Lecture 2: Principles of Magnetic Sensing

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1. Basic Concepts in Magnetism
2. Sensor Principles

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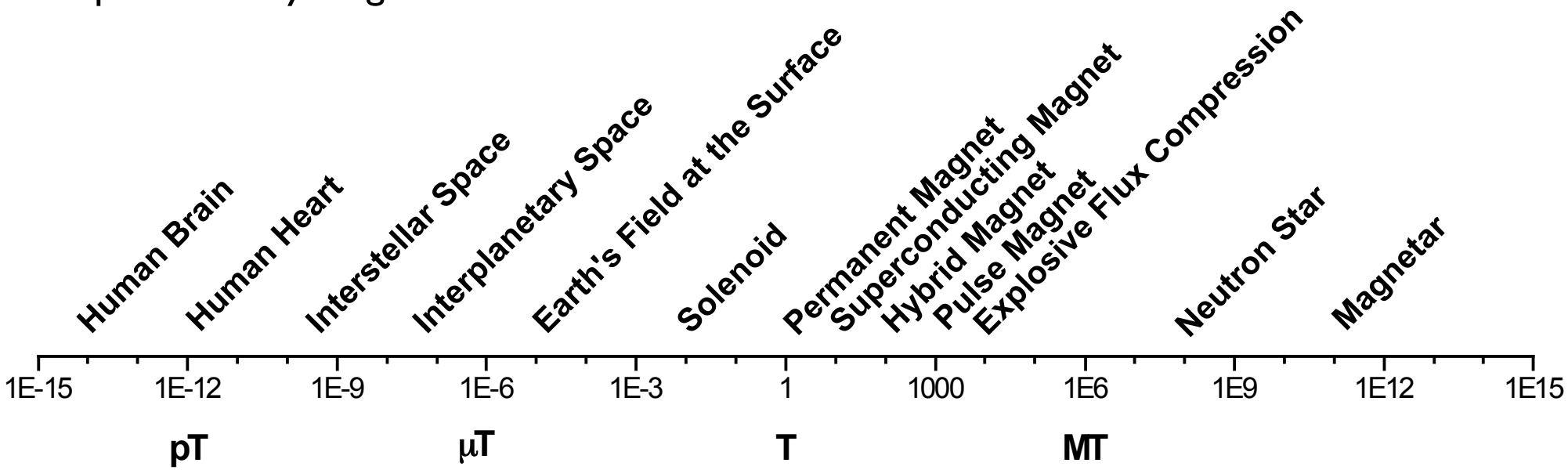
www.tcd.ie/Physics/Magnetism

2.1 Basic concepts in Magnetism

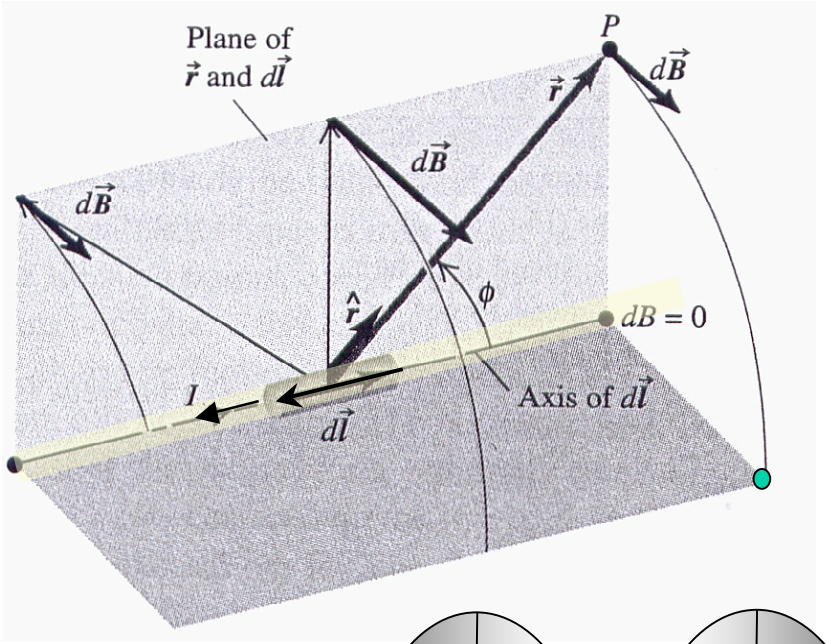
Magnetic field sources are

- distributions of electric current (including moving charged particles)
- time-varying electric fields
- permanently magnetized material



Magnetic fields

Two sources of ***H*** - currents

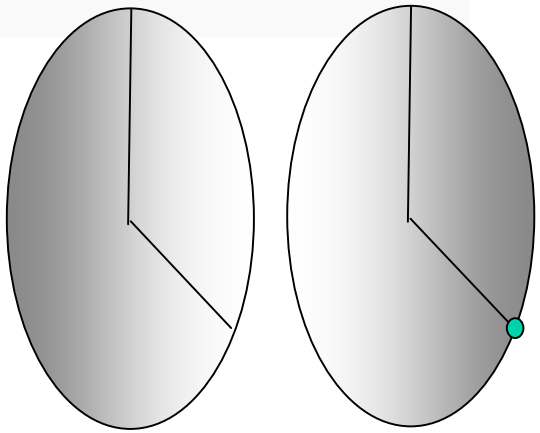


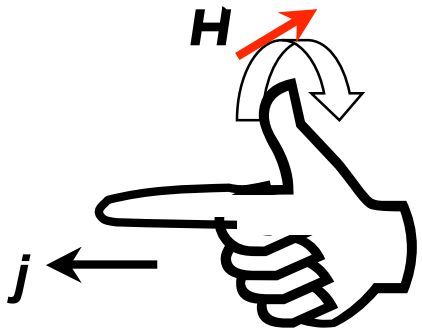
Biot-Savart Law

$$\delta \boldsymbol{H} = -\frac{1}{4\pi}\frac{\boldsymbol{r} \times \boldsymbol{j}}{|r^3|}\delta V$$

$$\delta \boldsymbol{H} = -\frac{1}{4\pi} I \frac{\boldsymbol{r} \times \delta \ell}{|r^3|}$$

Unit of *H* - Am^{-1}





Right-hand corkscrew

Scalar product

$\boldsymbol{m.r}$

$m_x x + m_y y + m_z z$

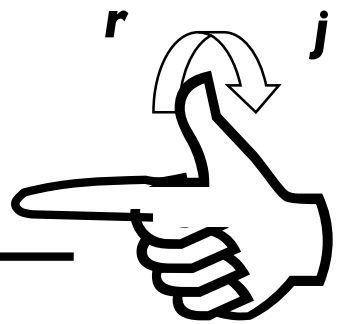
$\nabla.\boldsymbol{B}$

$\partial B_x/\partial x + \partial B_y/\partial y + \partial B_z/\partial z$

The 'divergence' of $\boldsymbol{B}$

$\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

Vector product



$\boldsymbol{r} \times \boldsymbol{j}$

Right-hand rule

$$\begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ x & y & z \\ j_x & j_Y & j_Z \end{vmatrix}$$

$$(yj_z - zj_y)\mathbf{e}_x - (xj_z - zj_x)\mathbf{e}_y + (xj_y - yj_x)\mathbf{e}_z$$

$\nabla \times \boldsymbol{B}$

The 'curl' of $\boldsymbol{B}$

$$\begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x & B_Y & B_Z \end{vmatrix}$$

$$(\partial B_z/\partial y - \partial B_y/\partial z)\mathbf{e}_x - (\partial B_z/\partial x - \partial B_x/\partial z)\mathbf{e}_y + (\partial B_y/\partial x - \partial B_x/\partial y)\mathbf{e}_z$$

In a steady state (no time-dependent electric field)

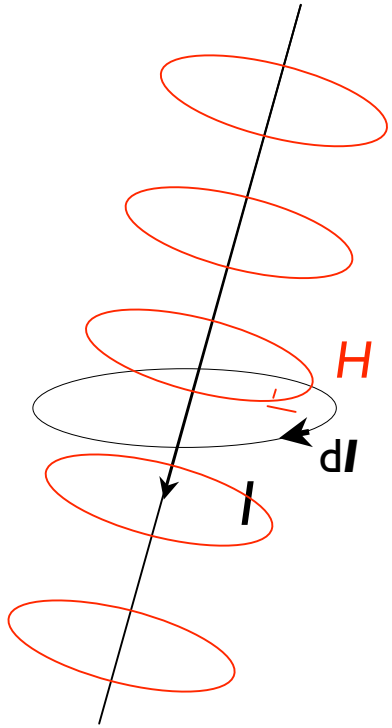
$$\nabla \times \boldsymbol{H} = \boldsymbol{j}$$

$$\int (\nabla \times \boldsymbol{H}) d\mathcal{A} = \int \boldsymbol{j}\, d\mathcal{A}$$ Stokes theorem →

$$\oint \boldsymbol{H}.d\boldsymbol{l} = I$$

$$H = I/2\pi r$$

The field at a distance 5cm m from a wire carrying a current of 1 A is ~ 3 A m^{-1}

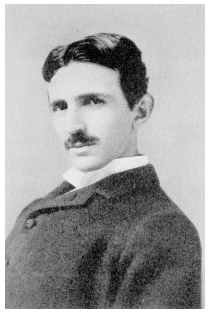


In free space $\boldsymbol{B} = \mu_0 \boldsymbol{H}$

Unit of B - Tesla

Unit of μ_0 T/Am^{-1}

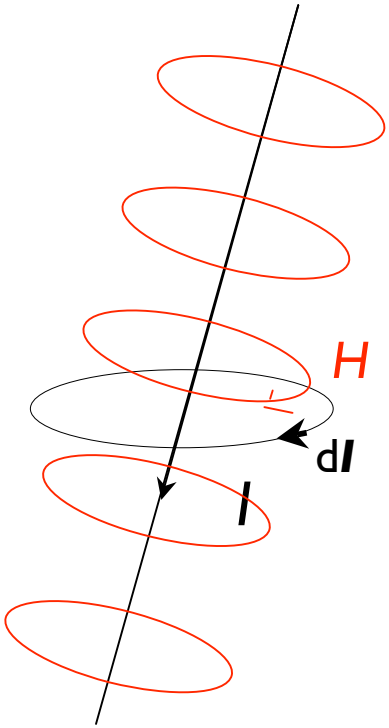
$\mu_0 = 4\pi\ 10^{-7}\ T/Am^{-1}$



1 T = $10^7/\ 4\pi \approx 800{,}000\ Am^{-1}$

$B = \mu_0 I/2\pi r$

The field at a distance 5cm m from a wire carrying a current of 1 A is 4 μT



Two sources of ***H*** - magnets

The magnetic moment ***m*** is the elementary quantity in solid state magnetism.

Define a local moment density - magnetization - ***M***(r,t) which fluctuates wildly on a sub-nanometer and a sub-nanosecond scale.
Define a mesoscopic average magnetization ***M***, averaging over a few nm and ~ 1μs

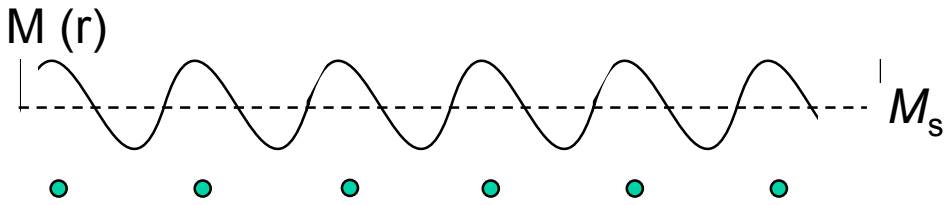
$$\delta m = M \delta V$$

The continuous medium approximation

M can be the spontaneous magnetization $\boldsymbol{M}_s$ within a ferromagnetic domain

A macroscopic average magnetization is the domain average

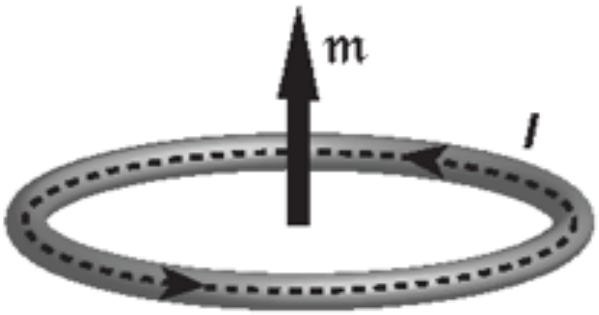
$$\boldsymbol{M} = \Sigma_i \boldsymbol{M}_i V_i / \Sigma_i V_i$$



The mesoscopic average magnetization

The magnetic moment

Ampère: A magnetic moment *m* is equivalent to a current loop.



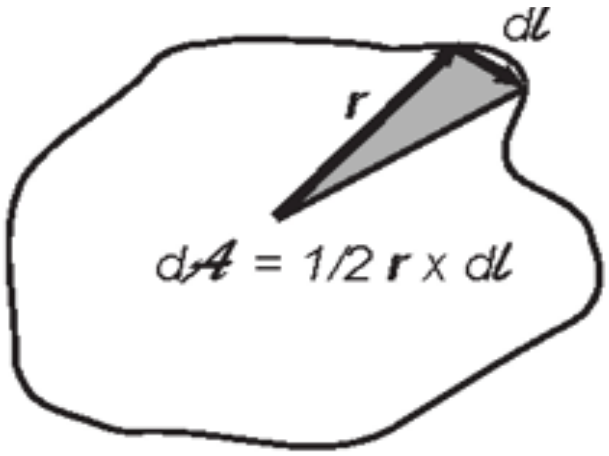
Provided the current flows in a plane

$$\boldsymbol{m} = I\mathcal{A}$$

Units: Am^2

In general:

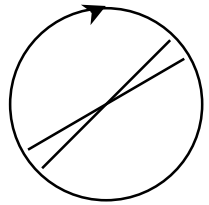
$$\boldsymbol{m} = (1/2)\int \boldsymbol{r} \times \boldsymbol{j}(\boldsymbol{r})\, d^3r$$



where $\boldsymbol{j}$ is the current density; $I = \boldsymbol{j}.\mathcal{A}$

so $\boldsymbol{m} = 1/2\int \boldsymbol{r} \times I d\boldsymbol{l} = I\int d\mathcal{A} = \boldsymbol{m}$

Inversion	Space	Time
Polar vector	$-\boldsymbol{r}$	$\boldsymbol{r}$
Axial vector	$\boldsymbol{m}$	$-\boldsymbol{m}$



Magnetization

The local moment density ***M*** is the magnetization

Units: A m^{-1}

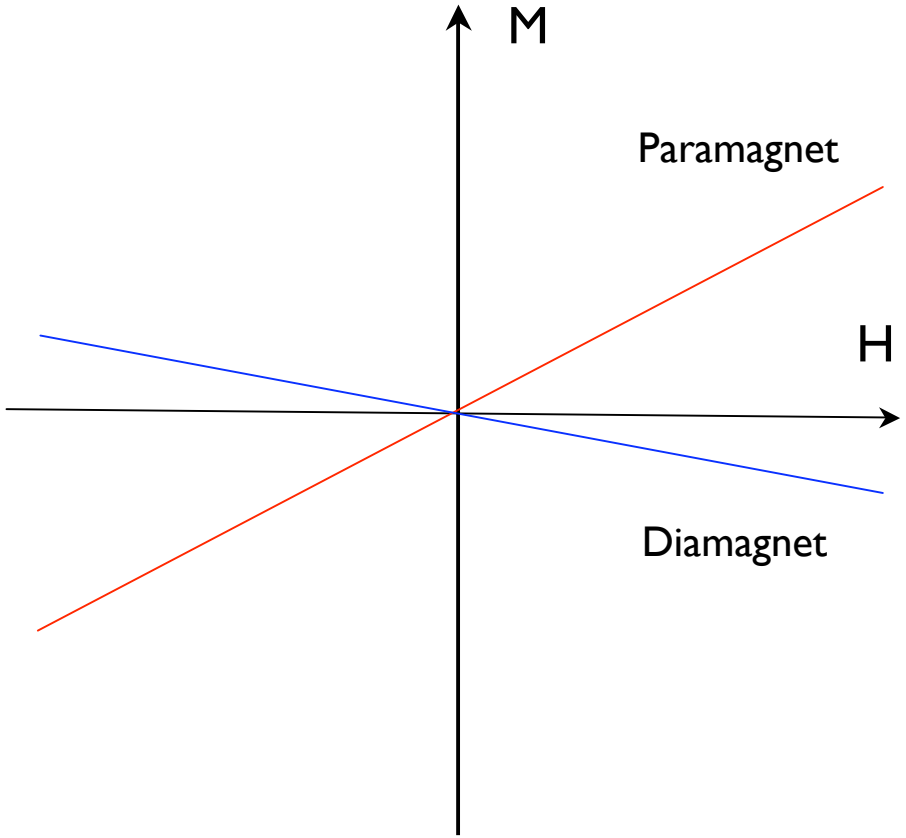
e.g. for iron M = 1710 kA m^{-1}; for $BaFe_{12}O_{19}$ M = 380 kA m^{-1}

