency	Field (T)	Noise	Comments
Coil	Faraday's law	dΦ/dt	10^{-3} - 10^{9}	10^{-10} - 10^{2}	100 nT	bulky ,absolute
Fluxgate	saturation	H	dc - 10^{3}	10^{-10} -10^{-3}	10 pT	bulky
Hall probe	Lorentz f'ce	B	dc - 10^{5}	10^{-5} - 10	100 nT	thin film
MR	Lorentz f'ce	B^{2}	dc - 10^{5}	10^{-2} -10	10 nT	thin film
AMR	spin-orbit int	H	dc - 10^{7}	10^{-9} -10^{-3}	10 nT	thin film
GMR	spin accum.n	H	dc - 10^{9}	10^{-9} -10^{-3}	10 nT	thin film
TMR	tunelling	H	dc - 10^{9}	10^{-9} -10^{-3}	1 nT	thin film
GMI	permability	H	dc - 10^{4}	10^{-9} -10^{-2}		wire
MO	Kerr/Faraday	M	dc - 10^{5}	10^{-9} -10^{2}	1 pT	bulky
SQUID lt	flux quanta	Φ	dc - 10^{9}	10^{-15} -10^{-2}	1 fT	cryogenic
SQUID ht	flux quanta	Φ	dc - 10^{4}	10^{-15} -10^{-2}	30 fT	cryogenic
NMR	resonance	B	dc - 10^{3}	10^{-10} -10	1 nT	Very precise