# Lecture 3: Signal and Noise

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Ireland.

1. Detection techniques
2. Random processes
3. Noise mechanisms
4. Thermodynamics
5. Noise reduction

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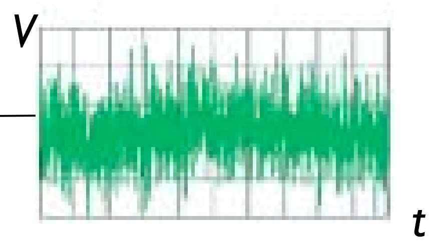
# 3 Signal and Noise

The critical figure of merit for any sensor is the *signal/noise ratio*.

Signal source → Detector + electronics →

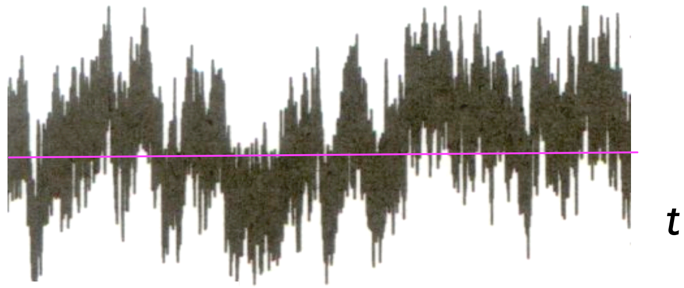



$$\langle V \rangle = \lim_{T \to \infty} (1/T) \int_{-T/2}^{T/2} V(t)\,dt$$

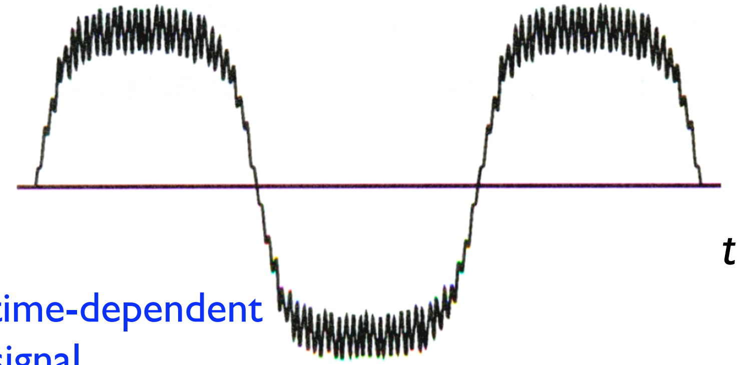
- Intrinsic signal noise
- Noise introduced by detector, filters and amplifier

Electronic noise is an uncontrollable random fluctuation in an electrical signal $V(t)$. Some noise is unavoidable, some can be eliminated by improving the detector design.

The effect of noise may be reduced by averaging the signal over time.




static signal




time-dependent signal

A *stationary* process is one where the noise has a fixed probability distribution, invariant in time.

A *non-stationary* process is one where the noise is derived from a time-varying distribution (e.g. domain structure which evolves with time).

When measuring magnetic fields, the sensitivity of the detector is conventionally quoted for bandwidth $\Delta f$ of 1 Hz. eg in nT/√Hz. Resolution $\neq$ accuracy.

To measure rapidly-varying signals, the bandwidth $\Delta f$ must be greater than the signal frequency. For example, the sensitivity is reduced 1000 fold at 1 MHz

By averaging the measurement of a steady signal for longer times, the accuracy increases as $\sqrt{t_m}$

## Signal/noise ratio

The signal/noise ratio is defined as the ratio of signal power to noise power:

$$SNR = P_{signal}/P_{noise}$$

or, equivalently $SNR = (A_{signal}/A_{noise})^2$ where A is the root-mean-square amplitude.

It can be expressed on a logarithmic scale in decibels

$$SNR = 10 \log_{10}(P_{signal}/P_{noise})$$

or

$$SNR = 20 \log_{10}(A_{signal}/A_{noise})$$

e.g. A SNR ratio of 0 db means that the amplitude of the signal and the noise fluctuations are similar; 60 db means that the rms amplitude of the signal is 1000 times that of the noise.

## Noise figure

The performance of an *amplifier* can be discussed in terms of the additional noise at the output compared to the noise present at the input.

$$NF = 10 \log_{10}(P_{noise\text{-}out}/P_{noise\text{-}in})$$

$$NF = 10 \log_{10}\{(\langle V^2_{noise}\rangle + \langle V^2_{amp}\rangle)/\langle V^2_{noise}\rangle\}$$

$$NF = 10 \log_{10}\{1 + \langle V^2_{amp}\rangle)/\langle V^2_{noise}\rangle\}$$

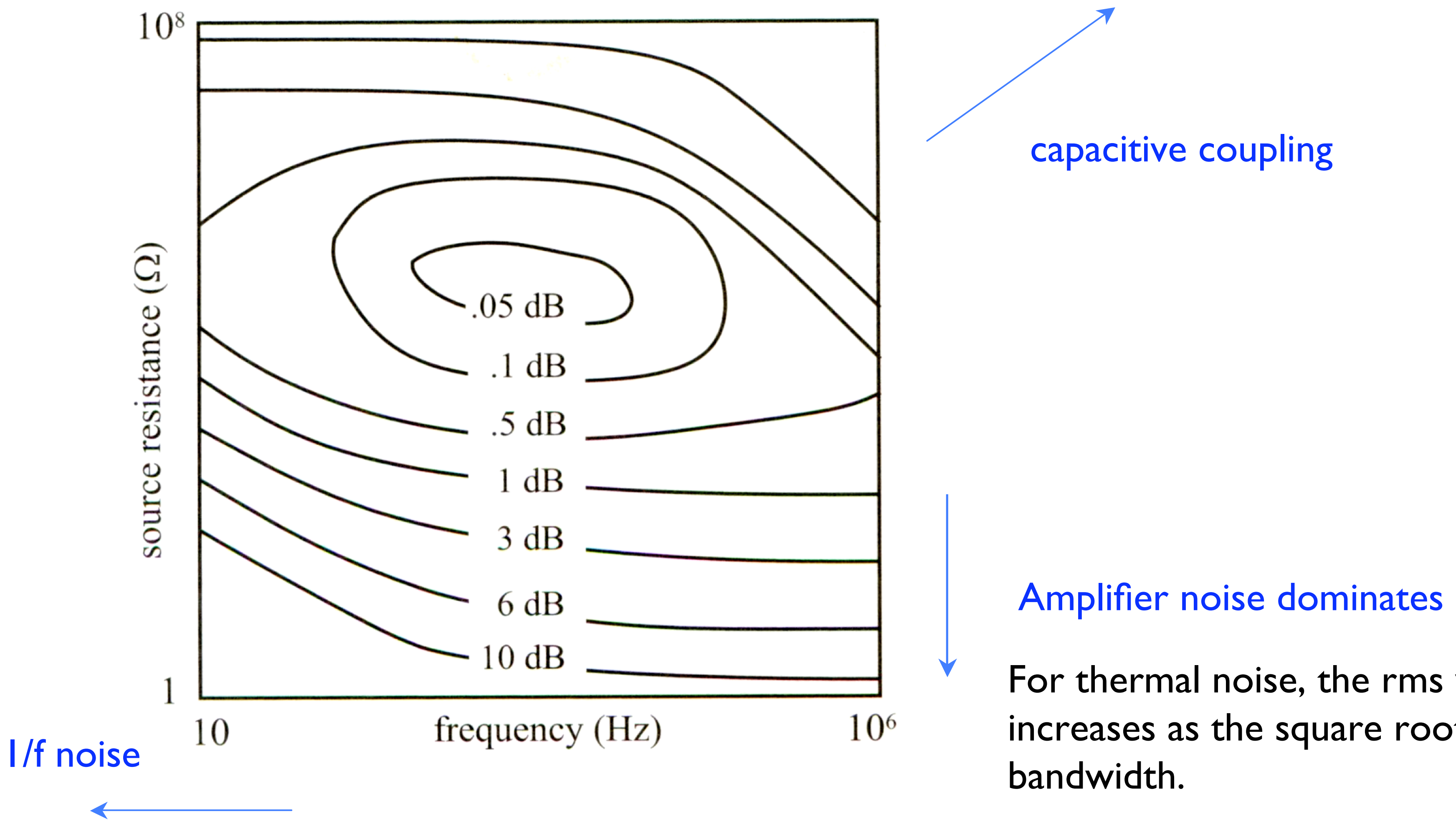
Here $V_{noise}$ is the noise present in the source, and $V_{amp}$ is the noise added by the amplifier.

The noise produced by the source can be approximated as that due to its impedance $V^2_{noise} = 4kT`R_{source}\Delta f$.

The amplifier noise can then be defined in terms of a noise temperature, to which the input impedance would have to be raised for its thermal noise to match that of the amplifier;

With high-mobility transistors, the noise temperature may be only a few K (even without cooling the amplifier)

$$NF = 10 \log_{10}(1 + T_{noise}/T_{source})$$



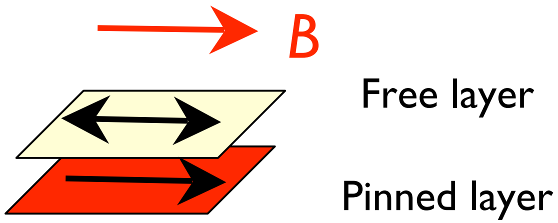

Noise contours for a low-noise preamp.
There is a sweet spot in the middle

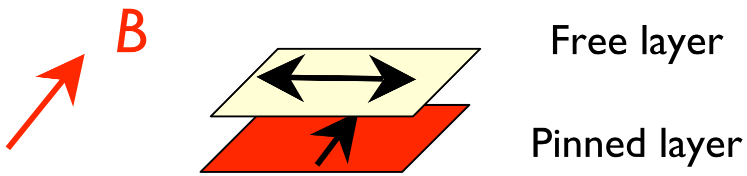
For thermal noise, the rms value increases as the square root of the bandwidth.