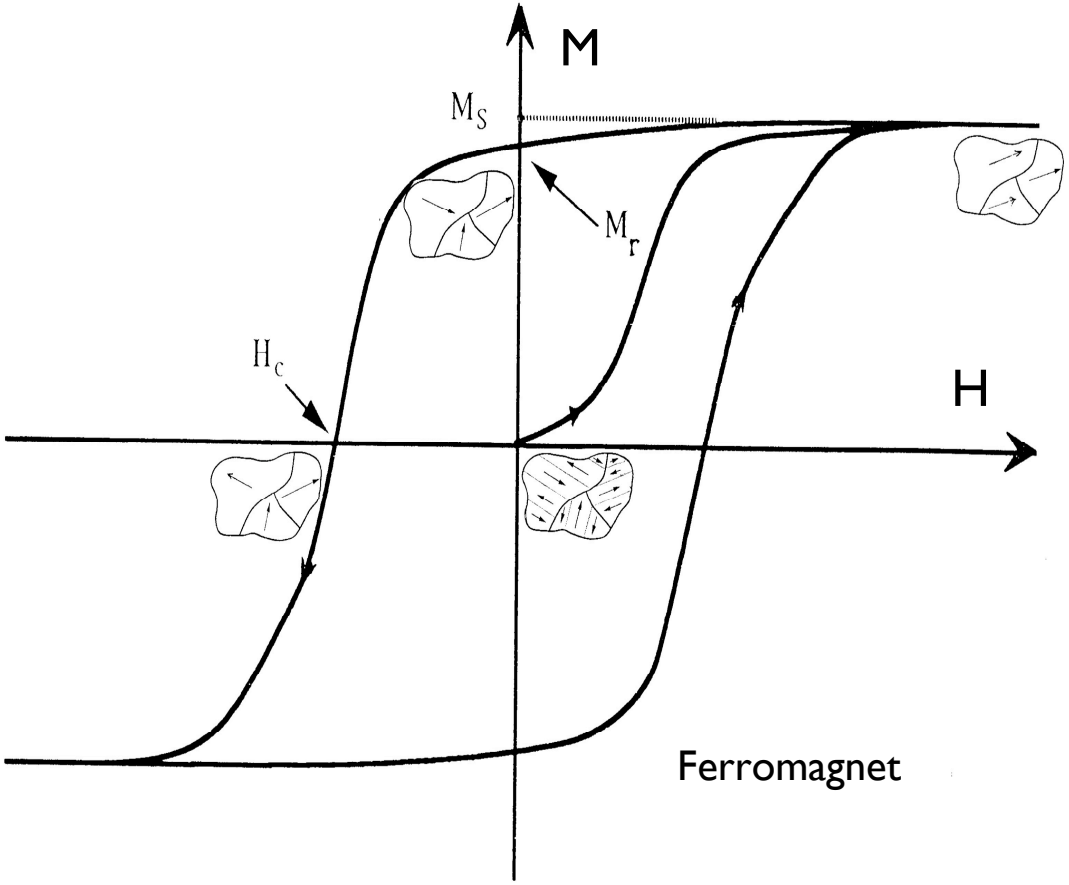
e.g. for a 2.5 cc $BaFe_{12}O_{19}$ fridge magnet (M = 380 kA m^{-1}, V = 2.5 10^{-6} m^3),
$m \approx 1$ A m^2

Magnetization ***M*** can be induced by an applied field *or* it can arise spontaneously within a ferromagnetic domain, $\boldsymbol{M}_s$. A macroscopic average magnetization is the domain average

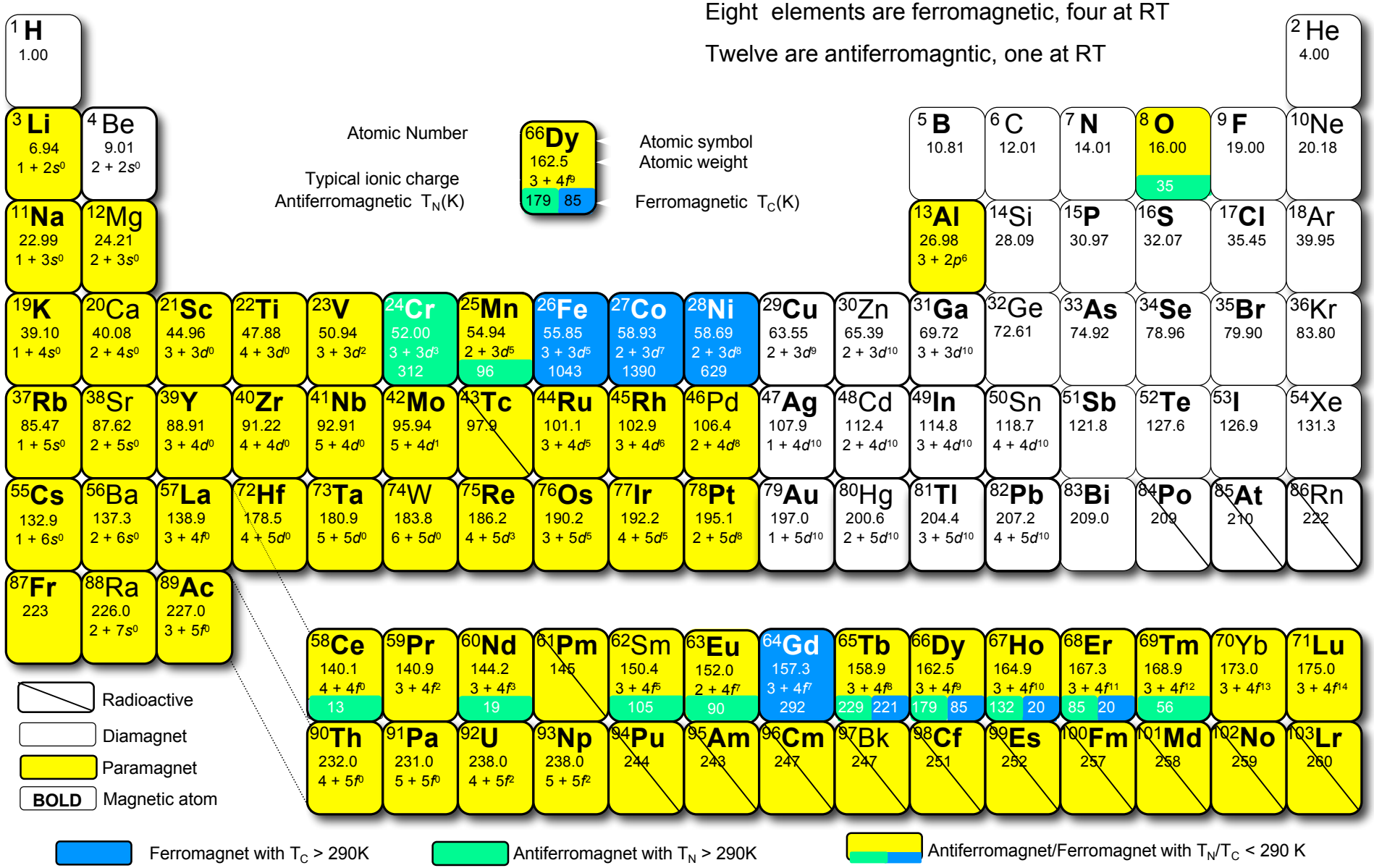
The equivalent Amperian current density is

$$\boldsymbol{j}_M = \nabla \times \boldsymbol{M}$$

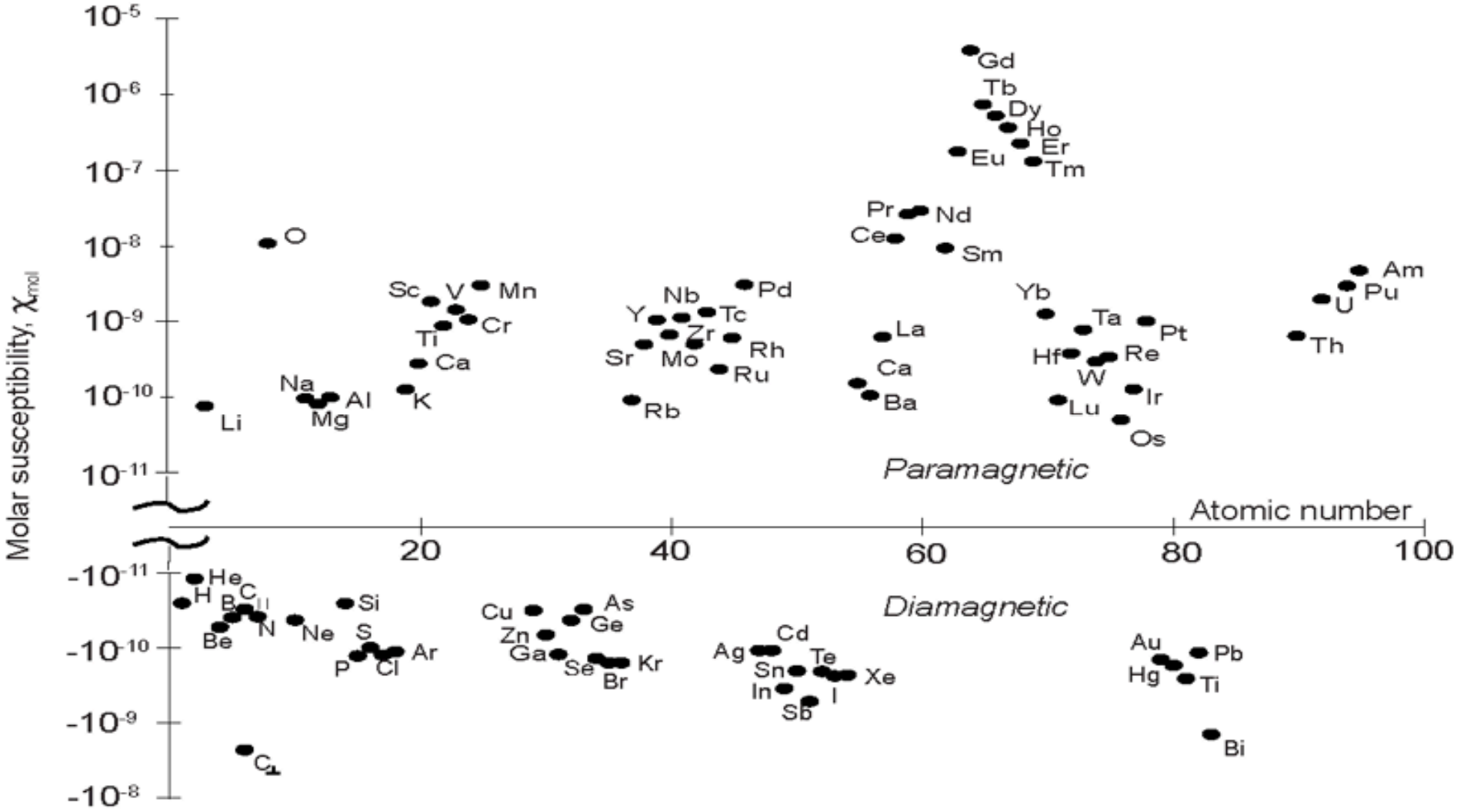

M
H
Paramagnet
Diamagnet


M
H
M_S
M_r
H_c
Ferromagnet

The Magnetic Periodic Table



Susceptibilities of the elements



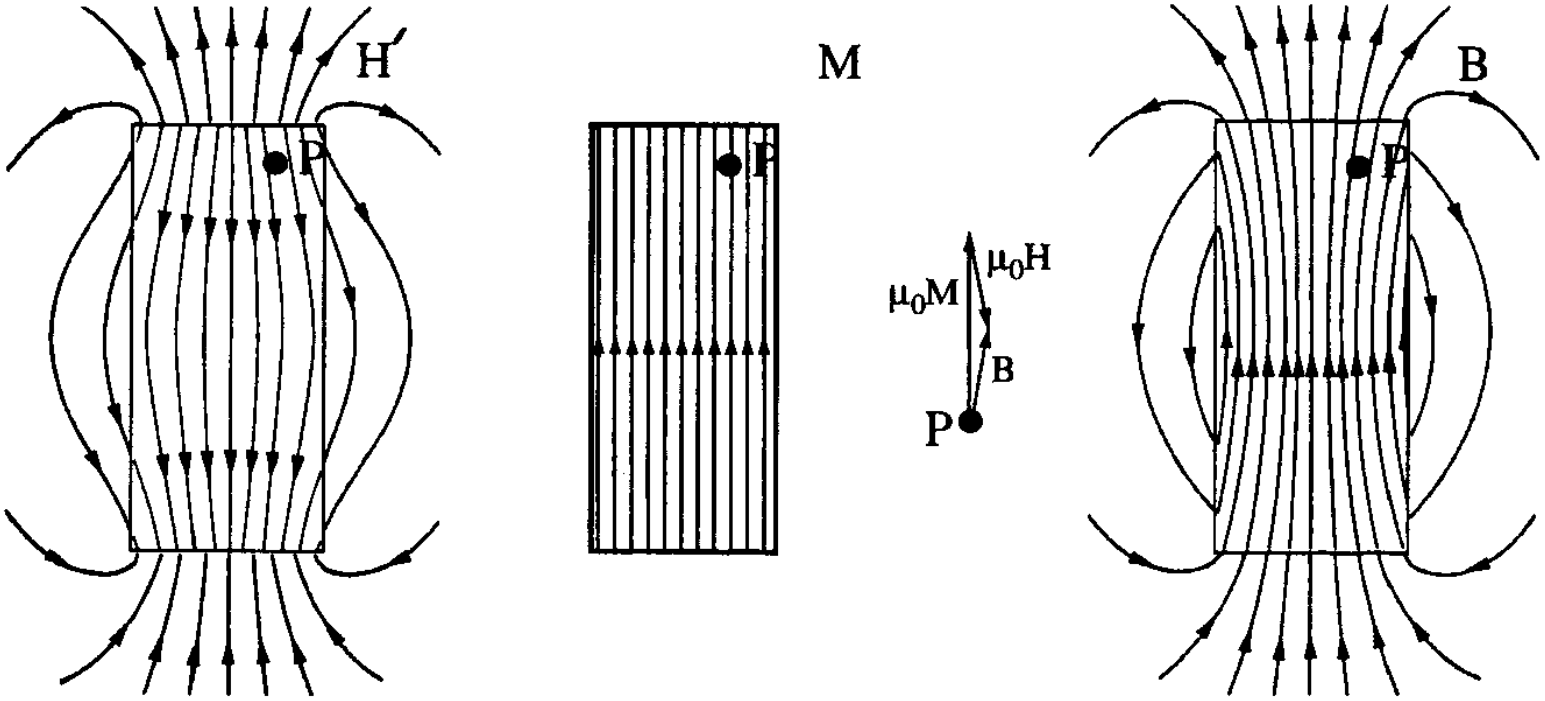
B , *H* and *M*

The equation used to *define* ***H*** is $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ i.e. $\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$

We call the *H*-field due to a magnet; — *stray field* outside the magnet

— *demagnetizing field,* H_d, inside the magnet

Units: Am^{-1}



The total *H*-field at any point is $\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_m$ where $\boldsymbol{H}'$ is the applied field

The ***B*** field - magnetic induction/magnetic flux density

$\nabla . \boldsymbol{B} = 0$ Significance; It is the *fundamental* magnetic field

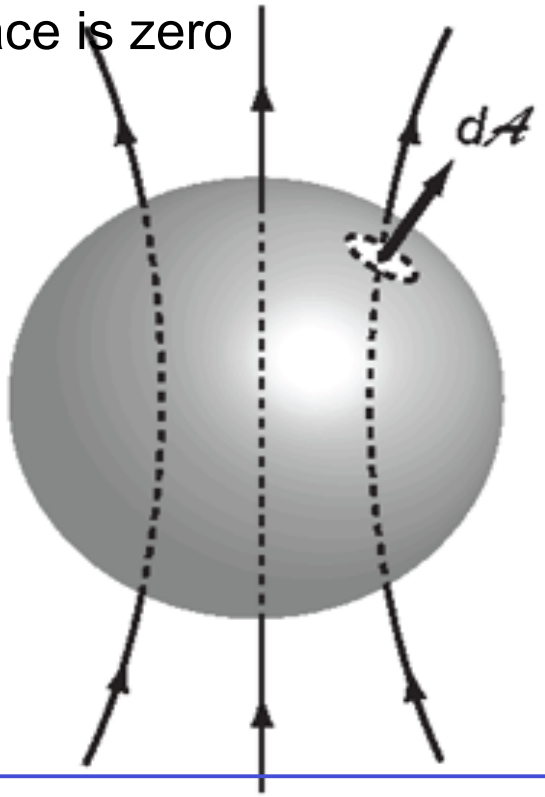
There are no sources or sinks of ***B*** i.e *no monopoles*

Magnetic vector potential $\boldsymbol{B} = \nabla \times \boldsymbol{A}$

The gradient of any scalar ϕ, $\nabla\phi$ may be added to ***A*** without altering ***B***

Gauss's theorem: The net flux of B across any closed surface is zero

$$\int_S \boldsymbol{B}.d\boldsymbol{\mathcal{A}} = 0$$



Magnetic flux $d\Phi = \boldsymbol{B}.d\boldsymbol{\mathcal{A}}$ Units: Weber (Wb)

Flux quantum $\Phi_0 = 2.07\ 10^{15}$ Wb

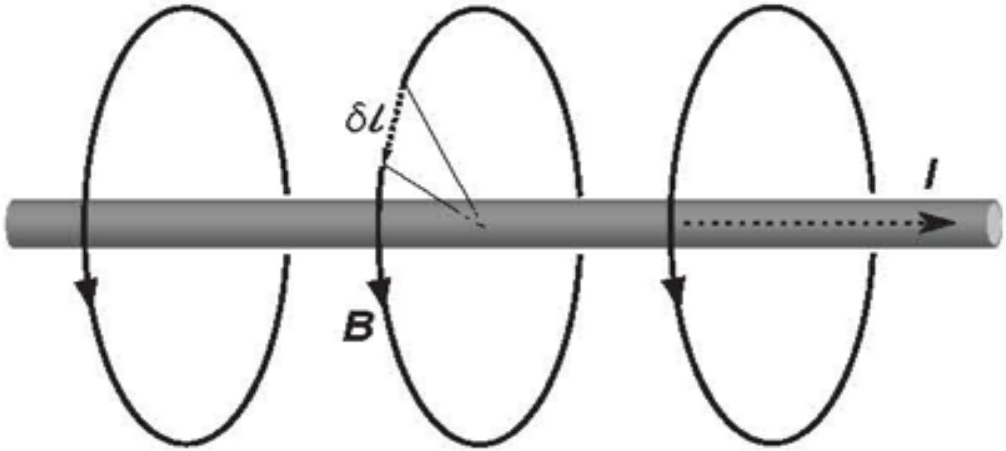
The equation $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}$ valid in static conditions gives:

Ampere's law $\int \boldsymbol{B}.\mathrm{d}\boldsymbol{l} = \mu_0 I$ for any closed path

Good for calculating the field for very symmetric current paths.