A low-noise voltage preamp has a noise figure of order 1 nV/√Hz; a low noise current preamp, of order 1pA /√Hz

## 3.1 Detection techniques

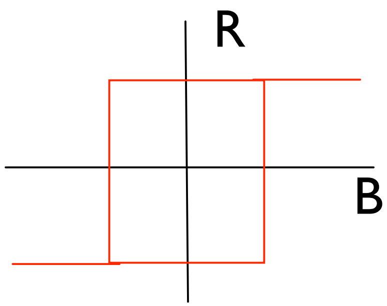
### 3.1.1 Spin valve sensor design (GMR or TMR).

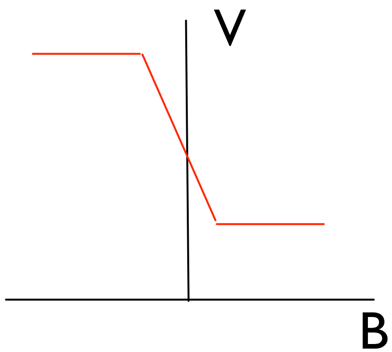



Memory cell

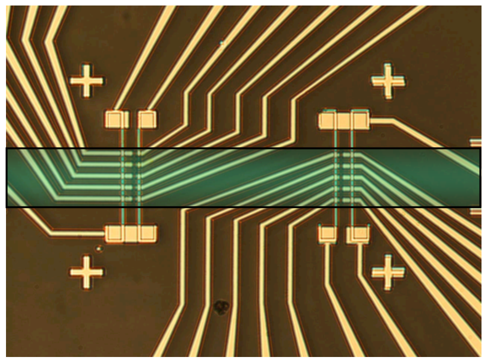
Linear sensor



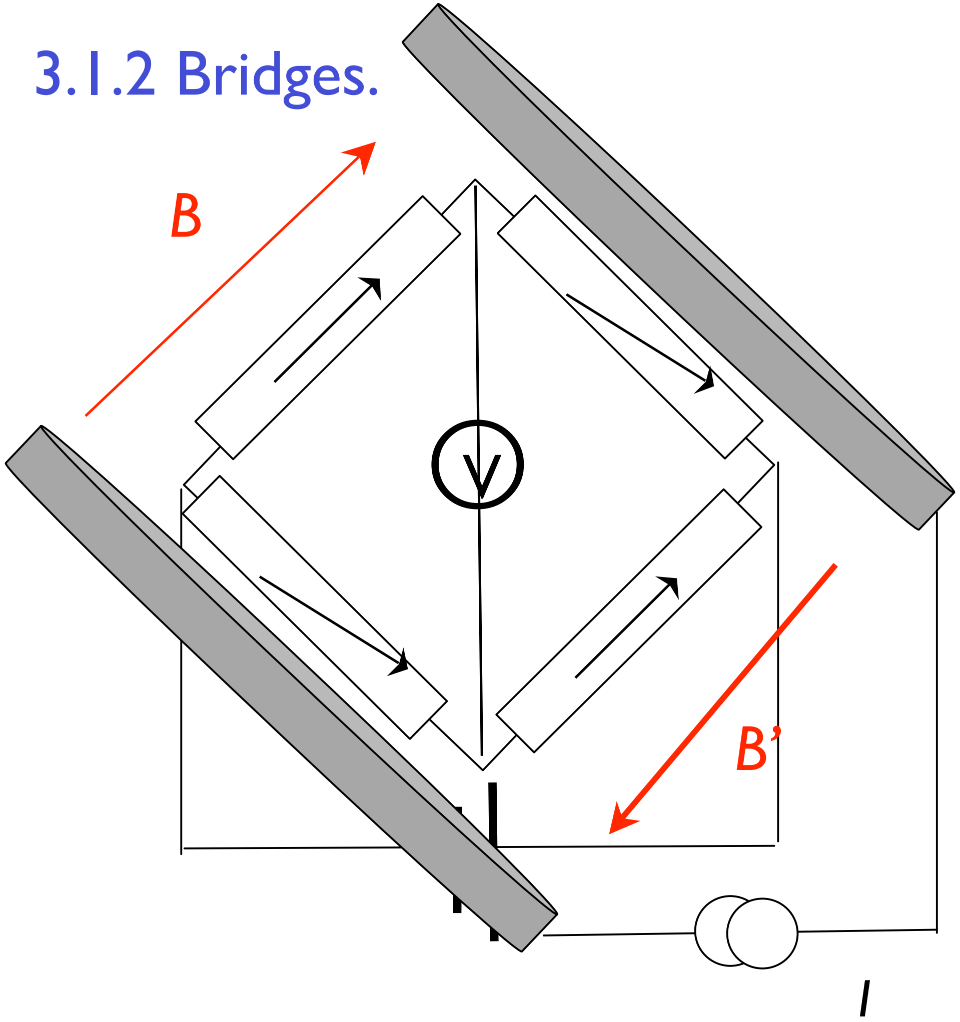

Transfer curve;

$V$ is linear in $B$ at constant current



Two pairs of yoke-type spin valve sensors for a microfluidic channel

## 3.1.2 Bridges.

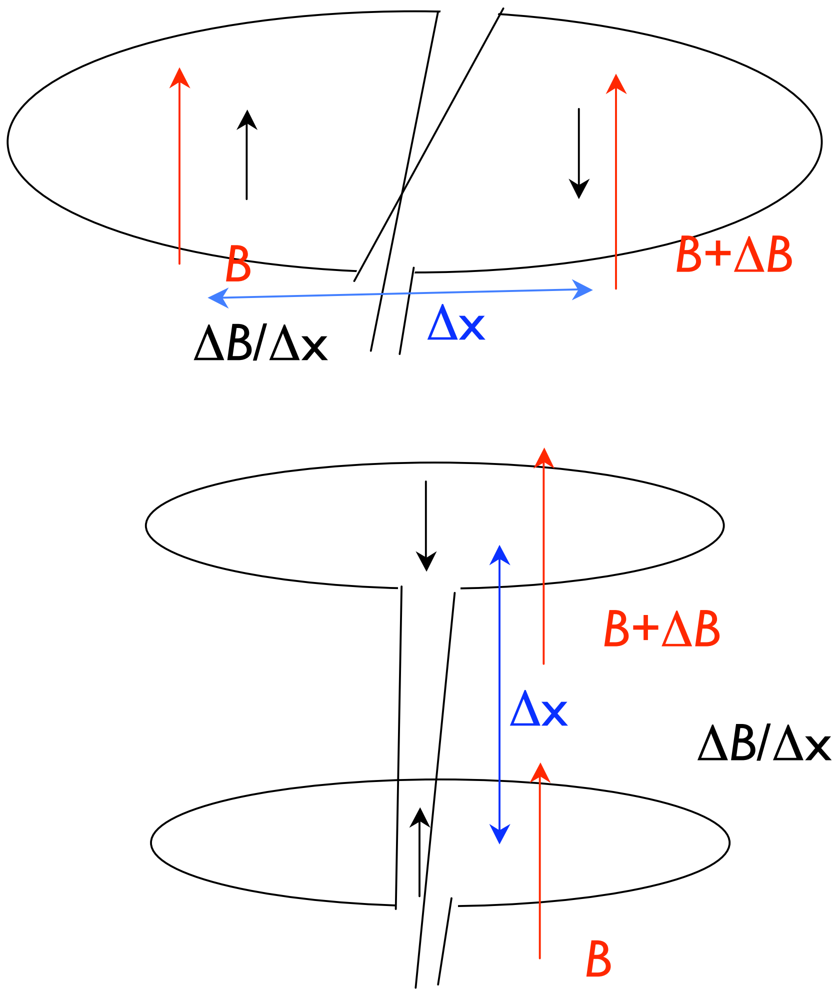



$B' = (4/5)^{3/2}\mu_0 NI/a$

One bridge can be used to measure each component of the magnetic field.

To make the sensor linear over a wide range of field, a *field-locked loop* can be added. The current in the coil is adjusted to create a field that just balances the applied field, and keeps the bridge at its null point.

## 3.1.3 Gradiometers and magnetometers




It is often required to distinguish local field variations from static or time-dependent fluctuations of the ambient field (e.g. due to a car parking outside)

Compensation is done with a quadrupole coil, either in a plane (gradiometer) or along an axis (magnetometer)

## 2.5 Flux concentrators.

It is possible to amplify the field to be sensed using a *flux concentrator.*
Two approaches are:

- Soft iron flux concentrator

- Superconducting loop with a constriction

## 3.2 Random processes

Consider a fluctuating quantity $V(t)$ such as the output of a noisy preamp. If $V$ is a random variable, it is drawn from a probability distribution $\mathrm{P}(V)$. The instantaneous value cannot be predicted, but averages can be precisely defined.

The expectation value of a function of $V$, $\mathrm{f}(V)$, can be defined as an integral over time, or over the distribution.

$$\langle \mathrm{f}(V) \rangle = \lim_{T \to \infty} (1/T)\int_{-T/2}^{T/2} \mathrm{f}(V(t))\,\mathrm{d}t$$

$$\langle \mathrm{f}(V) \rangle = \int \mathrm{f}(V)\mathrm{P}(V)\,\mathrm{d}V$$

Taking $\mathrm{f}(V) = 1$, the distribution $\mathrm{P}(V)$ must be normalized, $\int \mathrm{P}(V)\,\mathrm{d}V = 1$.