Example: the field at a distance r from a current-carrying wire

$B = \mu_0 I/2\pi r$



B interacts with any *moving* charge:

Lorentz force $\boldsymbol{f} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$

The ***H*** field

Significance; The magnetization of a solid reflects the local value of ***H.***

In free space $\boldsymbol{B} = \mu_0 \boldsymbol{H}$; $\quad \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}_c$

In a medium $\boldsymbol{B} = \mu_0 \mu_r \boldsymbol{H}$ (linear isotropic media only!)

$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ (general case)

$\nabla \times \boldsymbol{B} = \mu_0(\boldsymbol{j}_c + \boldsymbol{j}_M)$,

but $\boldsymbol{j}_M = \nabla \times \boldsymbol{M}$

Define $\boldsymbol{H} = (1/\mu_0)\, \boldsymbol{B}$ - M.

hence $\boxed{\nabla \times \boldsymbol{H} = \boldsymbol{j}_c}$ This is *useful.* We cannot measure $\boldsymbol{j}_M$

$$\boxed{\oint \boldsymbol{H}.d\boldsymbol{l} = \boldsymbol{I}_c}$$

Ampere's law for ***H***

The ***H*** field

Significance; The magnetization of a solid reflects the local value of ***H.***

In free space $\boldsymbol{B} = \mu_0 \boldsymbol{H}$.

$\nabla \times \boldsymbol{B} = \mu_0(\boldsymbol{j}_c + \boldsymbol{j}_M)$, $\boldsymbol{j}_M = \nabla \times \boldsymbol{M}$ hence $\nabla \times \boldsymbol{H} = \boldsymbol{j}_c$ $\int \boldsymbol{H}.d\boldsymbol{l} = \boldsymbol{I}_c$

Ampere's law for ***H***

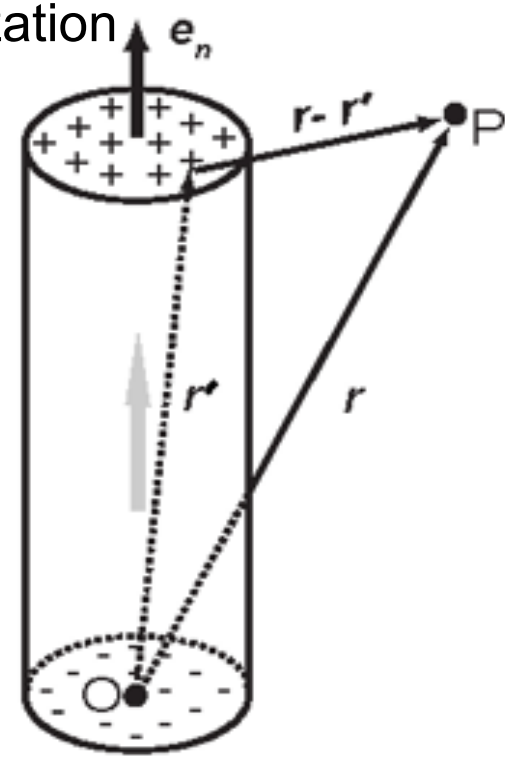
Coulomb approach to calculate H

H has sources and sinks associated with nonuniform magnetization

$$\nabla.\boldsymbol{H} = -\nabla.\boldsymbol{M}$$

Imagine ***H*** due to a distribution of magnetic charges q_m. (Am)

Field of a point 'charge' $\boldsymbol{H} = q_m \boldsymbol{e}_r / 4\pi r^2$



Magnetization distribution is replaced by

- surface charge distribution $\sigma_m = \boldsymbol{M}.\boldsymbol{e}_n$
- volume charge distribution $\rho_m = -\nabla.\boldsymbol{M}$

Magnetic scalar potential

When H is due only to magnets i.e $\nabla \times \boldsymbol{H} = 0$
we can define a scalar potential φ_m (Units are Amps)
Such that

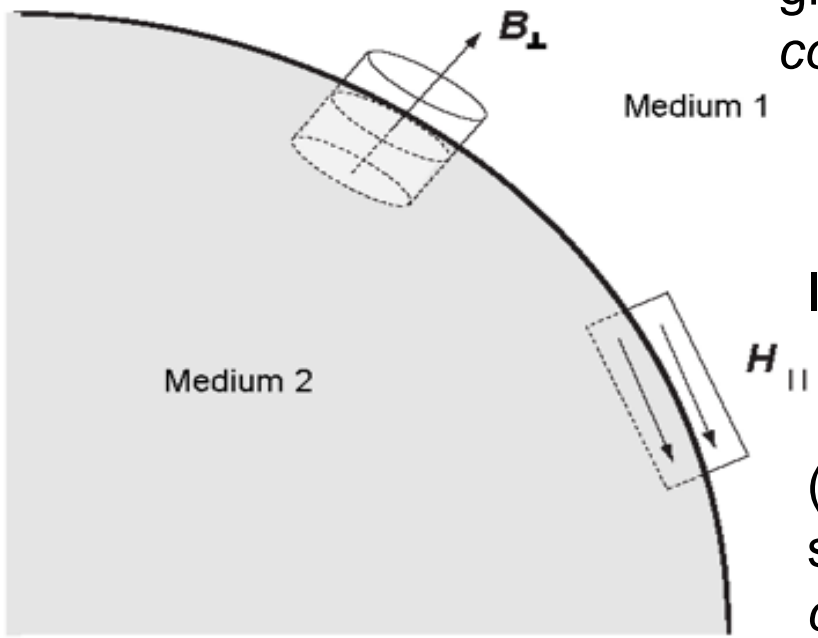
$\boldsymbol{H} = -\nabla\varphi_m$

The potential of charge q_m is $\varphi_m = q_m / 4\pi r$

If currents are present, this cannot be done.

Poisson's equation $\nabla^2\varphi_m = \nabla . \boldsymbol{M}$

2.4 Boundary conditions



Gauss's law $\int_S \boldsymbol{B}.d\boldsymbol{\mathcal{A}} = 0$
gives that *the perpendicular component of* ***B*** *is continuous.*

$$(\boldsymbol{B}_1-\boldsymbol{B}_2).\boldsymbol{e}_n=0$$

It follows from from Ampère's law

$$\int_{loop} \boldsymbol{H}.d\boldsymbol{l} = I_c = 0$$

(there are no conduction currrents on the surface) that *the parallel component of H is continuous.*

$$(\boldsymbol{H}_1-\boldsymbol{H}_2) \times \boldsymbol{e}_n=0$$

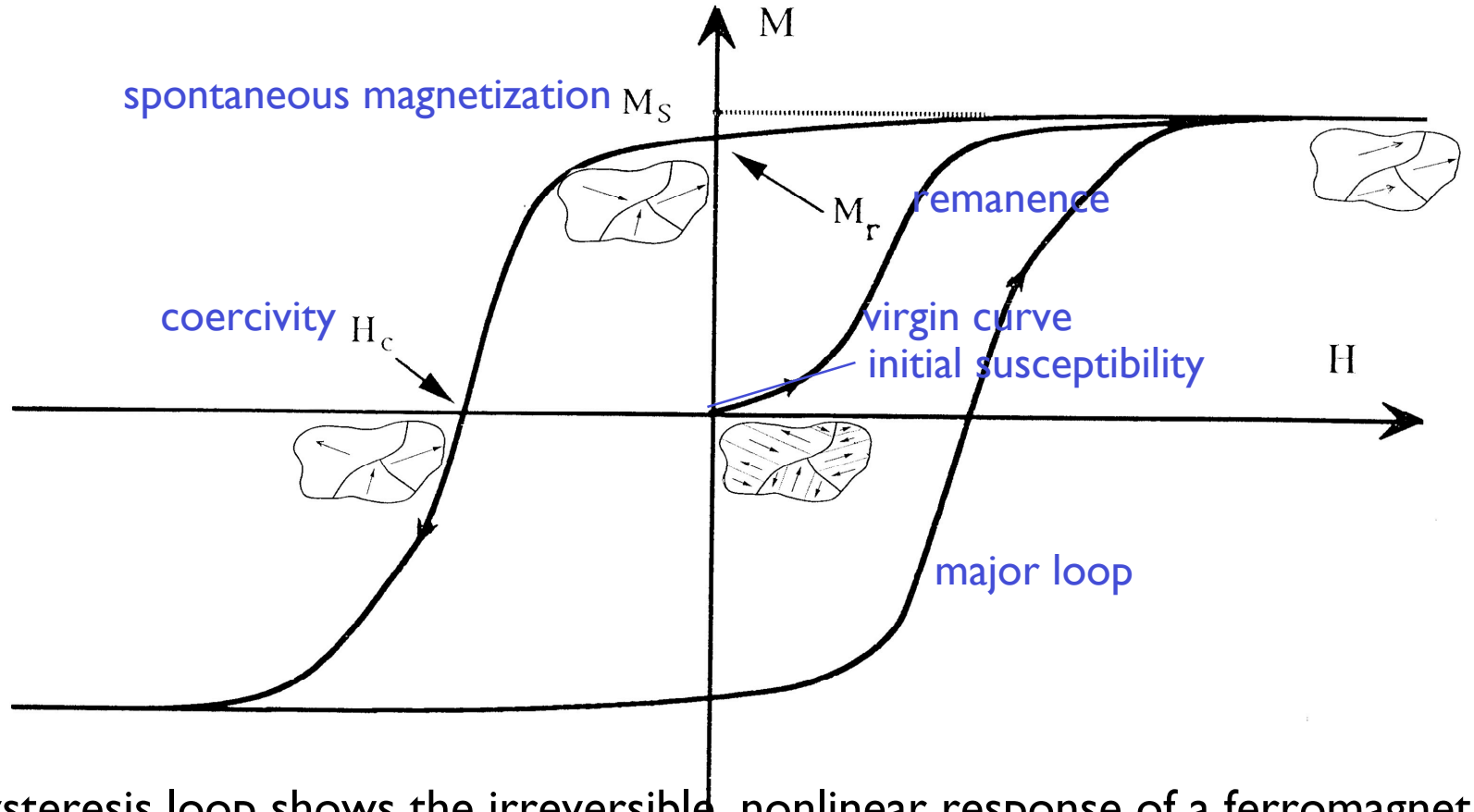
Conditions on the potentials

Since $\int_S \boldsymbol{B}.d\boldsymbol{\mathcal{A}} = \int_{loop} \boldsymbol{A}.d\boldsymbol{l}$ (Stoke's theorem)

$$(\boldsymbol{A}_1 - \boldsymbol{A}_2) \times \boldsymbol{e}_n = 0$$

The scalar potential is continuous $\varphi_{m1} = \varphi_{m2}$

Hysteresis



The hysteresis loop shows the irreversible, nonlinear response of a ferromagnet to a magnetic field $\boldsymbol{M} = \boldsymbol{M}(\boldsymbol{H})$. It reflects the arrangement of the magnetization in ferromagnetic *domains*. The magnet cannot be in thermodynamic equilibrium anywhere around the open part of the curve! It reflects the arrangement of the magnetization in ferromagnetic *domains*. The $\boldsymbol{B} = \boldsymbol{B}(\boldsymbol{H})$ loop is deduced from the relation $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$.

Maxwell's equations in a material medium

$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \cdot \boldsymbol{D} = \rho$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{j}_c + \partial \boldsymbol{D}/\partial t$$
$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$$

Written in terms of the four fields, these are valid in any medium.
In vacuum $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$, $\boldsymbol{H} = \boldsymbol{B}/\mu_0$,
ρ is charge density (C m^{-3}), $\boldsymbol{j}_c$ is conduction current density (A m^{-2})

In m*agnetostatics* there is no time-dependence of $\boldsymbol{B}$, $\boldsymbol{D}$ or ρ

Conservation of charge $\nabla . \boldsymbol{j} = -\partial\rho/\partial t$. In a steady state $\partial\rho/\partial t = 0$

Magnetostatics: $\nabla . \boldsymbol{j} = 0$; $\nabla . \boldsymbol{B} = 0$ $\nabla \times \boldsymbol{H} = \boldsymbol{j}_c$

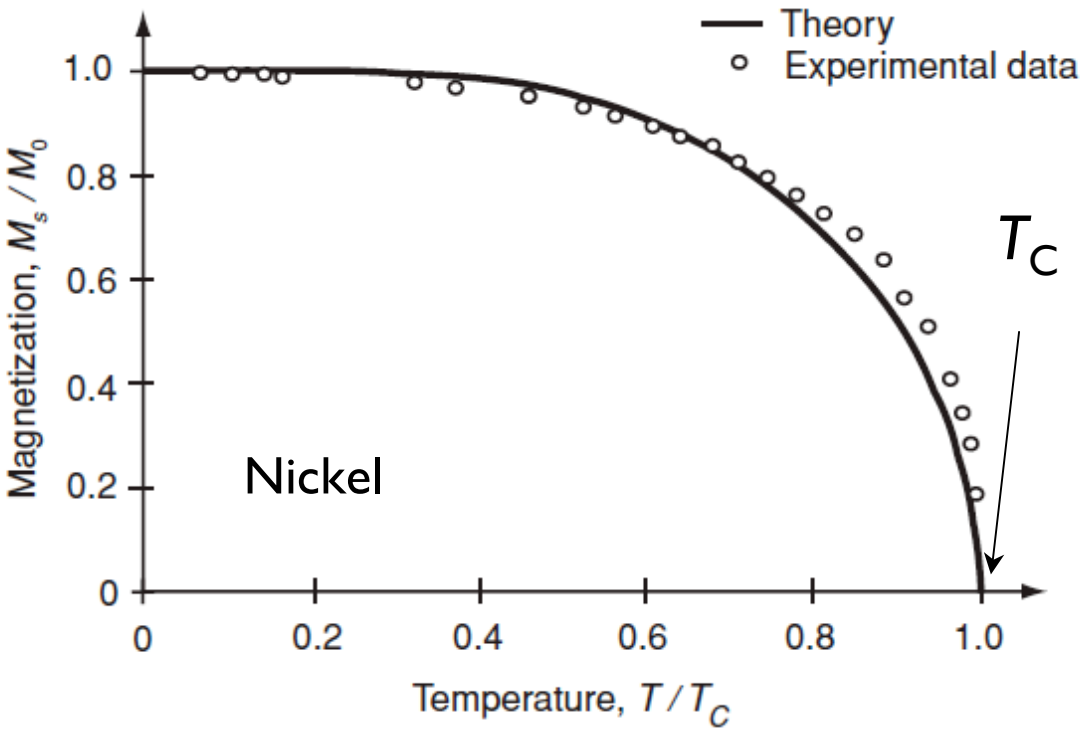
Constituent relations: $\boldsymbol{j}_c = \boldsymbol{j}(\boldsymbol{E})$; $\boldsymbol{P} = \boldsymbol{P}(\boldsymbol{E})$; $\boldsymbol{M} = \boldsymbol{M}(\boldsymbol{H})$

Magnetic materials

Ferromagnets and ferrimagnets have spontaneous magnetization within a domain.

The magnetization falls with increasing temperature, first gradually, then abruptly at the Curie point T_C.

Material	T_C (K)	M_s kAm^{-1}
Fe	1044	1710
Ni	628	488
$Ni_{80}Fe_{20}$	843	830
Co	1360	1443
Fe_3O_4	860	480
$BaFe_{12}O_{19}$	740	380
$SmCo_5$	1020	856
$Nd_2Fe_{14}B$	588	1290



Iron Fe

bcc; $a_0 = 287$ pm

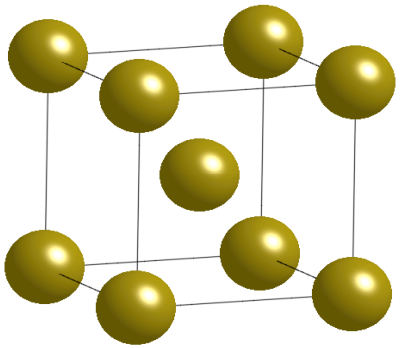
The most important ferromagnetic material.

Main constituent of the whole Earth, 5 wt % of crust.

Usually alloyed with 6 at% Si and fabricated in 300 µm rolled laminations (isotropic or grain oriented), castings or reduced powder,

Mainly used in electrical machines (motors, transformers) and magnetic circuits.

Production 5 Mt/yr for magnetic purposes (8 B€)



J_s = 2.0 T (Si-Fe)

M_s = 1.71 MA m^{-1} (Fe)

T_C = 1044 K (Fe)

K_1 = 48 kJ m^{-3} (Fe)

λ_s = -8 10^{-6}

Permalloy $Fe_{20}Ni_{80}$

fcc; a_0 = 324 pm

Multipurpose soft magnetic material, with near-zero anisotropy *and* magnetostriction

Sometimes alloyed with Mo, Cu …

Sputtered or electrodeposited films, sheet, powder.

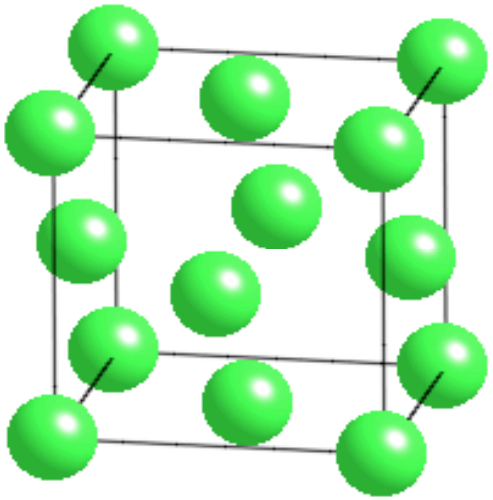
Uses: magnetic recording; write heads, read heads (AMR), magnetic shields, transformer cores

J_s = 1.0 T M_s = 0.8 MA m^{-1}

$K_1 \approx 2$ kJ m^{-3} $\lambda_s = 2\ 10^{-6}$

T_C = 843 K

Compositions near $Fe_{50}Ni_{50}$ have larger J_s but greater anisotropy



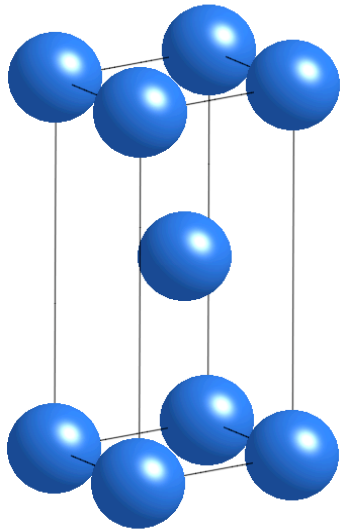
Cobalt Co

hcp; a = 251 pm, c = 407 pm

Highest-T_C ferromagnet, anisotropic, expensive (¤50 /kg), strategic.

Useful alloying addition

Sputtered nanocrystalline thin films (with Cr, Pt, B additions) are used as media for hard discs



J_s = 1.8 T M_s = 1.44 MA m^{-1}

K_1 = 530 kJ m^{-3}

T_C = 1360 K

Magnetite, Fe_3O_4

spinel; a_0 = 839 pm

Most common magnetic mineral, source of rock magnetism, main constituent of lodestones..

A ferrimagnet. with Fe^{2+} and Fe^{3+} disordered on B -site above the Verwey transition at T_v = 120 K, ordered below; A-B superexchange is the main magnetic interaction

$$[Fe^{3+}]_{tett}\ \{Fe^{2+}\ Fe^{3+}\}_{oct}\ O_4$$

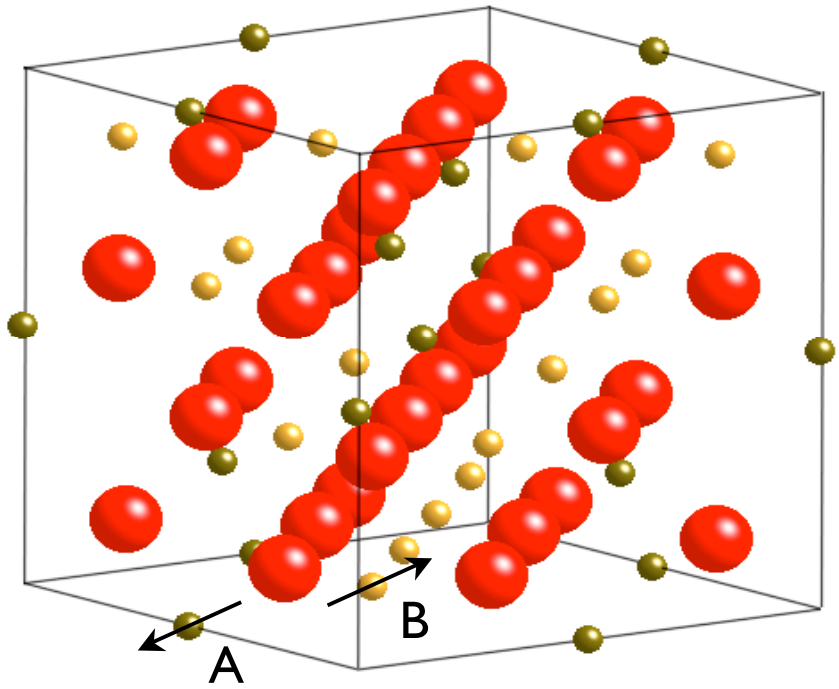
$$\downarrow \qquad \uparrow \qquad \uparrow$$

$$-5\ \mu_B \ + 4\ \mu_B \ +5\ \mu_B \ = \ 4\ \mu_B$$

A half-metal. Fe(B); ↓ electrons hop in a t_{2g} band

Used as toner, and in ferrofluids.

Potential for spin electronics..



$[A]\{B_2\}O_4$

J_s = 0.60 T $\qquad\qquad$ m_0 = 4.0 μ_B / fu

K_1 = -13 kJ m^{-3} $\qquad$ λ_s = 40 10^{-6}

T_C = 843 K

$BaFe_{12}O_{19}$; Hexaferrite

magnetoplumbite; a = 589 pm c = 2319 pm

An hcp lattice of oxygen and Ba, with iron in octahedral (12k, $4f_2$, 2a) tetrahedral ($4f_1$) and trigonal bipyramidal (2b) sites.

Brown ferrimagnetic insulator. All magnetic ions are Fe^{3+}. Also $SrFe_{12}O_{19}$ and La/Co substitution.

Structure is 12k↑2a↑2b↑$4f_1$↓$4f_2$↓

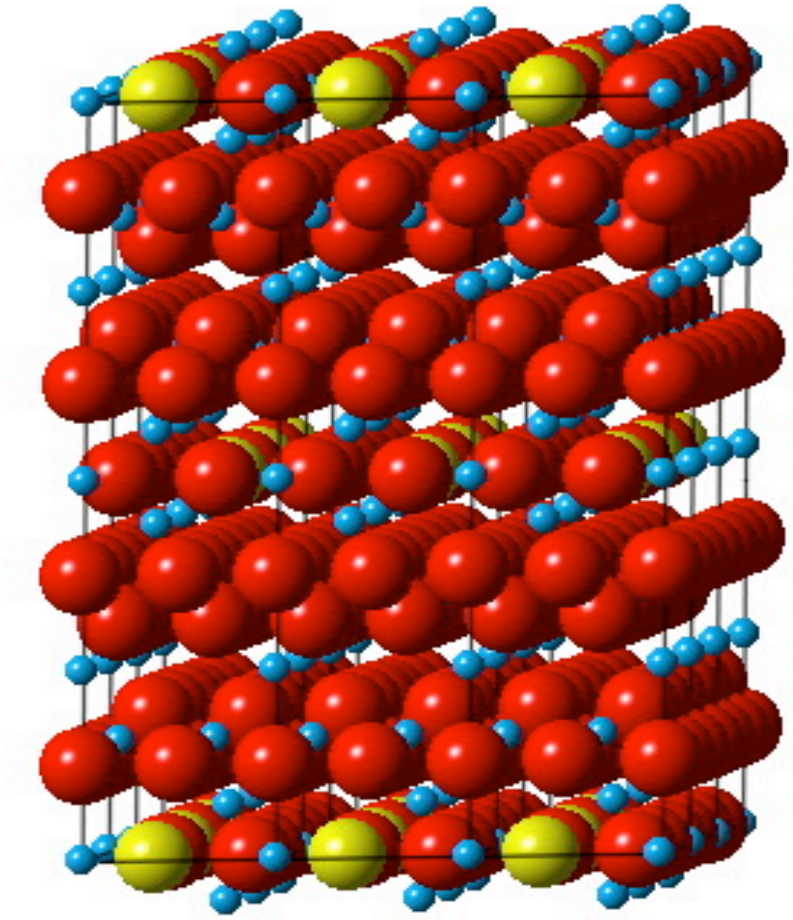
T_C = 740 K.

m_0 = 20 μ_B / fu

Low-cost permanent magnet, the first magnet to break the 'shape barrier'. 98% of all permanent magnets by mass are Ba or Sr ferrite. Found on every fridge door and in innumerable catches, dc motors, microwave magnetrons, …

80g manufactured per year for everyone on earth

J_s = 0.48 T . K_1 = 450 kJ m^{-3}. B_a = 1.7 T



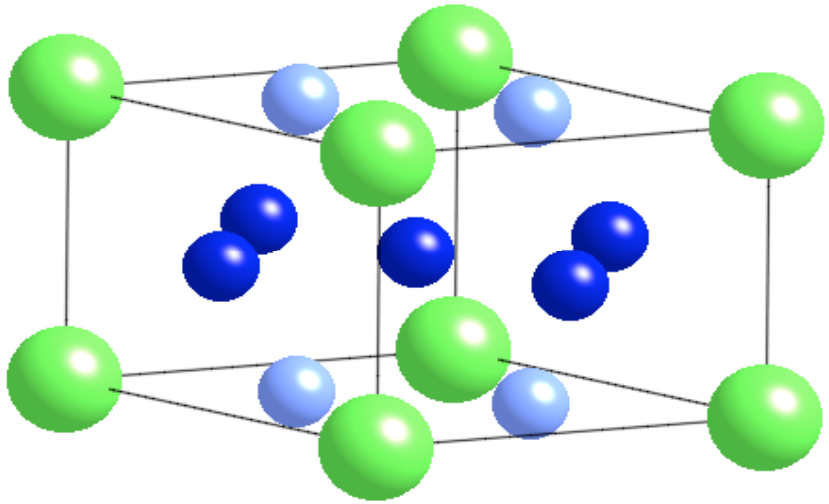
Samarium-cobalt $SmCo_5$

hexagonal; a = 499 pm c = 398 pm

Versatile, high-temperature permanent magnet.
Cellular intergrowth with Sm_2Co_{17} in
$Sm(Co, Fe, Zr, Cu)_{7.6}$ alloys provides
domain-wall pinning
Dense sinterered oriented material.
Uses: specialised electrical drives
Expensive (≈150 ¤/kg)

J_r = 1.0 T $(BH)_{max}$ = 200 kJ/m^3
K_1 = 17 MJ m^{-3} B_a = 30 T
T_C = 1020 K

R-T exchange is direct, between the 5d and 3d shells
This is antiferromagnetic; on-site coupling of 5d and 4f spins is ferromagnetic, hence moments couple parallel for light rare earths (J = L - S) and antiparallel for heavy rare earths (J = L + S).
Useful alloys are of Pr, Nd, Sm with Fe, Co, Ni



Neomax, $Nd_2Fe_{14}B$

tetragonal; a = 879 pm, c = 1218 pm

The highest-performance permanent magnet.

Discovered in 1983 by Sagawa (sintered) and by Croat and Herbst (melt spun)

Dy, Co .. substitutions

Dense sinterered oriented material, melt-spun isotropic flakes.

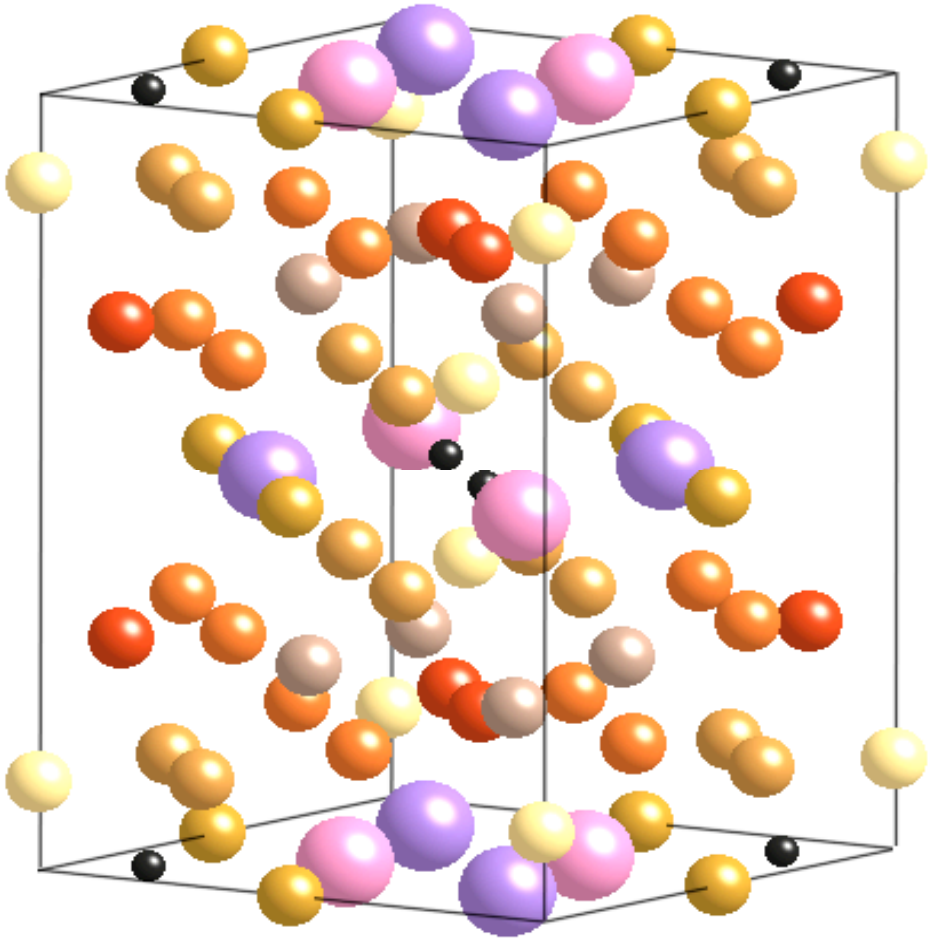
Voice-coil actuators, spindle motors, nmr imaging, flux sources ….

Cost ≈ 30 €/kg, Production 50 kT/yr (1.5 B€)

J_r = 1.4 T $(BH)_{max}$ = 200-400 kJ/m^3

K_1 = 4.9 MJ m^{-3} B_a = 7.7 T

T_C = 878 K

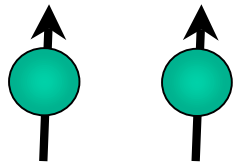


Exchange interactions.

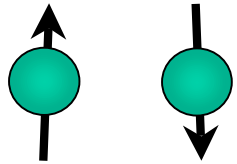
The interaction responsible for magnetic order is exchange. Basically it is a Coulomb interaction between the charges of electrons on adjacent ions 1, 2, subject to the symmetry constraints of quantum mechanics. It written in terms of their *spins*.

Heisenberg-Dirac Hamiltonian $\mathcal{H} = -2 J \boldsymbol{S}_1 . \boldsymbol{S}_2$

$J > 0$, *ferromagnetic*



$J < 0$, anti*ferromagnetic*.



Curie or Néel temperature $T_c \approx 2Z J S(S+1)/3k_B$

Exchange interactions.

The magnetic coupling in a ferromagnet can be represented by a 'magnetic stiffness' *A*

$$E_{ex} = \int A(\nabla \mathbf{e}_M)^2 d^3r$$

$\mathbf{e}_M = \mathbf{M}(\mathbf{r})/M_s$ $\quad (\theta,\phi)$

$$E_{ex} = \int A[(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2]d^3r$$

$(\nabla \mathbf{e}_{Mx})^2 + (\nabla \mathbf{e}_{My})^2 + (\nabla \mathbf{e}_{Mz})^2$

$$l_{ex} = \sqrt{\frac{A}{\mu_0 M_s^2}}$$

Exchange length

$A = kT_C/2a$

$A = 2JS^2Z_c/a_0$

$A \sim 10$ pJ m^{-1}

$l_{ex} \sim 2$ - 3 nm

Demagnetizing field

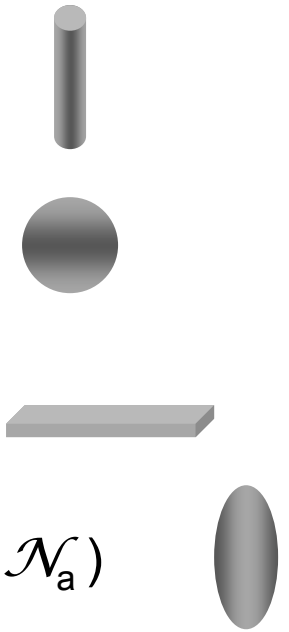
The H-field in a magnet depends $\boldsymbol{M}(\boldsymbol{r})$ and on the *shape* of the magnet.
$\boldsymbol{H}_d$ is uniform only in the case of a *uniformly-magnetized ellipsoid*.

$$(\boldsymbol{H}_d)_i = -\mathcal{N}_{ij}\boldsymbol{M}_j \qquad i,j = x,y,z$$

$$\mathcal{N}_x + \mathcal{N}_y + \mathcal{N}_z = 1$$

Demagnetizing factors for some simple shapes

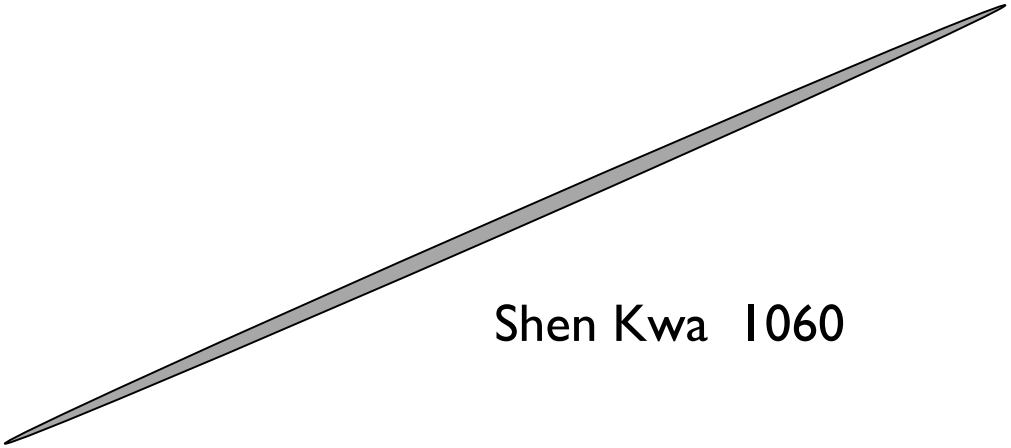
Long needle, $\boldsymbol{M}$ parallel to the long axis	0
Long needle, $\boldsymbol{M}$ perpendicular to the long axis	1/2
Sphere, $\boldsymbol{M}$ in any direction	1/3
Thin film, $\boldsymbol{M}$ parallel to plane	0
Thin film, $\boldsymbol{M}$ perpendicular to plane	1
General ellipsoid of revolution (a,a,c)	$\mathcal{N}_c = (1 - 2\mathcal{N}_a)$



The shape barrier.