The average is $\langle V \rangle = \int V\mathrm{P}(V)\,\mathrm{d}V$ and the variance

$$\sigma^2 = \langle (V - \langle V \rangle)^2 \rangle$$
$$= \langle V^2 - 2V\langle V \rangle + \langle V \rangle^2 \rangle$$
$$= \langle V^2 \rangle - \langle V \rangle^2$$

The standard variation $\sigma = [\langle V^2 \rangle - \langle V \rangle^2]^{1/2}$

## Time dependence

The probability distribution contains no information about the time variation $V(t)$

This can be described by the *autocovariance function* $\Psi_V(\tau)$

$$\Psi_V(\tau) = \langle V(t)\ V(t - \tau)\rangle = \lim_{T \to \infty} (1/T)\int_{-T/2}^{T/2} V(t)V(t - \tau)\mathrm{d}t$$

When the autocovariance is normailized by the variance $\sigma$, then it is called the *autocorrelation function*.

It ranges from 1 to -1.

The rate at which the autocorrelation function decays as a function of $\tau$, indicates how fast $V(t)$ varies with time.

The Fourier transform of the fluctuating quantity $V(t)$ is

$$\hat{V}(f) = \lim_{T \to \infty} \int_{-T/2}^{T/2} \exp(2\pi i f t) V(t) dt \qquad (1)$$

The inverse transform is

$$V(t) = \lim_{F \to \infty} \int_{-F/2}^{F/2} \exp(-2\pi i f t) \hat{V}(f) df \qquad (2)$$

The Fourier transform is also a random variable.

The *power spectral density S(f)* is defined in terms of the Fourier transform $\hat{V}(f)$

$$S(f) = \langle |\hat{V}(f)|^2 \rangle = \langle \hat{V}(f)\ \hat{V}^*(f) \rangle$$

$\hat{V}^*(f)$ is the complex conjugate i → - i

$$S(f) = \lim_{T \to \infty} \int_{-T/2}^{T/2} \exp(2\pi i f t) V(t) dt \int_{-T/2}^{T/2} \exp(-2\pi i f t') V(t') t'$$

The Fourier transform is defined for positive and negative frequency. If it is defined only for positive frequency, the lower limit is 0, and a factor 2 is introduced in (2).

The power spectrum $S(f)$ is related to the autocovariance function by the *Wiener-Khinchin theorem*: Its Fourier transform is equal to the autocovariance $\Psi_V(\tau)$ .

$$\int_{-\infty}^{\infty} S(f)\exp(-2\pi i f\tau)\, df = \langle V(t)\ V(t-\tau)\rangle \qquad (3)$$

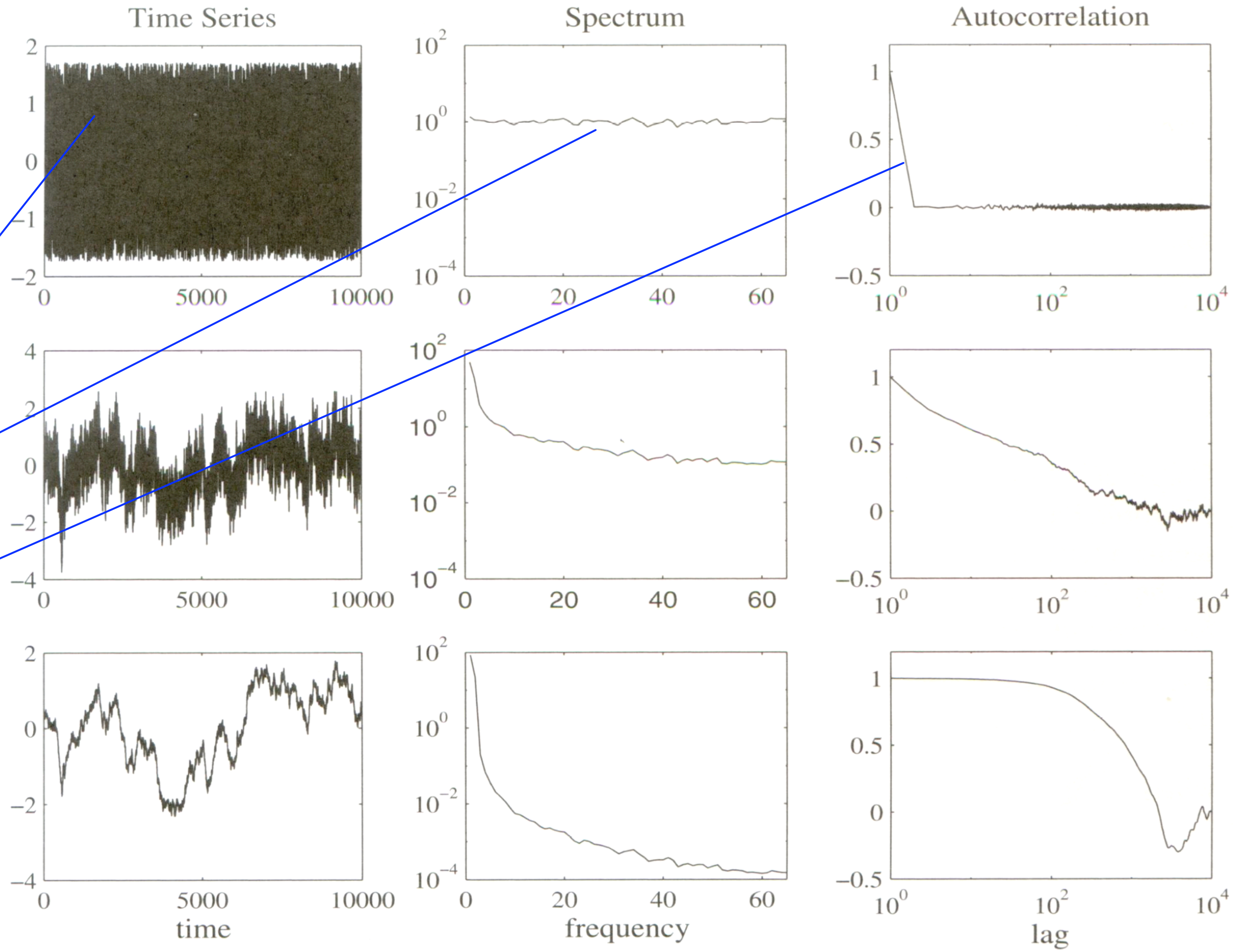
Conversely, the Fourier transform of the autocovariance gives the power spectrum.

As the autocorelation function decays more slowly, the the power spectrum decays faster and *vice versa*.

$V(t)$

$S(f)$

$\langle V(t)V(t-\tau)\rangle$




Taking $\tau = 0$ in the W-K theorem gives *Parseval's theorem*

$$\langle |V(t)|^2\rangle = \int_{-\infty}^{\infty} \langle |\hat{V}(f)|^2\rangle\, df$$

As an example of the use of the Wiener-Khinchin theorem, consider a relaxation process defined by a relaxation time $\tau_0$.

The relaxation is defined by an exponentially-decaying autocorrelation function

$$\Psi_V(\tau) = \Psi_V(0) \exp\left[-|\tau|/\tau_0\right]$$

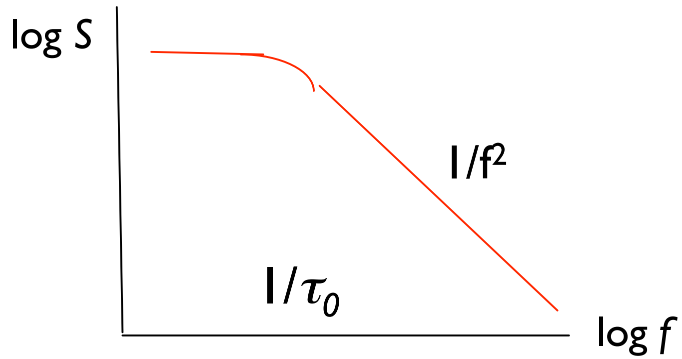
From the W-K theorem $\int_{-\infty}^{\infty} S(f) \exp(-2\pi i f\tau)\, df = \langle V(t)\ V(t-\tau)\rangle = \Psi_V(\tau)$

The inverse relation is $S(f) = \int_{-\infty}^{\infty} \Psi_V(\tau) \exp(2\pi i f\tau)\, d\tau$

$$= \int_{-\infty} \Psi_V(0) \exp\left[-|\tau|/\tau_0\right] \exp(2\pi i f\tau)\, d\tau$$

$$= \Psi_V(0)\{\tau_0/(1 + \tau_0^2 f^2)\}$$

The power spectral density corresponding to a single exponential decay is therefore a Lorentzian spectrum with a corner frequency of $1/\tau_0$, and a $1/f^2$ frequency dependence at higher frequency



## Probability distributions.

The noise is drawn from a probability distribution. Three common ones are the binomial, Poisson and Gaussian distributions.

- *Binomial distribution*; Applies when an event can have one of two possible outcomes, with probability $P$ and 1-$P$, respectively. The probability of finding x ‘heads’ and n-x ‘tails’ in n trials is

$$P_n(x) = \{n!/(n-x)!x!\} P^x(i-P)^{n-x} \qquad (1)$$

- *Poisson distribution*; Applies for rare events, like radioactive disintegrations that seldom occur. Divide time into small intervals, so that there is either one decay (‘heads’) or zero decays ‘tails’ in the interval. When $n$ is large and $P$ is small, (1) reduces to

$$P(x) = \exp(-N)N^x/x!$$

where $N$ = n$P$ is the average number of events.