Daniel Bernouilli
1743

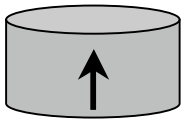
$N < 0.1$

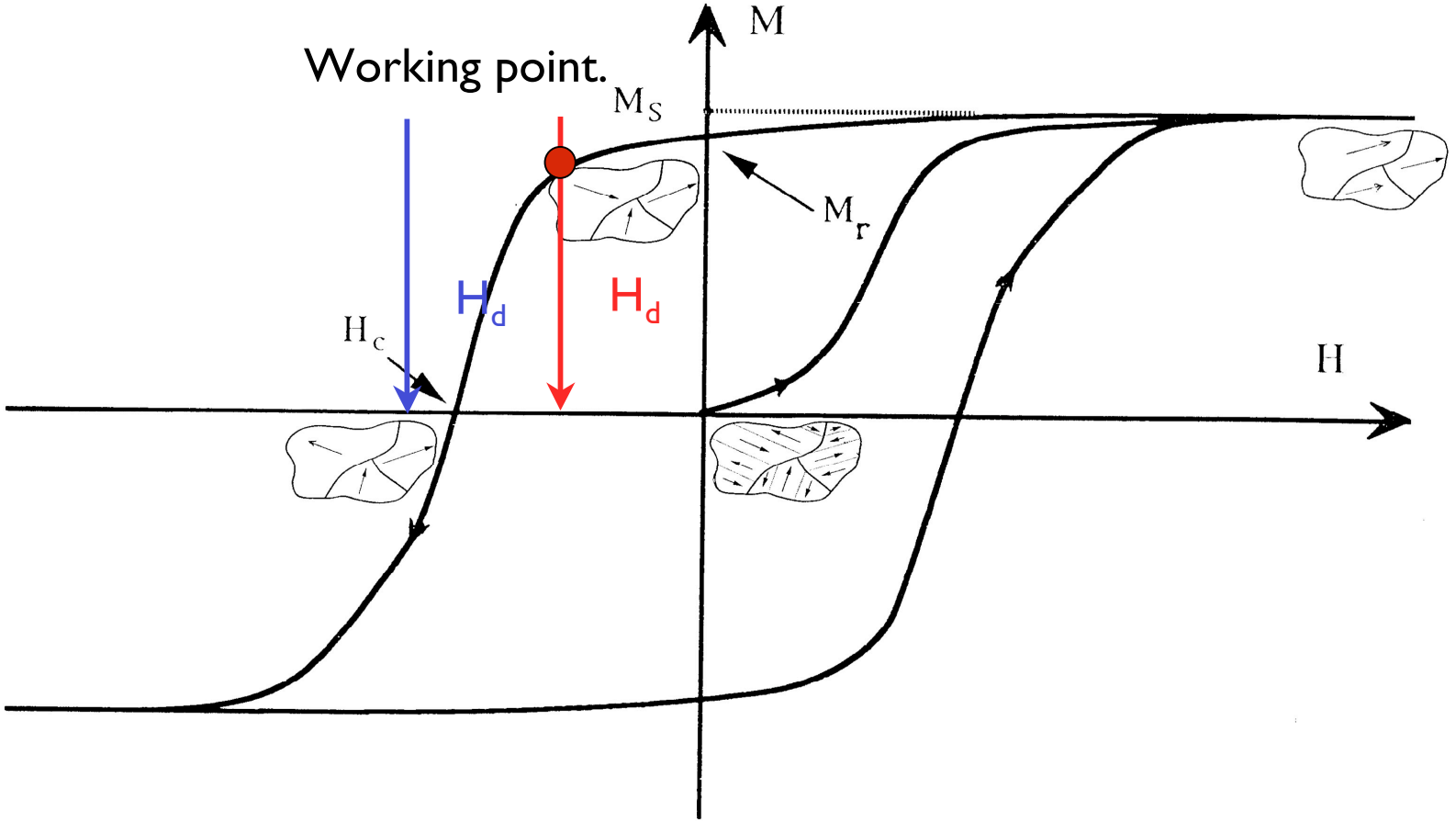


Shen Kwa 1060

Gowind Knight 1760

New icon for permanent magnets! ⇒



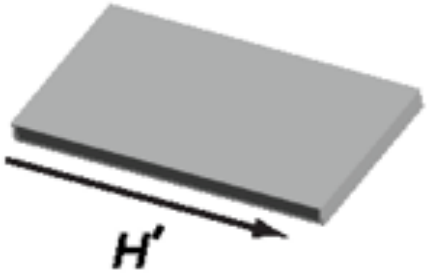

Working point.
M
M_S
M_r
H_c
H_d
H_d
H

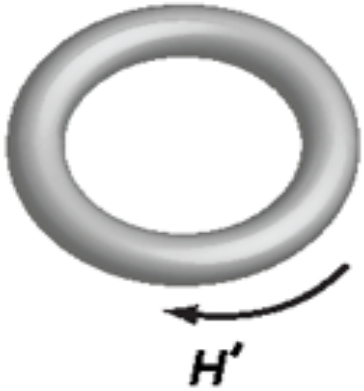
Measuring magnetization with no need for demagnetization correction

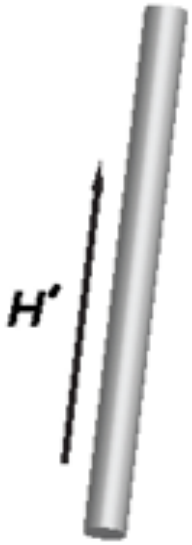
Apply a field in a direction where $\mathcal{N} = 0$

$\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_m$ $\qquad$ $(\boldsymbol{H}_d)_i = -\mathcal{N}_{ij}\boldsymbol{M}_j$

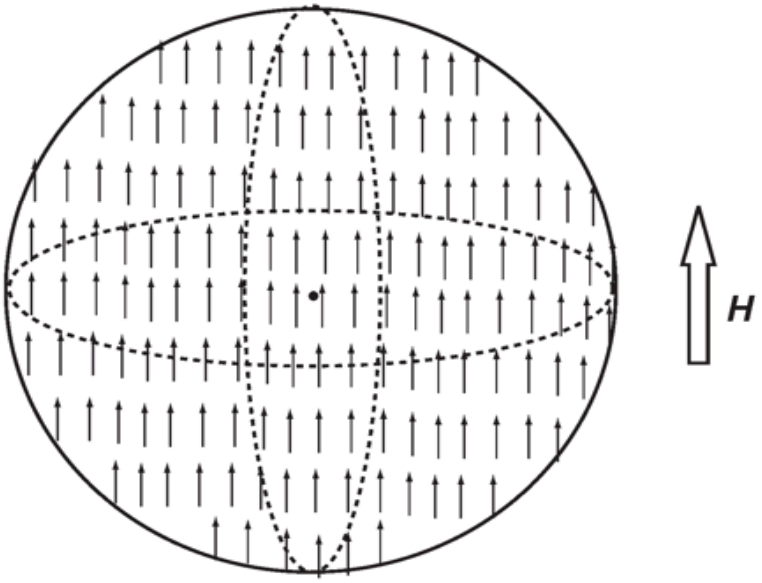
$$\boldsymbol{H} \approx \boldsymbol{H}' - \mathcal{N}\boldsymbol{M}$$

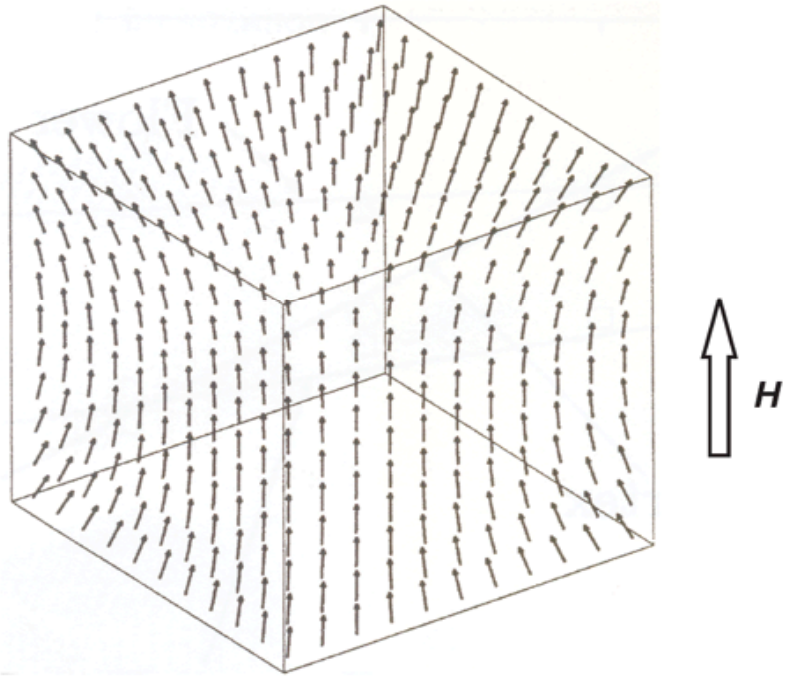






It is not possible to have a uniformly magnetized cube





When measuring the magnetization of a sample ***H*** *is the independent variable,* ***M*** *=* ***M****(****H****).*

Response to an applied field *H′*

Susceptibility of linear, isotropic and homogeneous (LIH) materials

$\boldsymbol{M} = \chi' \boldsymbol{H}'$ $\quad\quad$ χ' is external susceptibility

$\boldsymbol{M} = \chi \boldsymbol{H}$ $\quad\quad$ χ is internal susceptibility

It follows that from $\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_d$, dividing by $\boldsymbol{M}$, that

$$1/\chi = 1/\chi' - \mathcal{N}$$

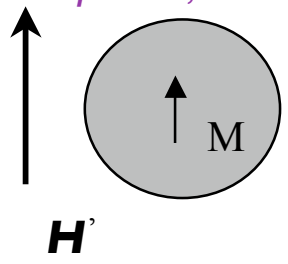
Typical paramagnets and diamagnets:

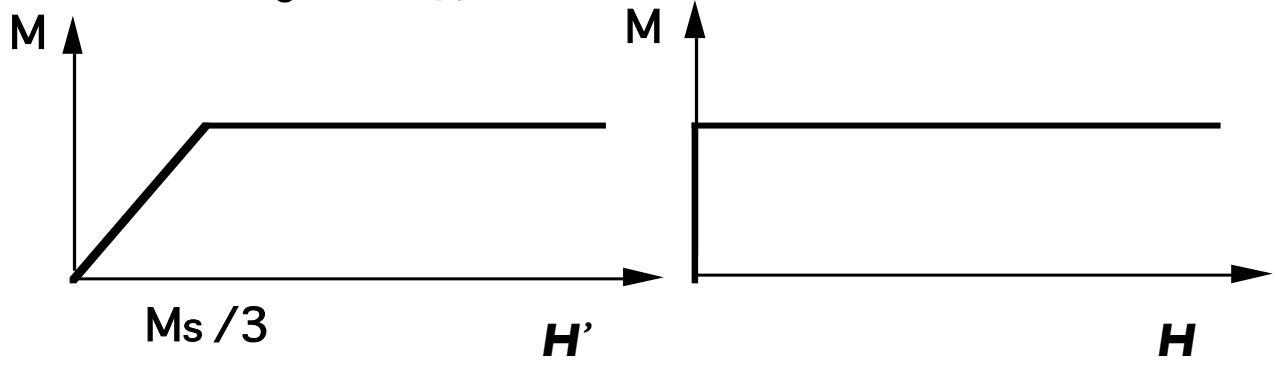
$\chi \approx \chi'$ $\quad$ (10^{-5} to 10^{-3})

Paramagnets close to the Curie point and ferromagnets:

$\chi >> \chi'$ $\quad\quad$ χ diverges as $T \rightarrow T_C$ but χ' never exceeds $1/\mathcal{N}$.

Ferromagnetic sphere, $\chi' = 3$





Permeability

In LIH meida $\boldsymbol{B} = \mu \boldsymbol{H}$ $\mu = B/H$ Units: $TA^{-1}m$

Relative permeability

$\mu_r = \mu/\mu_0$

$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ gives $\mu_r = 1 + \chi$

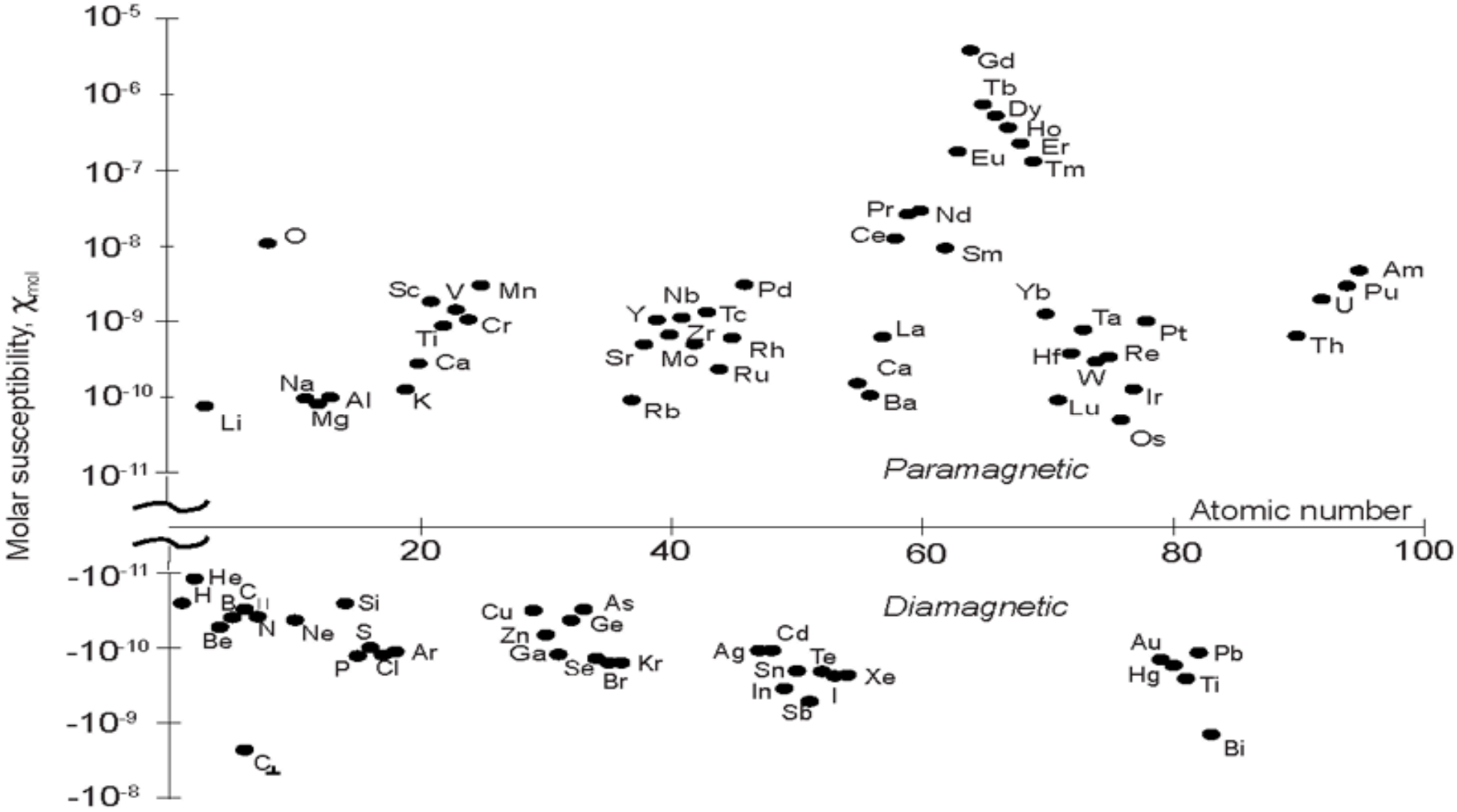
μ_0 is the *permeability of free space*.

- In practice it is often easier to measure the mass of a sample than its volume. Measured magnetization is usually $\sigma = M/\rho$, the magnetic moment per unit mass (ρ is the density).
- Likewise the mass susceptibility is defined as $\chi_m = \chi/\rho$
- Molar susceptibility is $\chi_{mol} = \mathcal{M}\,\chi_m/1000$ $\mathcal{M}$ is the molecular weight (g/mole)

Examples.

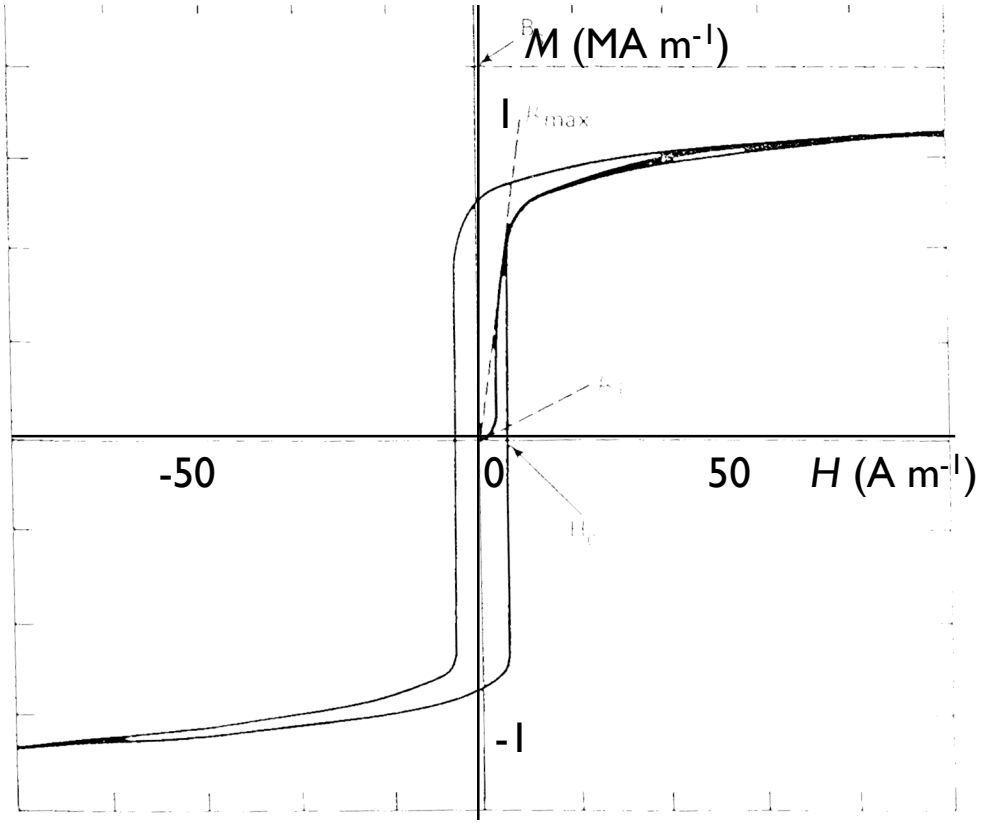
Susceptibility of representative materials					
Susceptibility	Units	H_2O	Al	$CuSO_4 \cdot 5H_2O$	$Gd_2(SO_4)_3 \cdot 8H_2O$
χ		-9.0×10^{-6}	2.1×10^{-5}	1.41×10^{-4}	2.6×10^{-3}
χ_m	$m^3\ kg^{-1}$	-9.0×10^{-9}	7.9×10^{-9}	6.2×10^{-8}	8.7×10^{-7}
χ_{mol}	$m^3\ mol^{-1}$	-1.62×10^{-10}	2.1×10^{-10}	1.57×10^{-8}	6.5×10^{-7}

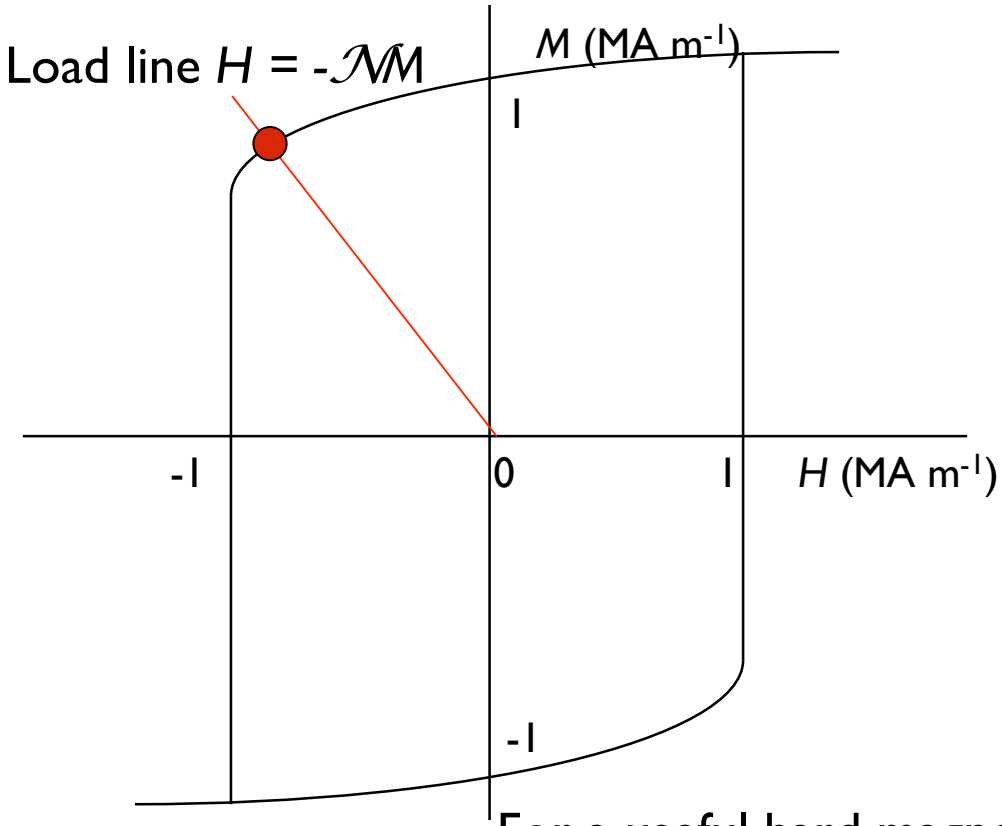
Susceptibilities of the elements



Soft and hard magnets.

The area of the hysteresis loop represents the energy loss per cycle. For efficient *soft* magnetic materials, this needs to be as small as possible.

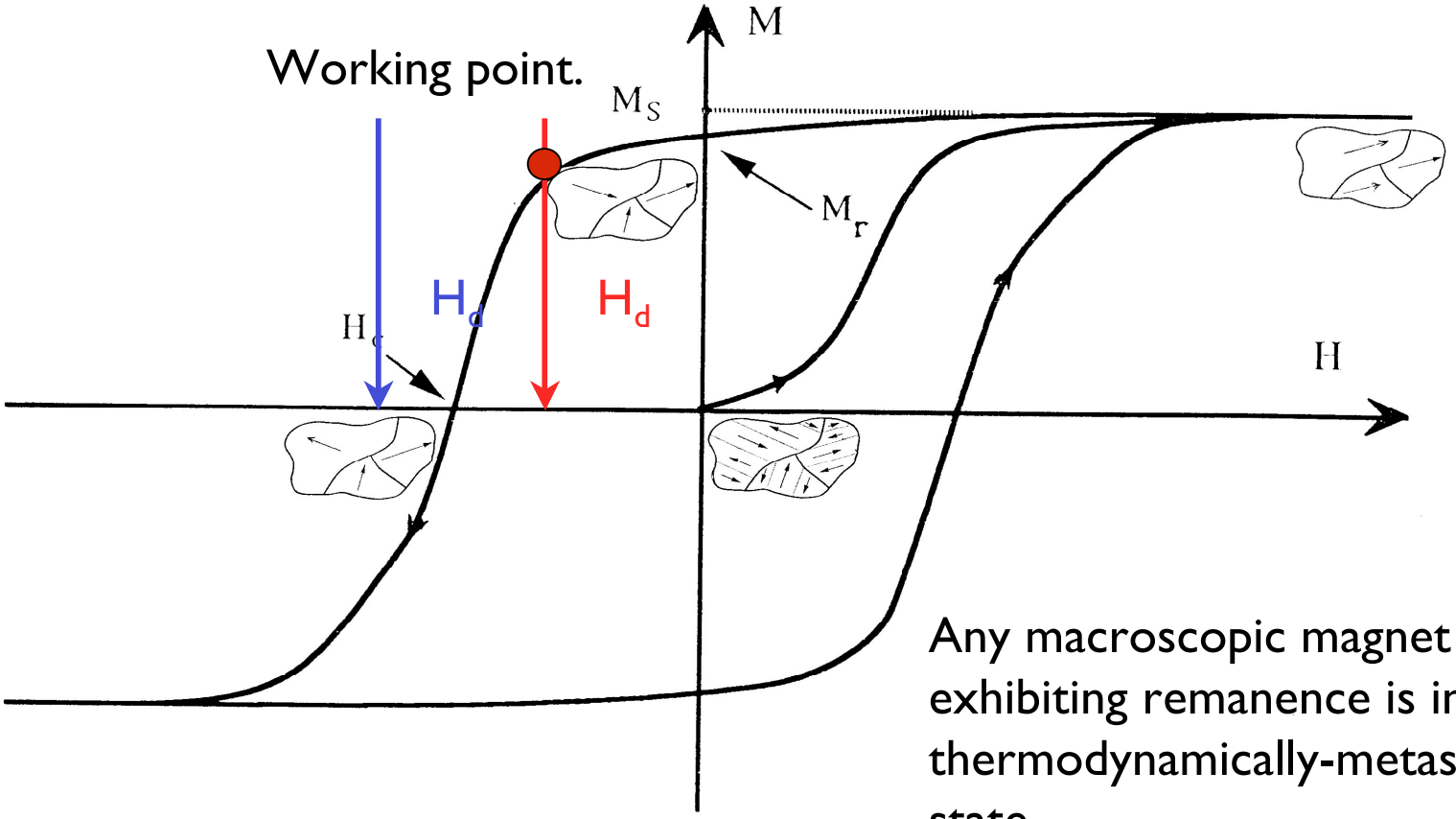




For a useful hard magnet.

$H_c > M_r/2$

Energy product.



Any macroscopic magnet exhibiting remanence is in a thermodynamically-metastable state.

Magnetostatic energy and forces

Energy of ferromagnetic bodies

• Magnetostatic (dipole-dipole) forces are long-ranged, but weak. They determine the magnetic microstructure.

$M \approx 1$ MA m^{-1}, $\mu_0 H_d \approx 1$ T, hence $\mu_0 H_d M \approx 10^6$ J m^{-3}

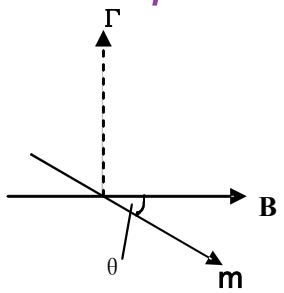
Products $\boldsymbol{B}.\boldsymbol{H}$, $\mathbf{B}.\mathbf{M}$, $\mu_0 \boldsymbol{H}^2$, $\mu_0 \boldsymbol{M}^2$ are all energies per unit volume.

• Magnetic forces *do no work* on moving charges $\boldsymbol{f} = q(\boldsymbol{v} \times \boldsymbol{B})$

• No potential energy associated with the magnetic force.

Torque and potential energy of a dipole in a uniform field

$$\boldsymbol{\Gamma} = \boldsymbol{m} \times \boldsymbol{B}$$



$$\varepsilon_m = -\boldsymbol{m}.\boldsymbol{B}$$

Force

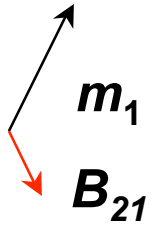
In a non-uniform field, $\boldsymbol{f} = -\nabla \varepsilon_m$ $\quad \boldsymbol{f} = \boldsymbol{m}.\nabla \boldsymbol{B}$

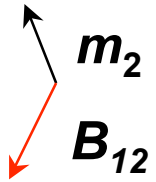
Reciprocity theorem

The interaction of a pair of dipoles, ε_p, can be considered as the energy of $\boldsymbol{m}_1$ in the field $\boldsymbol{B}_{21}$ created by $\boldsymbol{m}_2$ at $\boldsymbol{r}_1$ *or vice versa.*

$$\varepsilon_p = -\boldsymbol{m}_1.\boldsymbol{B}_{21} = -\boldsymbol{m}_2.\boldsymbol{B}_{12}$$

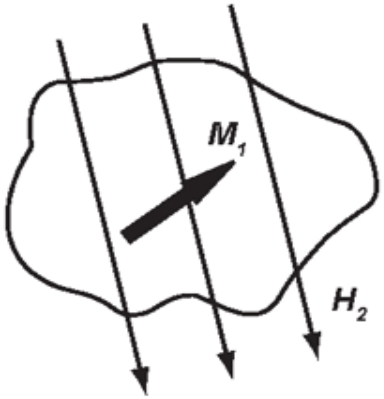
So $$\boxed{\varepsilon_p = -(1/2)(\boldsymbol{m}_1.\boldsymbol{B}_{21} + \boldsymbol{m}_2.\boldsymbol{B}_{12})}$$

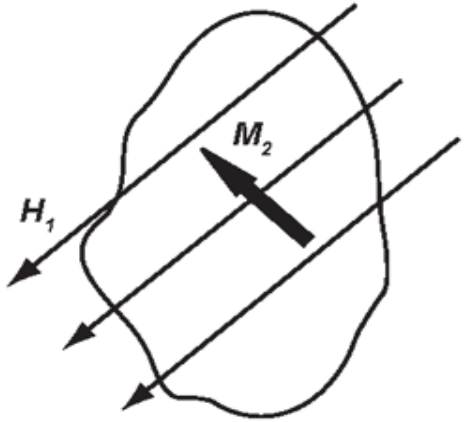




Extending to magnetization distributions:

$$\varepsilon = -\mu_0 \int \boldsymbol{M}_1.\boldsymbol{H}_2 \, d^3r = -\mu_0 \int \boldsymbol{M}_2.\boldsymbol{H}_1 \, d^3r$$





Magnetic energy terms

Self energy of a magnet in its demagnetizing field $E_m = -(1/2)\int \mu_0 H_d M d^3r$

$$E_m = (1/2)\int \mu_0 H_d^2 \, d^3r$$

Self energy of a uniformly magnetized sample $E_m = (1/2)\mu_0 \mathcal{N} M^2 V$

Energy associated with a magnetic field $E_m = (1/2)\int \mu_0 H^2 d^3r$

Energy product of a permanent magnet

Aim to maximize energy associated with the field created around the magnet, from previous slide:

$$\frac{1}{2}\int \mu_0 H_d^2 d^3r = -\frac{1}{2}\int_V \mu_0 \mathbf{H}_d.\mathbf{M}\, d^3r.$$

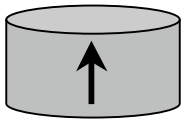
Can rewrite as:

$$\frac{1}{2}\int_o \mu_0 H_d^2 d^3r = -\frac{1}{2}\int_i \mu_0 H_d^2 d^3r - \frac{1}{2}\int_i \mu_0 \mathbf{M}.\mathbf{H}_d d^3r.$$

where we want to maximize the integral on the left. Since $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$,

Energy product: twice the energy stored in the stray field of the magnet is

$$-\mu_0 \int_i \boldsymbol{B}.\boldsymbol{H}_d \, d^3r$$



Optimum shape, $\mathcal{N} = 1/2$

Thermodynamics

(*U,Q,F,G* are in units of Jm^{-3})

First law: $dU = H_x dX + dQ$

$dQ = TdS$

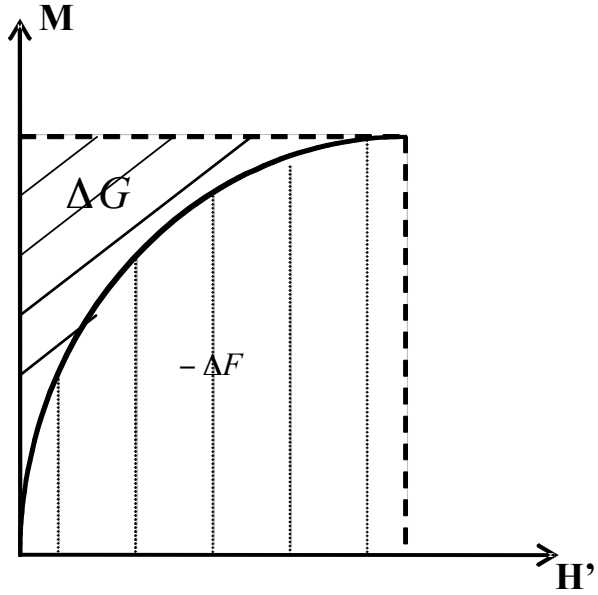
Four thermodynamic potentials;

U(X,S) *internal energy*

E(H_X,S) *enthalpy*

F(X,T) = U - TS *Helmholz free energ*
dF = HdX - SdT

G(H_X,T) = F - H_XX *Gibbs free energy*
dG = -XdH_X - SdT



Magnetic work is $H\delta B$ or $\mu_0 H'\delta M$

$S = -(\partial G/\partial T)_{H'}$ $\mu_0 M = -(\partial G/\partial H')_{T'}$

$dF = \mu_0 H'dM - SdT$
$dG = -\mu_0 MdH' - SdT$

Maxwell relations

$(\partial S/\partial H')_{T'} = -\mu_0(\partial M/\partial T)_{H'}$ etc.

Magnetostatic Forces

Force density on a uniformly magnetized body at constant temperature

$$\boldsymbol{F}_m = -\nabla G$$

$$\boldsymbol{F}_m = \nabla(\mu_0 \boldsymbol{H}'.\boldsymbol{M}) \qquad \nabla(\boldsymbol{H}'.\boldsymbol{M}) = (\boldsymbol{H}'.\nabla)\boldsymbol{M} + (\boldsymbol{M}.\nabla)\boldsymbol{H}'$$

Kelvin force

$$\boldsymbol{F}_m = \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}'$$

General expression, for when ***M*** is dependent on ***H*** is

$$\boldsymbol{F}_m = -\mu_0 \nabla \left[\int_0^H \left(\frac{\partial M \upsilon}{\partial \upsilon} \right)_{H,T} dH \right] + \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}.$$

$\nu = 1/d$ d is the density

Anisotropy.

shape, magnetocrystalline, induced, strain

The ferromagnetic axis lies in some particular direction determined by shape or some intrinsic anisotropy related to crystal or atomic structure.

$$E_a = K_1 \sin^2\theta + K_2 \sin^4\theta + ...$$

The *shape* contribution is derived from the energy expression

$$E_m = (1/2)\mu_0 \mathcal{N} M^2 V$$

The magnetization lies along the direction for which $\mathcal{N}$ is smallest - the axis of a long bar.

The magnetocrystalline contribution for uniaxial crystals is given by a similar expression with different K_1 .....

Shape anisotropy.