The average $\langle x \rangle = N$, and $\sigma = \sqrt{N}$, so the relative standard deviation is

$$\sigma/\langle x \rangle = 1/\sqrt{N}$$

- *Gaussian distribution*.

When fluctuations result from the sum of a large number of independent random events, we have a ‘normal’ (Gaussian) distribution of the fluctuating quantity x

$$P(x) = (1/\sqrt{2\pi\sigma^2}) \exp\{-(x - \mu)^2/2\sigma^2\}$$

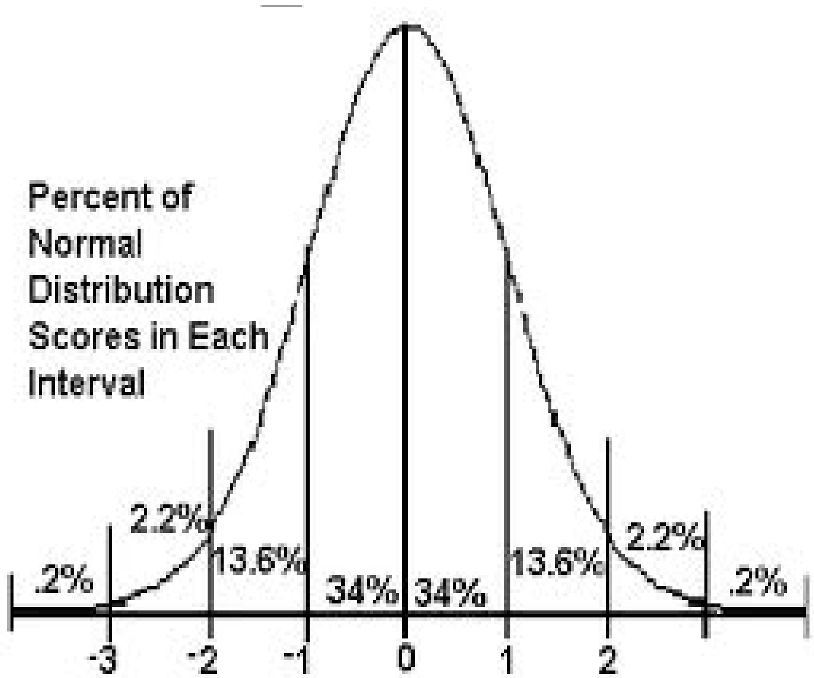
The distribution is normalized, with mean $\mu = \langle x \rangle$ and standard deviation $\sigma$.

The error function is the partial integral of the Gaussian distribution.
$(1/2)\mathrm{erf}(y/\sqrt{2\sigma^2}) = (1/\sqrt{2\pi\sigma^2})\int_0^y \exp(-x^2/2\sigma^2)dx$.

The Fourier transform of a Gaussian is itself a Gaussian with the inverse of the variance.

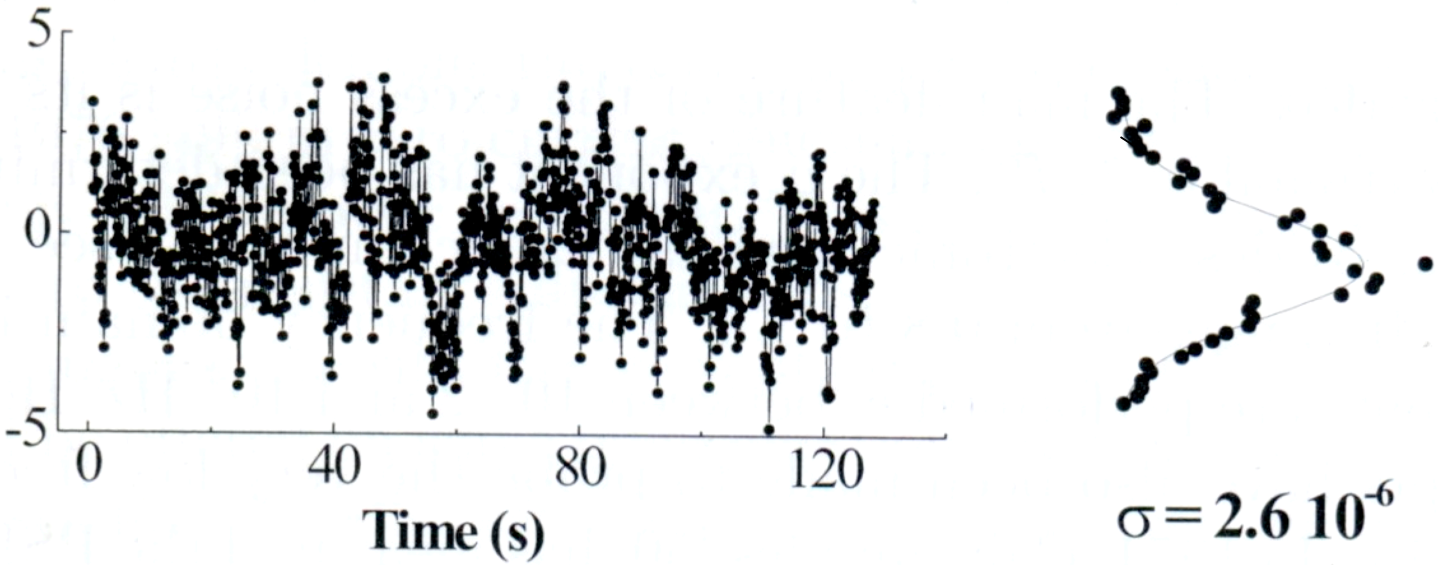
$(1/\sqrt{2\pi\sigma^2})\int_{\infty}^{\infty} \exp(-x^2/2\sigma^2)\exp(ikx)dx$

$= \exp-k^2\sigma^2/2$




The power spectral density contains all the characteristics of the Gaussian noise signal

## Gaussian noise



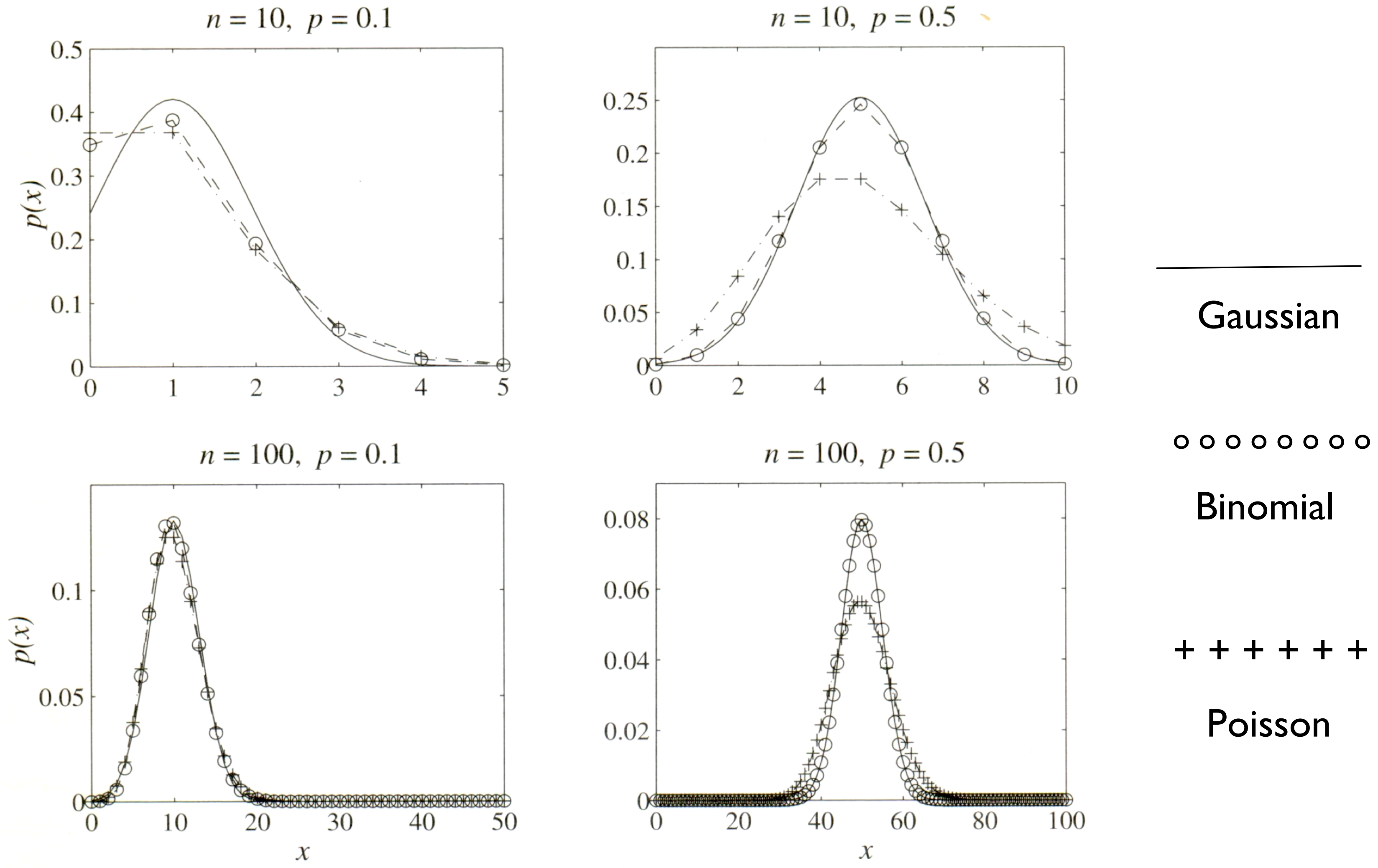



Figure 3.2. Comparison of the binomial (○), Poisson (+) and Gaussian (–) distributions: $n$ is the number of trials, and $p$ is the probability of seeing an event. By definition, the binomial distribution is correct. For a small probability of seeing an event, the Poisson distribution is a better approximation (although the difference is small for a large number of events), while for a large probability of seeing an event the Gaussian distribution is closer.