The *shape* contribution is derived from the energy expression

$$E_m = (1/2)\mu_0 \mathcal{N} M^2 V$$

$\mathcal{N}$ is the demagnetizing factor for the easy direction - the axis of a long bar. (1/2)[1 - $\mathcal{N}$] is the demagnetizing factor for the perpendicular direction (assuming an ellipsoid)

Hence $\Delta E_m = (1/2)\ \mu_0 M^2 V\{(1/2)(1 - \mathcal{N}) - \mathcal{N}\}$

so $K_{sh} = (1/4)\mu_0 M^2 (1 - 3\mathcal{N})$

The biggest it can be is $(1/4)\mu_0 M^2$ J m^{-3}

~ 2 10^5 J m^{-3} for $\mu_0 M$ = 1 T

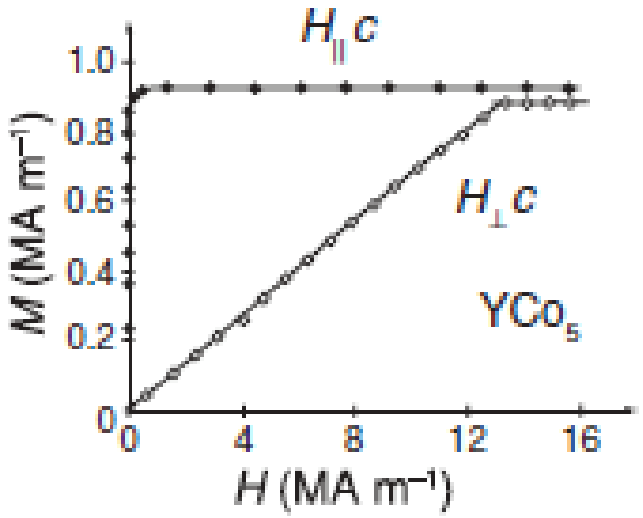
Magnetocrystalline anisotropy

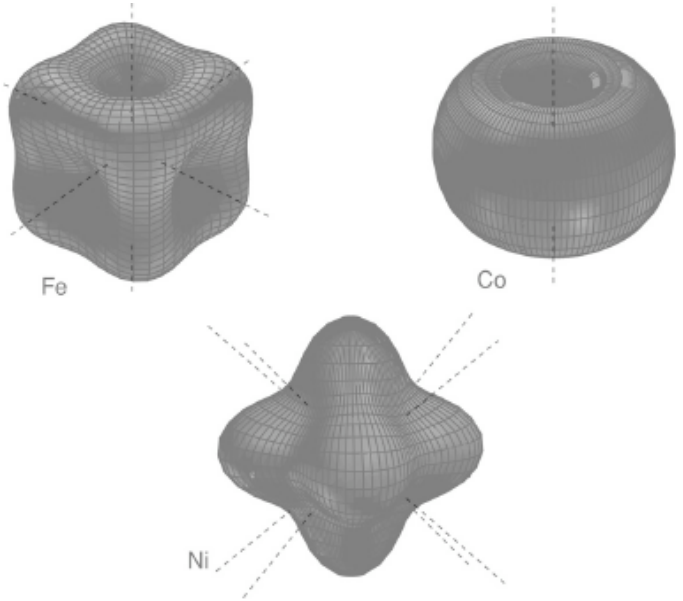
The magnetocrystalline contribution is ultimately caused bu spin-orbit coupling which connects the crystal structure (electron orbits) and magnetic moment direction.

$$\textit{Hexagonal:}\quad E_a = K_1 \sin^2\theta + K_2 \sin^4\theta + K_3 \sin^6\theta + K_3' \sin^6\theta \sin 6\phi,$$

$$\textit{Tetragonal:}\quad E_a = K_1 \sin^2\theta + K_2 \sin^4\theta + K_2' \sin^4\theta \cos 4\phi + K_3 \sin^6\theta + K_3' \sin^6\theta \sin 4\phi,$$

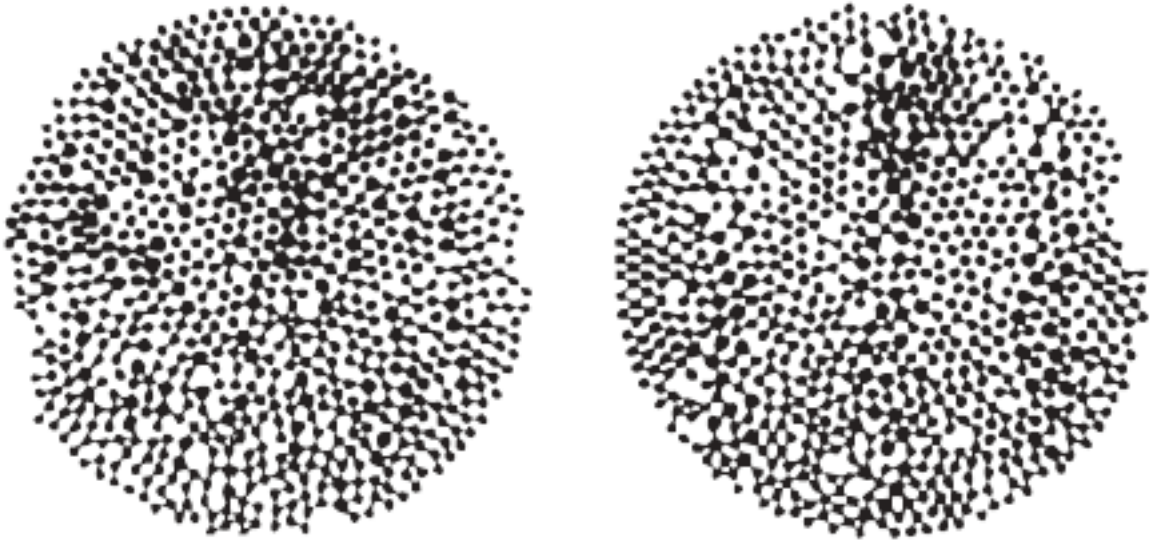
$$\textit{Cubic:}\quad E_a = K_{1c}\left(\alpha_1^2\alpha_2^2 + a_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2\right) + K_{2c}\left(\alpha_1^2\alpha_2^2\alpha_3^2\right),$$

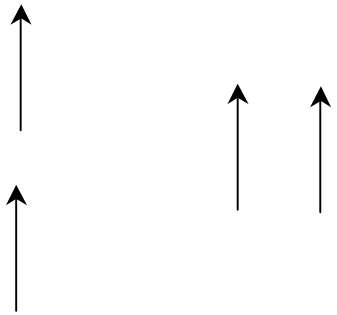


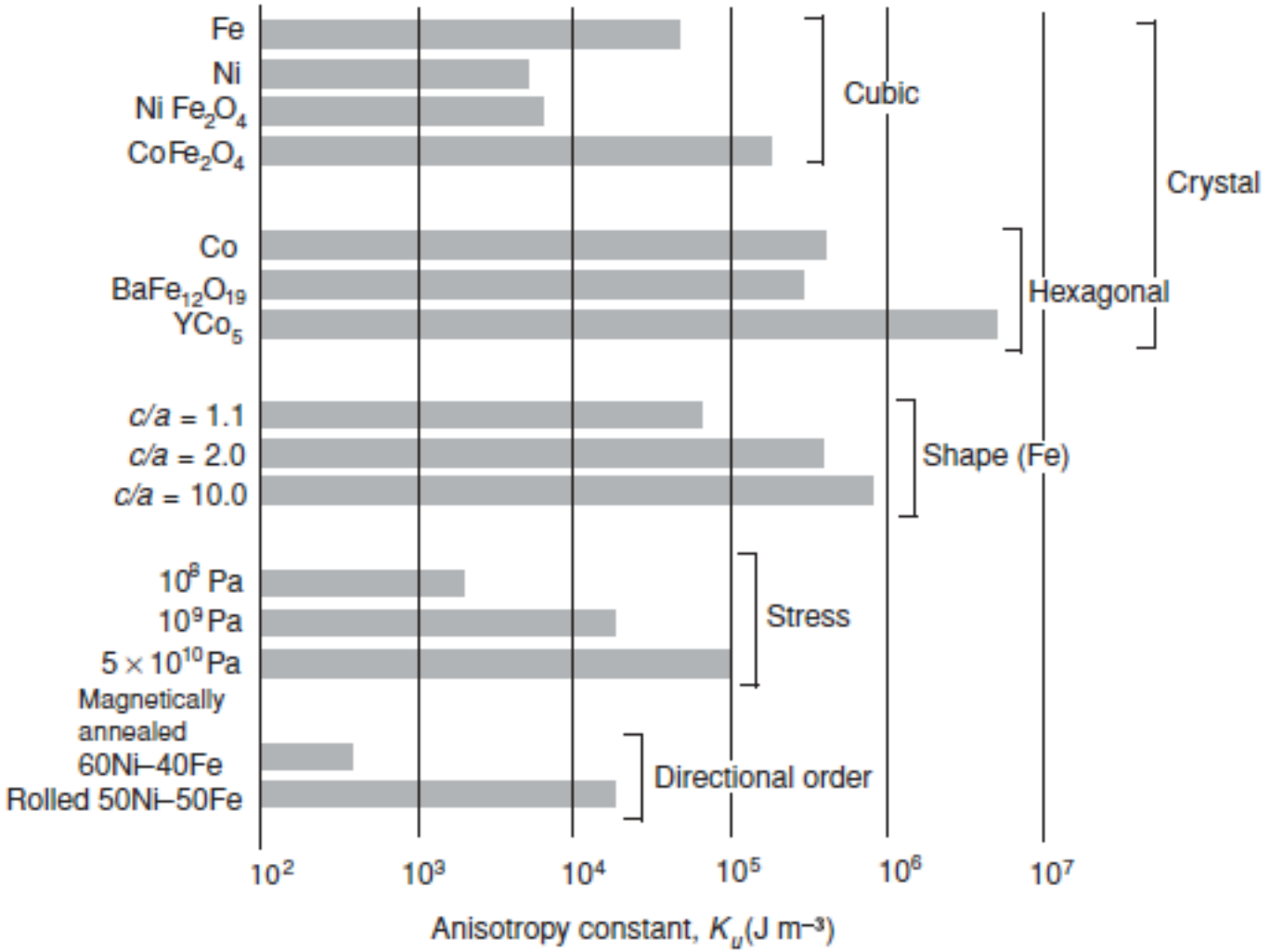


Anisotropy due to texture

Directional order of atomic constituents in a binary alloy can be induces by deposition on a magnetic field or by post depositional annealing.





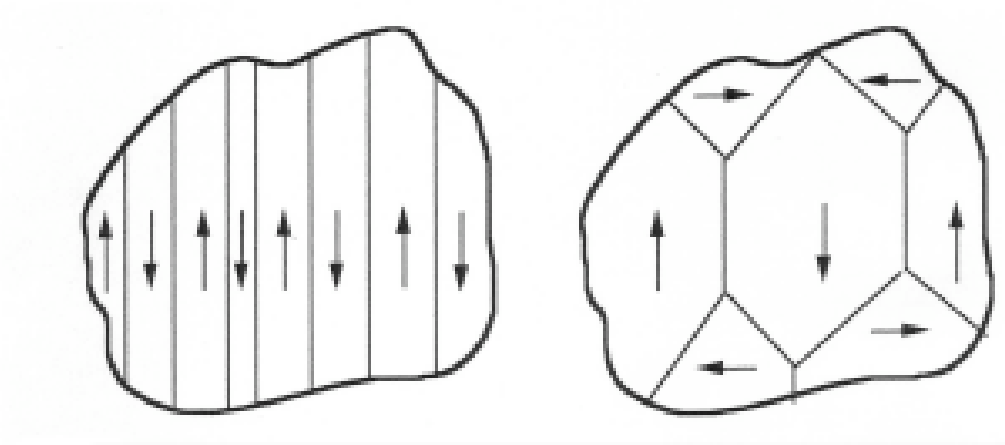

Fe
Ni
Ni Fe_2O_4
$CoFe_2O_4$
Cubic
Crystal
Co
$BaFe_{12}O_{19}$
YCo_5
Hexagonal
c/a = 1.1
c/a = 2.0
c/a = 10.0
Shape (Fe)
10^8 Pa
10^9 Pa
5×10^{10} Pa
Stress
Magnetically annealed
60Ni–40Fe
Rolled 50Ni–50Fe
Directional order
10^2
10^3
10^4
10^5
10^6
10^7
Anisotropy constant, K_u (J m^{-3})

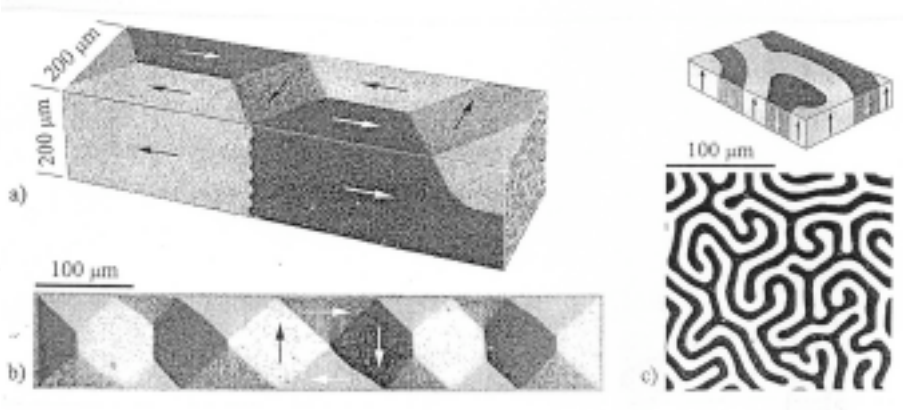
Domains

Micromagnetic energy, wall width and structure

Domains form to minimize the dipolar energy E_d

$$E_d = -\frac{1}{2}\int \mu_0 \boldsymbol{H}_d . \boldsymbol{M} d^3 r$$





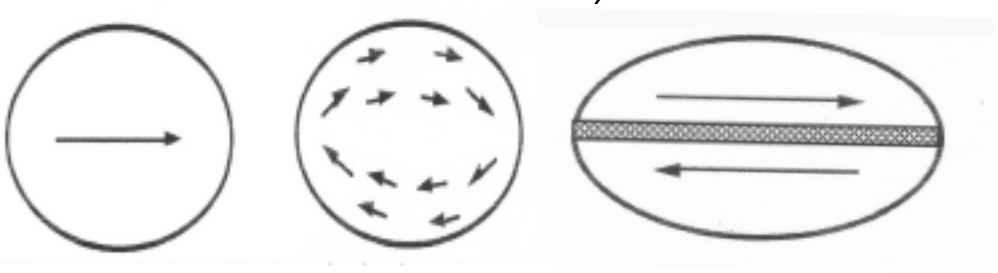
Micromagnetic energy

$$E_{tot} = E_{ex} + E_K + E_d + E_m + E_{stress} + E_{ms}$$

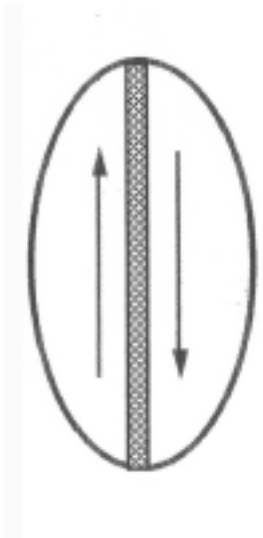
$$E_{tot} = \int \{A(\nabla \boldsymbol{M}/M_s)^2 - K_1(\mathbf{e}_m \cdot \mathbf{e}_n)^2 - \ldots - \frac{1}{2}\mu_0 \boldsymbol{M}.\boldsymbol{H}_d - \mu_0 \boldsymbol{M}.\boldsymbol{H}\} d^3r$$

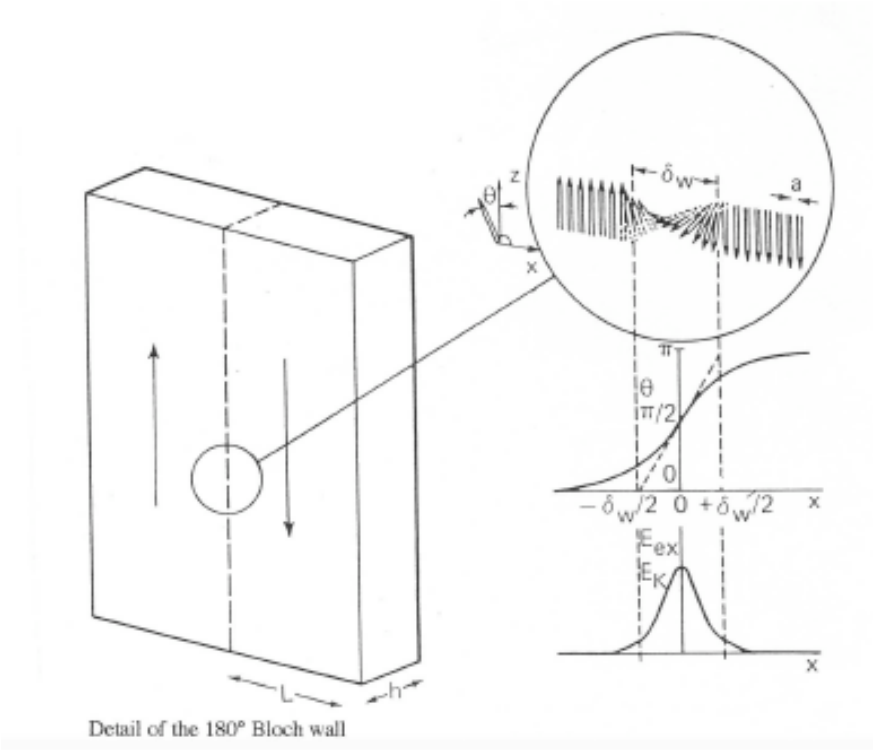
exchange | anisotropy (leading term) | magnetostatic | Zeeman

Minimizing E_{tot} gives the mesoscale magnetic structure of the sample (monodomain, multidomaindomains or vortex)



Domain walls





Detail of the 180° Bloch wall

Minimizing E_{tot} for two oppositely magnetized regions gives the domain wall width

$$\delta_w = \pi\sqrt{(A/K_1)}$$

A ~ 10 pJ m^{-1}; $K \sim 10^5$ jm^{-3}; δ_w ~ 30 nm.