## 3.3 Noise mechanisms

Noise in a magnetoresistive sensor has two origins

- electrical noise
- noise of magnetic origin, associated with domains or magnetic modes.

The frequency spectrum of the time sequence is $\widehat{V}(f)$

$$\hat{V} = 2\int_0^\infty V(t)\exp(-2\pi i f t)\mathrm{d}t.$$

The average *power* dissipated in the fluctuations (in unit resistance) during a measuring time $t_m$ that tends to infinity is

$$P = \lim_{t_m\to\infty}\frac{1}{t_m}\int_{-t_m/2}^{t_m/2}|V(t)|^2\,\mathrm{d}t = \lim_{t_m\to\infty}\int_0^\infty \frac{2|\hat{V}(f)|^2}{t_m}\mathrm{d}f.$$

The power spectrum of the fluctuating signal is defined as

$$S_V(f) = \lim_{t_m \to \infty} \frac{2|\hat{V}(f)|^2}{t_m} (0 < f < \infty)$$

The voltage spectrum is conventionally quoted for a bandwidth $\Delta f$ = 1 Hz units are $V^2$/Hz

Four types of noise:

- Johnson (thermal) noise
- Shot noise
- 1/f (flicker) noise
- Random telegraph noise

## Johnson (thermal) noise. 1927

### Thermal Agitation of Electricity in Conductors.

ORDINARY electric conductors are sources of spontaneous fluctuations of voltage which can be measured with sufficiently sensitive instruments. This property of conductors appears to be the result of thermal agitation of the electric charges in the material of the conductor.

The effect has been observed and measured for various conductors, in the form of resistance units, by means of a vacuum tube amplifier terminated in a thermocouple. It manifests itself as a part of the phenomenon which is commonly called 'tube noise.' The part of the effect originating in the resistance gives rise to a mean square voltage fluctuation $V^2$ which is proportional to the value $R$ of that resistance. The ratio $V^2/R$ is independent of the nature or shape of the conductor, being the same for resistances of metal wire, graphite, thin metallic films, films of drawing ink, and strong or weak electrolytes. It does, however, depend on temperature and is proportional to the absolute temperature of the resistance. This dependence on temperature demonstrates that the component of the noise which is proportional to $R$ comes from the conductor and not from the vacuum tube.

$$V^2 \propto RT$$

A similar phenomenon appears to have been observed and correctly interpreted in connexion with a *current sensitive* instrument, the string galvanometer (W. Einthoven, W. F. Einthoven, W. van der Horst, and H. Hirschfeld, *Physica*, 5, 358-360, No. 11/12, 1925). What is being measured in these cases is the effect upon the measuring device of continual shock excitation resulting from the random interchange of thermal energy and energy of electric potential or current in the conductor. Since the effect is the same for different conductors, it is evidently not dependent on the specific mechanism of conduction.

The amount and character of the observed noise depend upon the frequency-characteristic of the amplifier, as would be expected from experience with the small-shot effect. The apparent input power originating in the resistance is of the order $10^{-18}$ watt at room temperature. The corresponding output power is proportional to the area under the graph of *power amplification—frequency*, at least in the range of audio frequencies. The magnitude of the 'initial noise,' when the quietest tubes are used without input resistance, is about the same as that produced by a resistance of 5000 ohms at room temperature in the input circuit. For the technique of amplification, therefore, the effect means that the limit to the smallness of voltage which can be usefully amplified is often set, not by the vacuum tube, but by the very matter of which electrical circuits are built.

J. B. JOHNSON.
Bell Telephone Laboratories, Inc.,
New York, N.Y., Nov. 17.

- Johnson (thermal) noise.

$$S_V(f) = 4k_B T R$$

There are voltage fluctuations with no imposed current:

$$\langle V^2 \rangle = 4k_B T R \Delta f$$

e.g. The rms voltage fluctuations across a 1 MΩ resistor in a 1 kHz bandwidth at room temperature ($4k_B T \approx 1/10$ eV = 16 $10^{-21}$ J) is 4μV.


This is white noise. It often sets the noise floor. It can be reduced by reducing the sensor resistance, or by cooling

- Shot noise. A non-equilibrium effect associated with electric current

$$S_I(f) = 2eI$$

There are current fluctuations, first seen in vacuum tubes

$$I_{shot} = (2eI\Delta f)^{1/2}$$

Operating a TMR sensor at a high bias, to increase the signal also increases the noise.

Shot noise depends on the discrete nature of the electrons in the electric current. The current is a random sequence of electrons.

◆ 1/f noise. A ubiquitous and remarkable effect exhibited by many natural and man-made phenomena - heartbeat (< 0.3 Hz); water level of the Nile; pop music stations

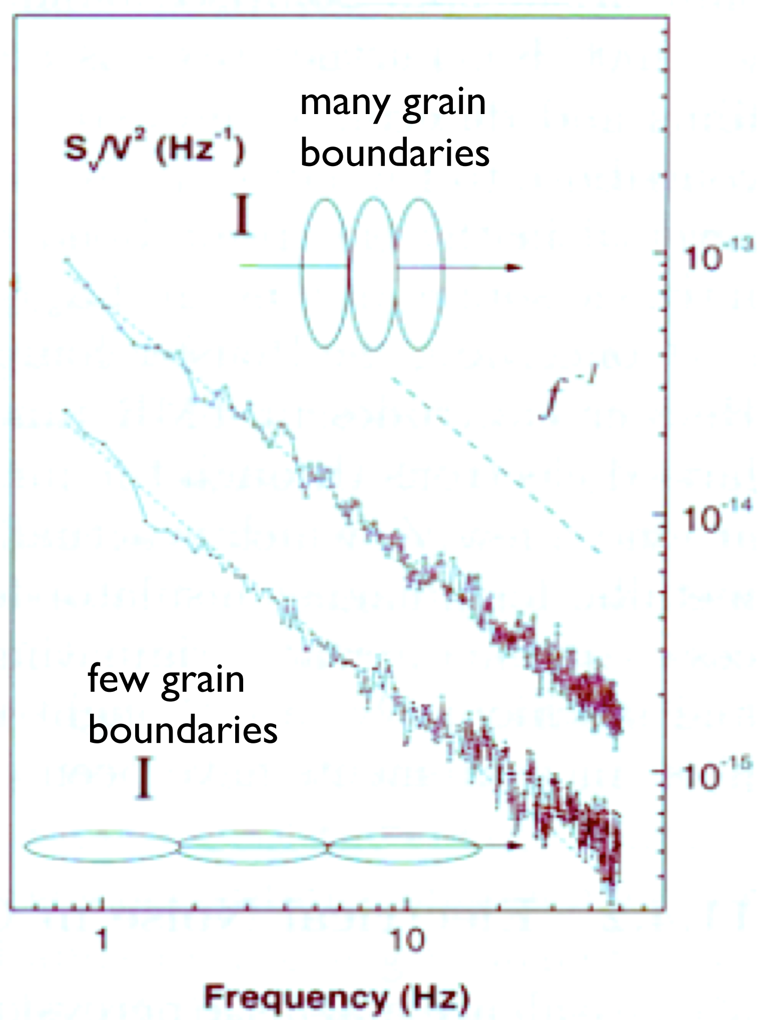
$$S_V(f) = Cf^{\alpha} \qquad \alpha \approx -1$$

The power spectral density is

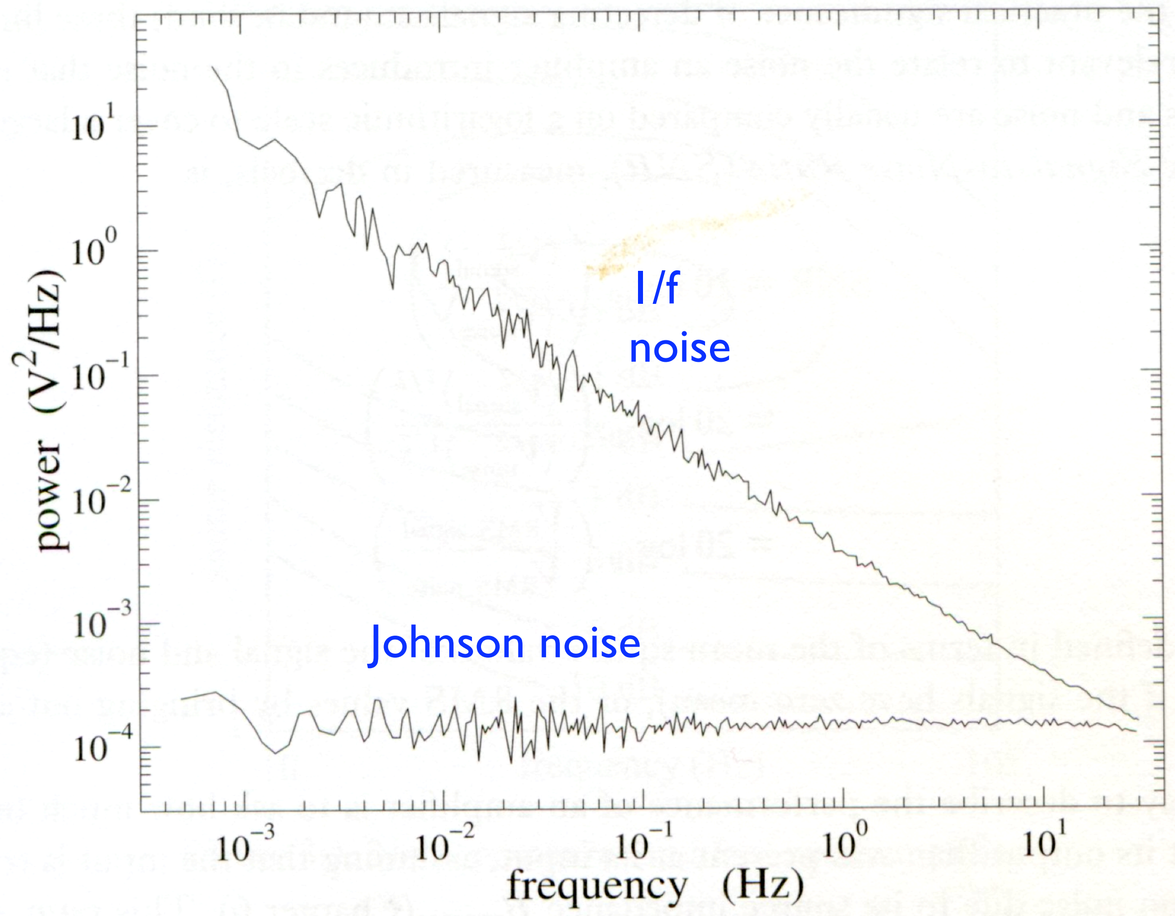
$$S_V(f) = \gamma_H v_a / N_e f$$

Hooge constant

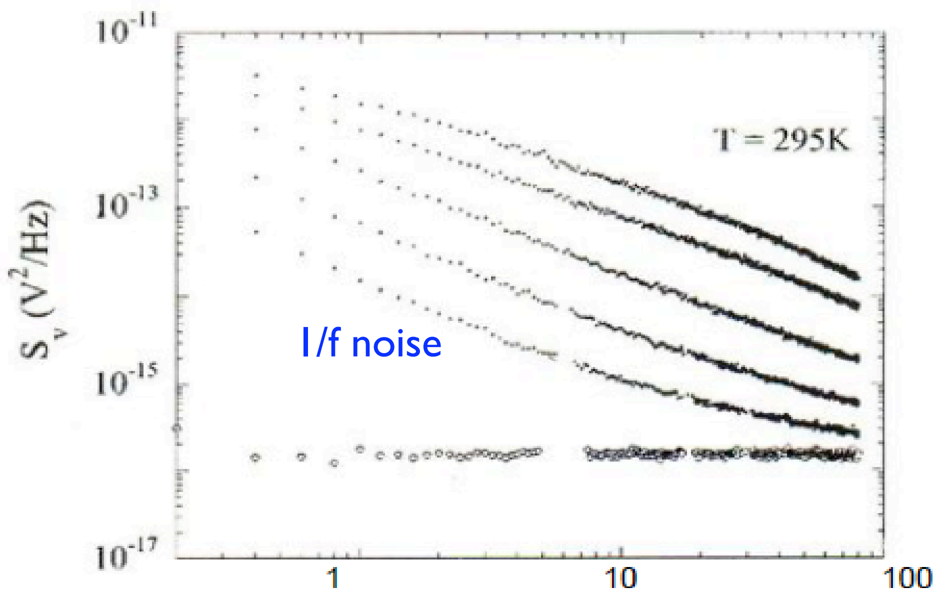
$\gamma_H = 10^{-3}$ for pure metals and semiconductors.
It can be as high as $10^3$ in some magnetic films



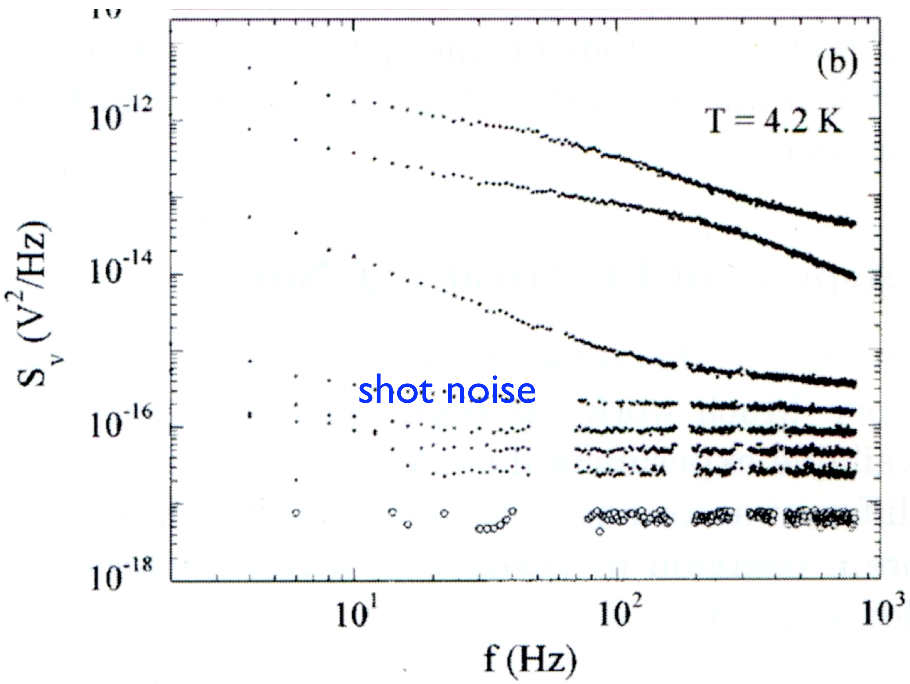

1/f noise in $CrO_2$




Noise spectrum of a 50 Ω resistor, with and without a passing current.




Noise in a CoFe/AlO$_x$/CoFe MTJ; Currents range from 0 to 36 microamps.



## Dutta-Dimmon-Horn model

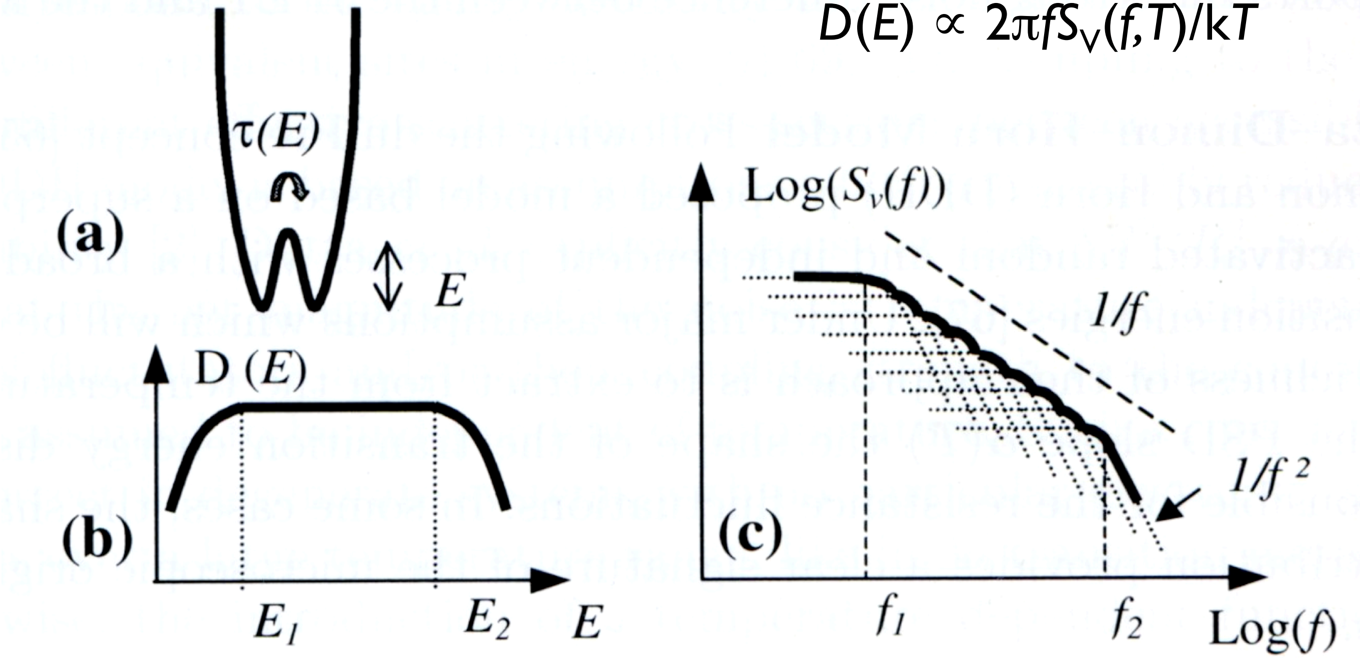
A phenomenological model based on a distribution of 2-level fluctuators.
Assume a distribution in energy $D(E)$. Each fluctuator involves an exponential time decay of the autocorrelation function.

$$\text{If } \tau_0 = \tau' \exp(E/kT)$$

$$S(f) = \int \Psi_V(0)\{\tau_0/(1 + \tau_0^2 f^2)\} D(E)\, dE$$

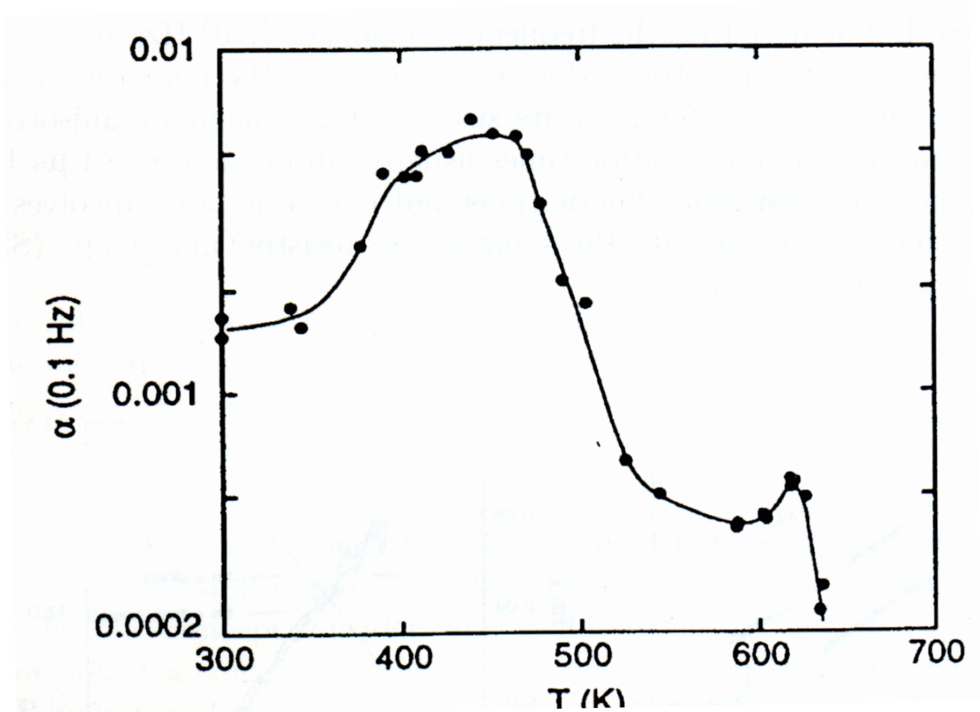
Superposition of the $1/f^2$ Lorentzian tails gives a $1/f$ spectrum

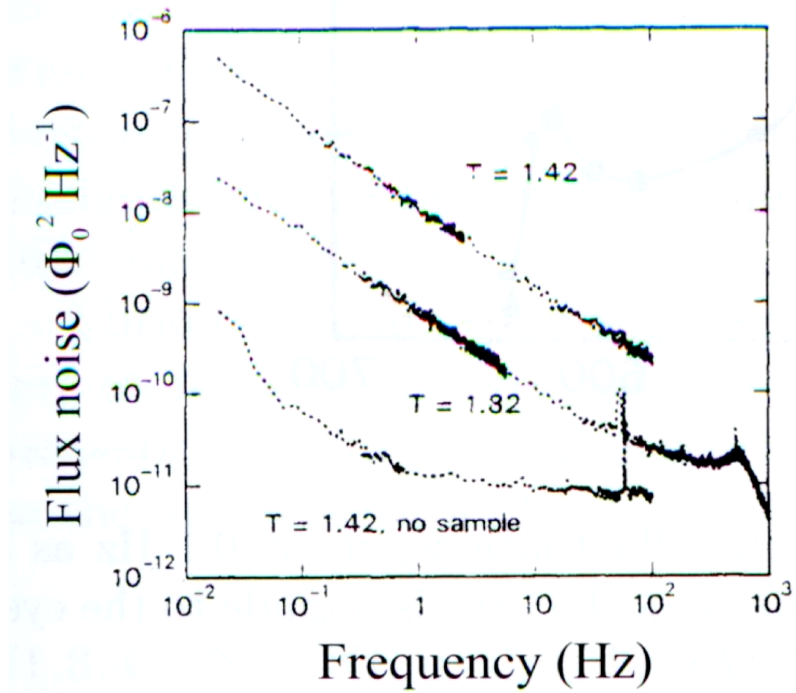
$$D(E) \propto 2\pi f S_V(f,T)/kT$$



## Magnetic 1/f noise

In ferromagnets, the fluctuators may have a magnetic origin, especially when there are many domains present, and the system has a large magnetoresistance.



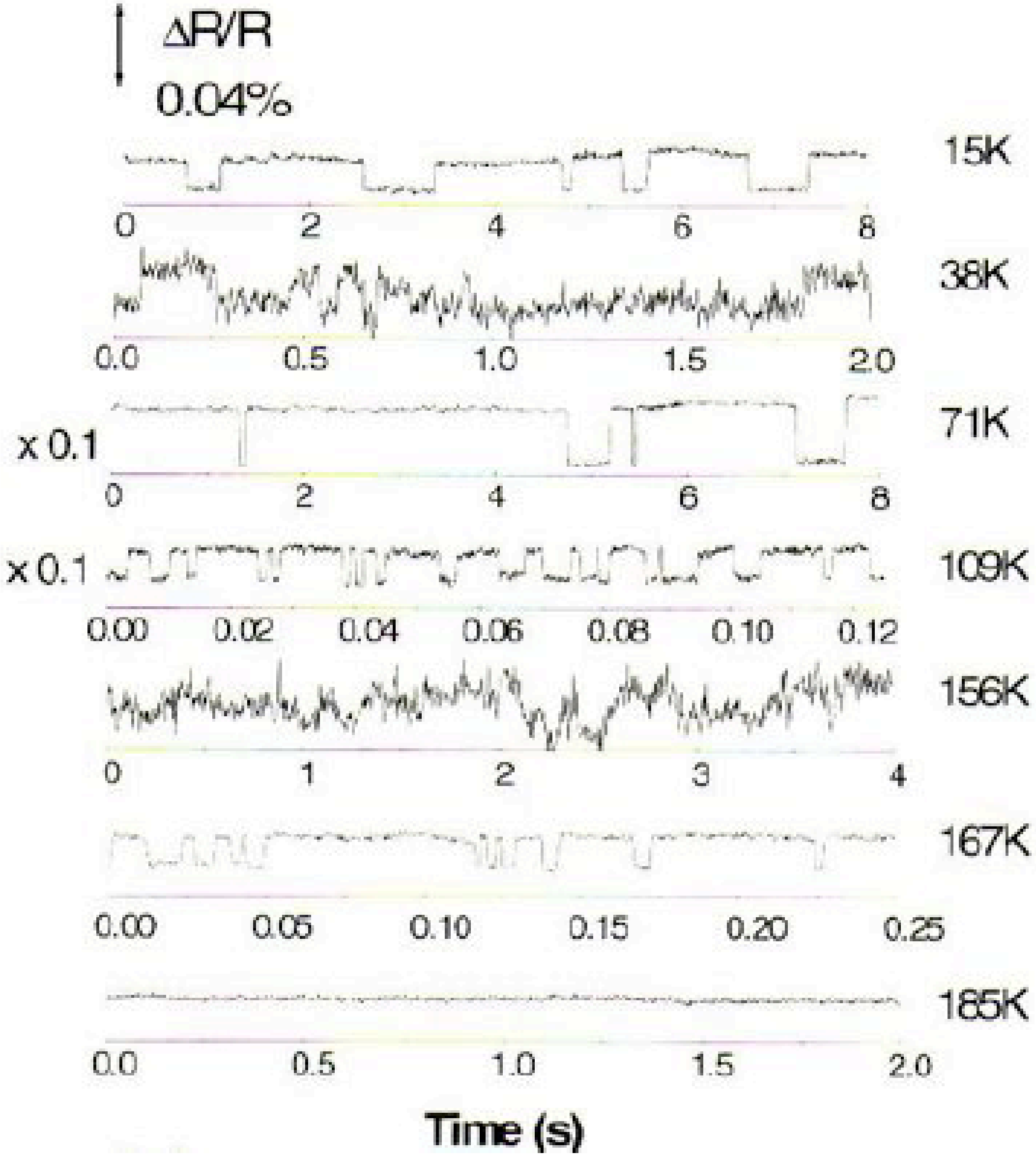

Normalized noise power for a nickel thin film

Flux noise power for a spin glass ($T_f$ = 1.50 K)

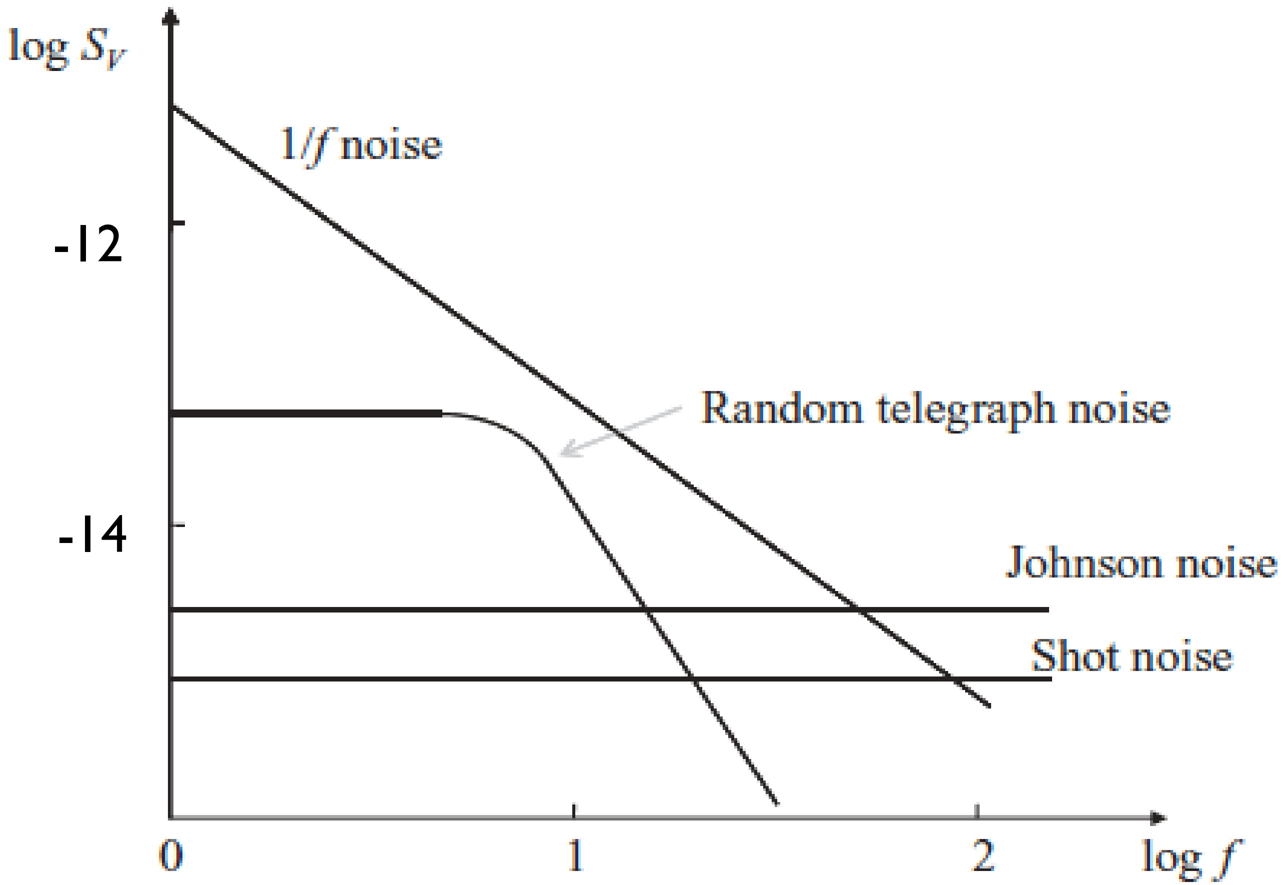
Domain-related noise can be controlled by micromagnetic engineering, such as use of a yoke-shaped soft magnetic free layer, or hard bias with permanently-magnetized elements

- Random telegraph noise.

Fluctuations between two distinct levels. The noise presents itself as a broad peak in the noise spectrum.




Resistance fluctuations of a $La_{0.67}Ca_{0.33}MnO_3$ thin film.




Types of noise

## 3.4 Thermodynamics

Thermodynamic quantities in equilibrium can be expressed in terms of the partition function *Z*. If *P* (X) is the probability of observing the system in a state X with energy *E(X)*

$$Z = \Sigma_i \exp(-E_i(X)/kT)$$

$$P(X) = (1/Z)\exp -(-E_i(X)/kT)$$

$$F = -kT \, ln\,(Z)$$

$$M = -(1/\mu_0)\partial F/\partial H$$

$$\chi = -(1/\mu_0)\partial^2 F/\partial H^2$$

Generalized susceptibility $\chi(f) = \chi'(f) - i\,\chi''(f)$

*Total fluctuation theorem* $kT\,\chi = \langle M^2(X)\rangle - \langle M(X)\rangle^2$

Fluctuation dissipation relation

$$S_M = kT\,\chi''(f))/\pi f$$

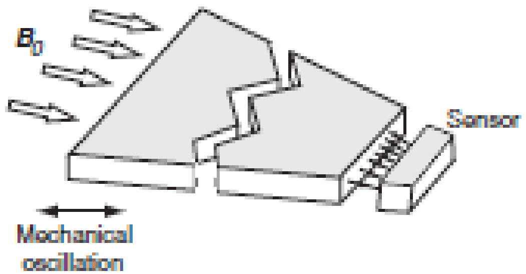
Kramers-Kronig relation

$$\chi'(0) = (2/\pi)\int_0^\infty (1/f)\chi''(f)df$$

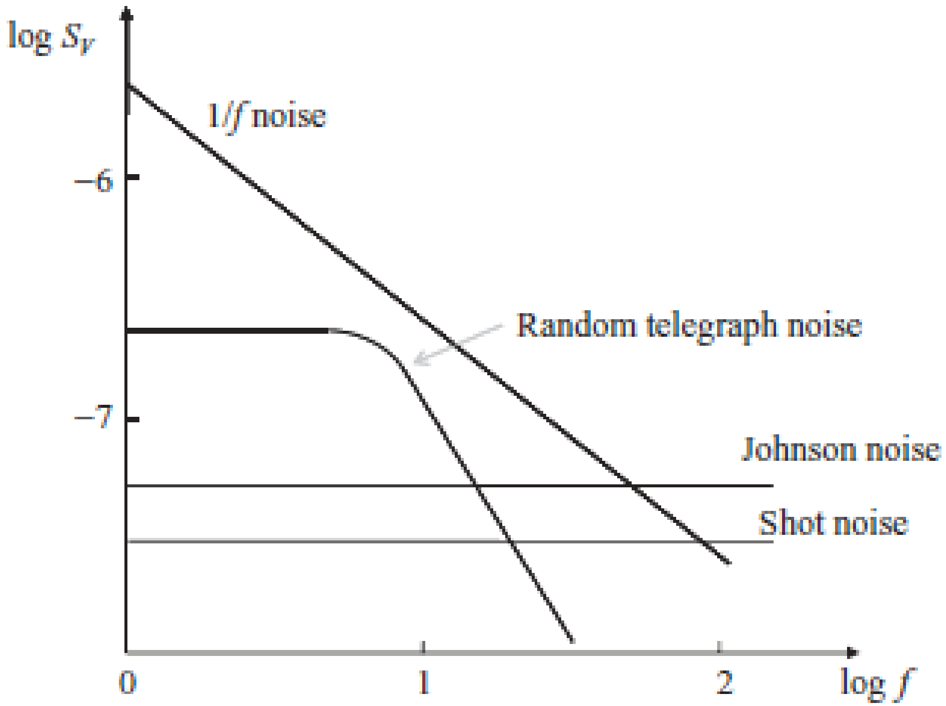
# 3.5 Noise reduction

## Reduction strategy for 1/f noise

In order to reduce the 1/f noise from a sensor, the approach is to modulate the signal at a high enough frequency (~ kHz) for the i/f noise to be reduced below the Johnson noise limit.



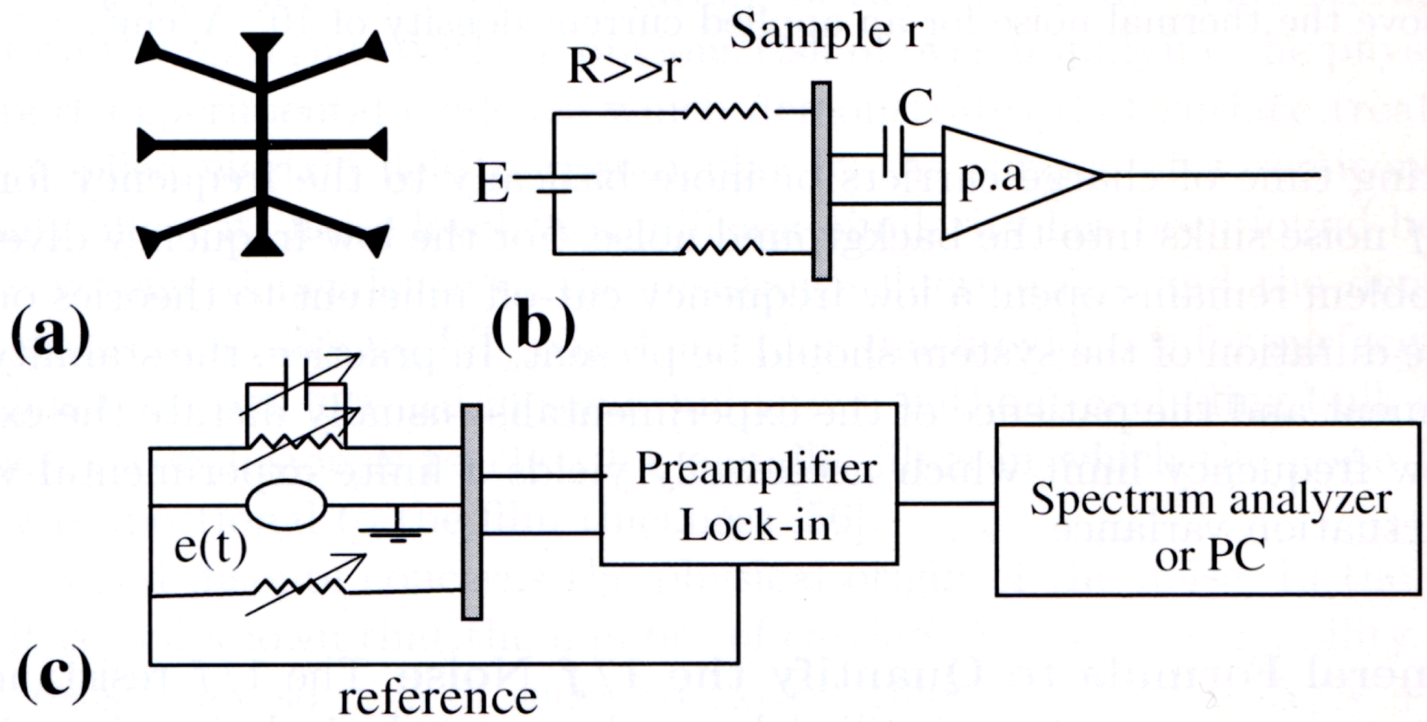

Soft magnetic flux concentrator and modulator. The signal is modulated to avoid the 1/f noise.




## Reduction strategy for Johnson noise

Cool the resistor

# 3.6 Experimental methods




Example of required sample geometry for electrical contacts outside the current lines. (b) Standard 4 probe noise measurement with a dc applied current (p.a = preamplifier). (c) Noise experiment in a 5 probe configuration with a ac applied current [17].