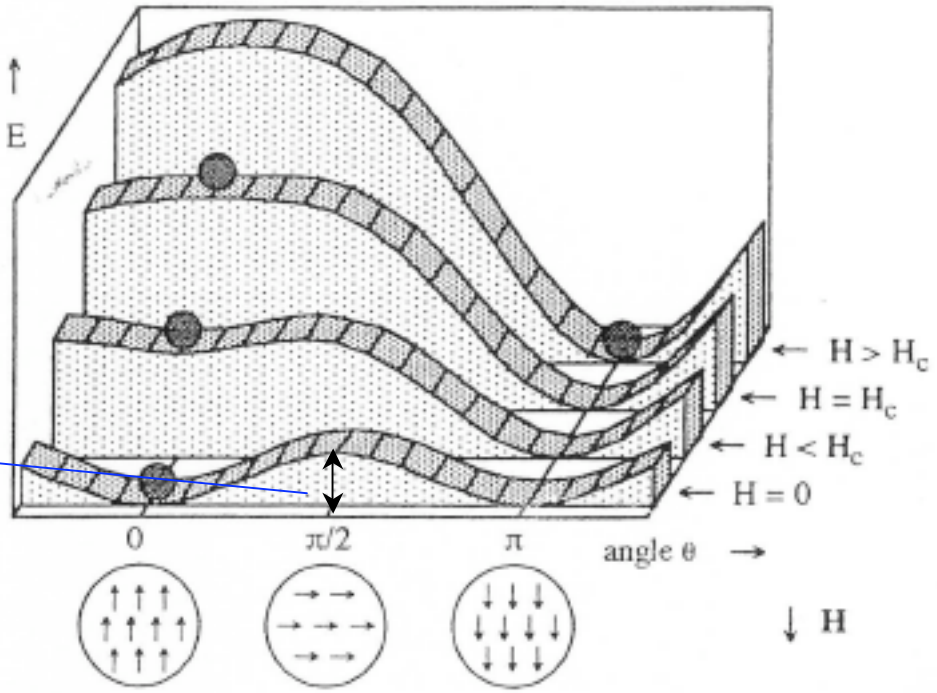
Superparamagnetism

Fine particles, blocking

$E_a = K\sin^2\theta$

$\Delta = KV$

the ratio Δ/kT is critical



Néel relaxation $\tau = \tau_0 \exp(\Delta/kT)$

Here τ_0 is an inverse attempt frequency, $10^{-9}s^{-1}$

If $\Delta/kT = 25$, $\tau = 70$ s.

2.2 Sensor Principles

Flux - Faraday's law

MR - Lorentz force

Hall effect - Lorentz force

AMR - Spin-orbit scattering

GMR - spin accumulation

TMR - spin-dependent tunelling

MO - Faraday effect

SQUID - Flux quantization

NMR - magnetic resonance

GMI - high-frequency permeability

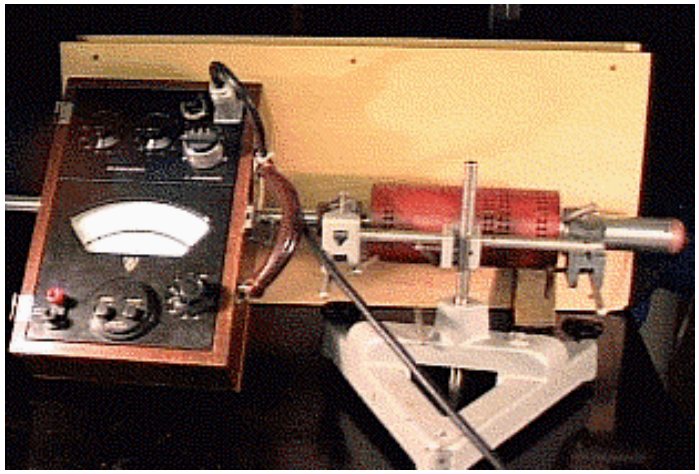
Sensor	Principle	Detects	Frequency	Field (T)	Noise	Comments
Coil	Faraday's law	$d\Phi/dt$	10^{-3} - 10^{9}	10^{-10} - 10^{2}	100 nT	bulky ,absolute
Fluxgate	saturation	H	dc - 10^{3}	10^{-10} - 10^{-3}	10 pT	bulky
Hall probe	Lorentz f'ce	B	dc - 10^{5}	10^{-5} - 10	100 nT	thin film
MR	Lorentz f'ce	B^2	dc - 10^{5}	10^{-2} - 10	10 nT	thin film
AMR	spin-orbit int	H	dc - 10^{7}	10^{-9} - 10^{-3}	10 nT	thin film
GMR	spin accum.n	H	dc - 10^{9}	10^{-9} - 10^{-3}	10 nT	thin film
TMR	tunelling	H	dc - 10^{9}	10^{-9} - 10^{-3}	1 nT	thin film
GMI	permability	H	dc - 10^{4}	10^{-9} - 10^{-2}		wire
MO	Kerr/Faraday	M	dc - 10^{5}	10^{-9} - 10^{2}	1 pT	bulky
SQUID lt	flux quanta	Φ	dc - 10^{9}	10^{-15} - 10^{-2}	1 fT	cryogenic
SQUID ht	flux quanta	Φ	dc - 10^{4}	10^{-15} - 10^{-2}	30 fT	cryogenic
NMR	resonance	B	dc - 10^{3}	10^{-10} - 10	1 nT	Very precise

2.2.1 Inductive sensors

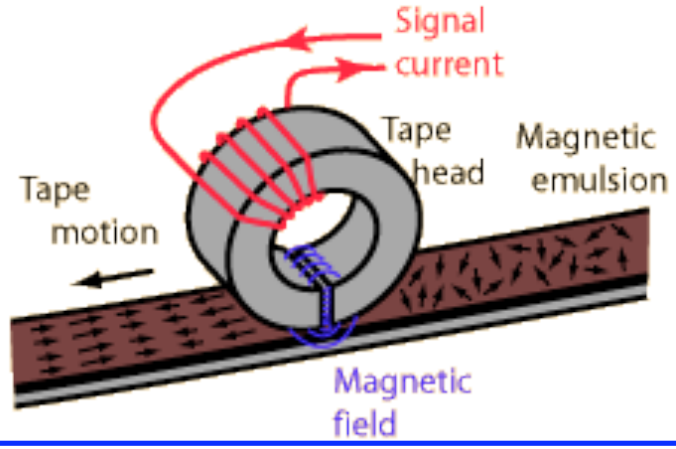
Inductive sensors detect an emf in a coil proportional to the rate of change of flux, according to Faraday's law:

$$\mathcal{E} = -d\Phi/dt$$

They provided an absolute measurement of $B = \Phi/n\mathcal{A}$, as in a search coil with an integrating voltmeter, or the rotating coil gaussmeter



Inductive read/write heads were widely used until 1990 in magnetic recording



Magnetic circuits

$\nabla \boldsymbol{B} = 0$

Assuming no flux leakage

$B_m \mathcal{A}_m = B_g \mathcal{A}_g$ (1)

Assuming ideal soft material $\mu = \infty$

Ampere's law $\int \mathbf{H}.d\mathbf{l} = 0$

$H_m \ell_m = -H_g \ell_g$ (2)

Multiplying $B_m H_m V_m = - B_g^2 V_g / \mu_0$

Dividing $-B_m/H_m = \mu_0 A_g l_m / A_m l_g$

The *permeance* coefficient

$P = 1/R_m$

reluctance

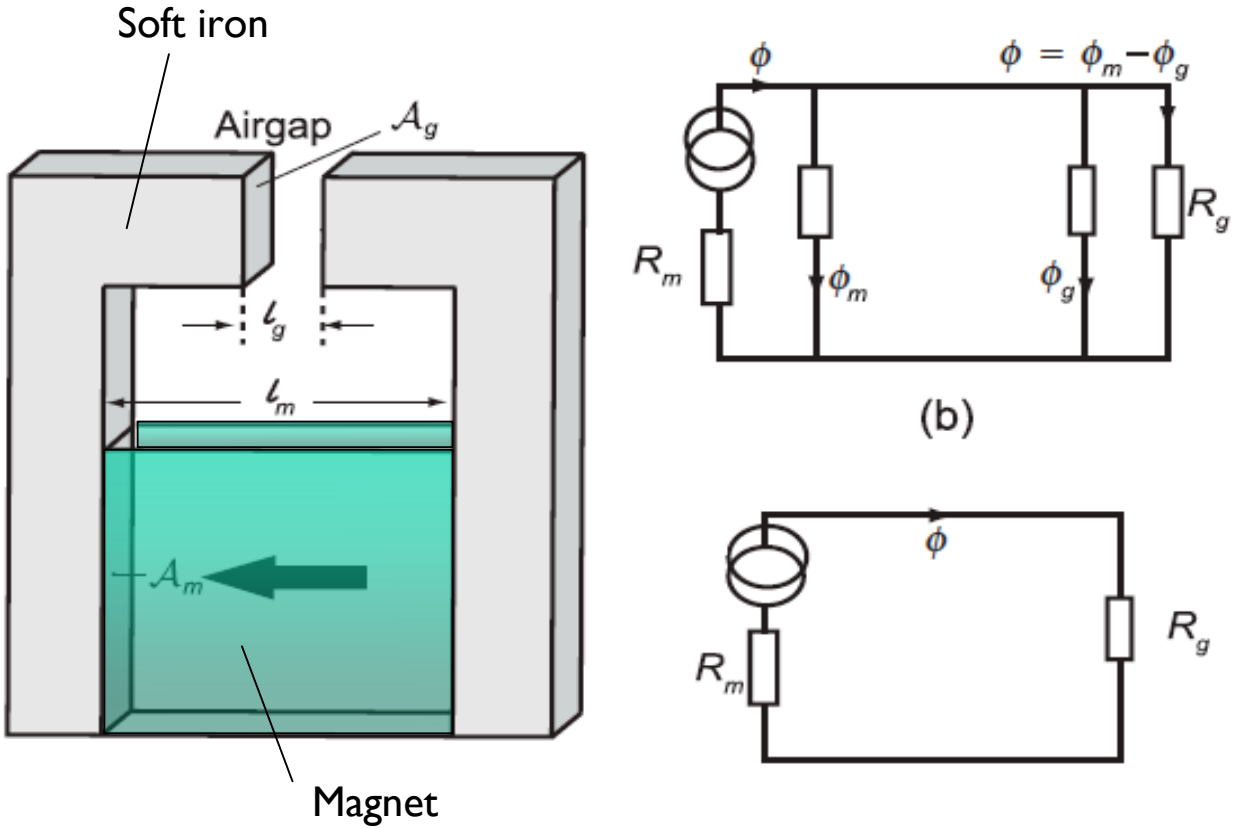
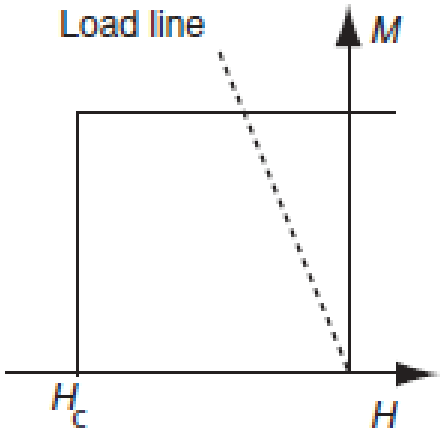
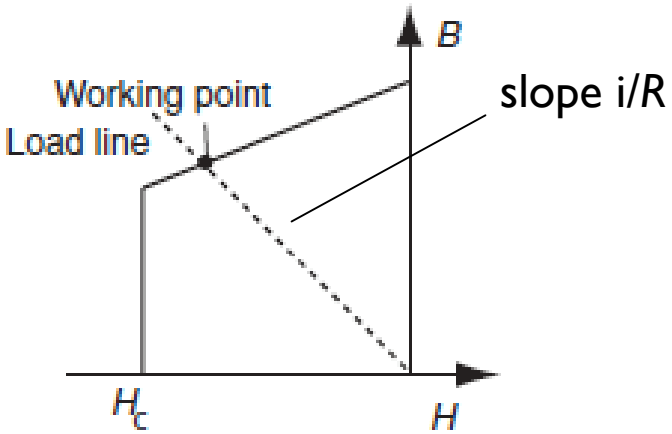


Figure A simple magnetic circuit, and its electrical equivalent, with and without losses.



Ideal $M(H)$ loop



Ideal $B(H)$ loop

Second-quadrant $\mu_0 M(H)$ and $B(H)$ for an ideal permanent magnet. The working point and load line for maximum energy product are indicated.

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$$

$$BH = \mu_0(H^2 - MH)$$

$$. \quad = \mu_0 M^2(\mathcal{N}^2 - \mathcal{N})$$

Maximizing $(BH)_{max} \rightarrow \mathcal{N} = 1/2$

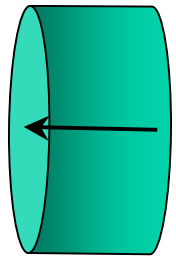
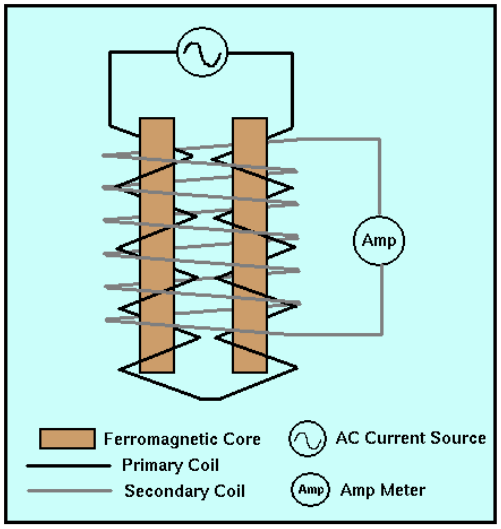


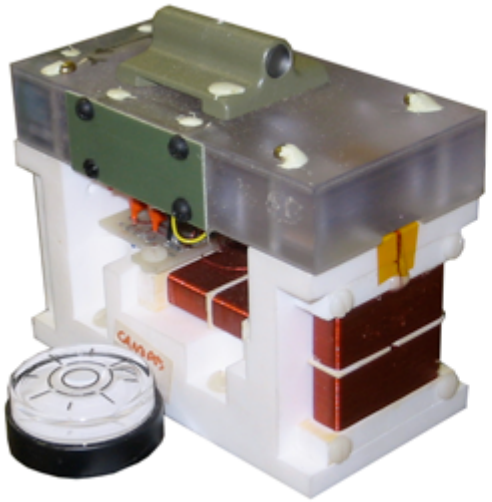
Table Analogy between electric and magnetic circuits

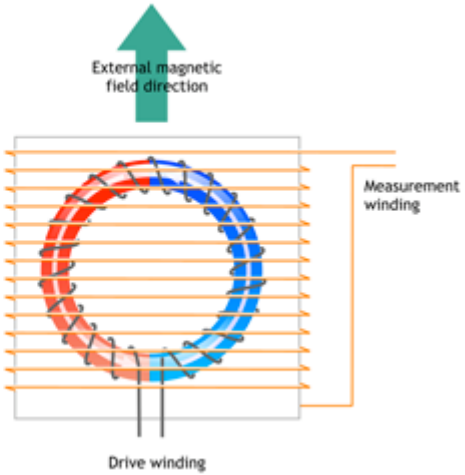
	Electric	Magnetic
Field	$\boldsymbol{E}$ (V m^{-1})	$\boldsymbol{H}$ (A m^{-1})
Potential	φ_e (V)	φ_m (A)
Current/flux density	$\boldsymbol{j}$ (A m^{-2})	$\boldsymbol{B}$ (T or Wb m^{-2})
Potential difference	$\varphi_e = \int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}$	$\varphi_m = \int \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}$
Continuity condition	$\nabla \cdot \boldsymbol{j} = 0$	$\nabla \cdot \boldsymbol{B} = 0$
Linear response law	$\boldsymbol{j} = \sigma \boldsymbol{E}$	$\boldsymbol{B} = \mu \boldsymbol{H}$
Current/flux	I (A)	Φ (T m^2 or Wb)
Resistance/reluctance	$R = \varphi_e / I$ (Ω)	$R_m = \varphi_m / \Phi$ (A Wb^{-1})
for a cylinder of section $\mathcal{A}$ and length l	$R = l/\mathcal{A}\sigma$	$R_m = l/\mathcal{A}\mu$
Conductance/permeance	$G = 1/R$	$P_m = 1/R_m$

2.2.2 Fluxgates

Fluxgates depend on the nonlinear saturation of the magnetization of a soft magnetic core. Two identical cores (or a single toroidal core) have oppositely wound ac field windings. A parallel applied field leads to saturation of one of the cores, producing an ac signal linear in H.



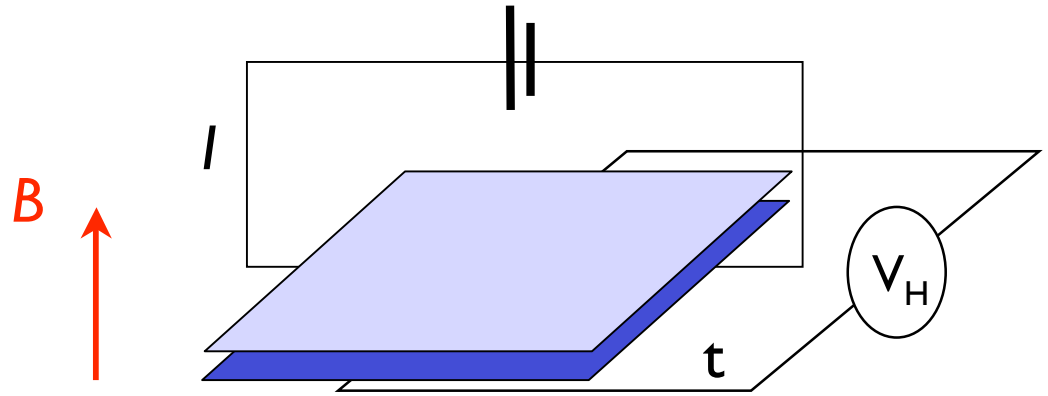




Fluxgates are bulky but sensitive, reliable and impervious to radiation. Used, for example, in space.

2.2.3 Hall sensors

Effect discovered by Edwin Hall in 1879



$V_H/t = R_0 jB$ $\quad R_0 = (1/ne)$

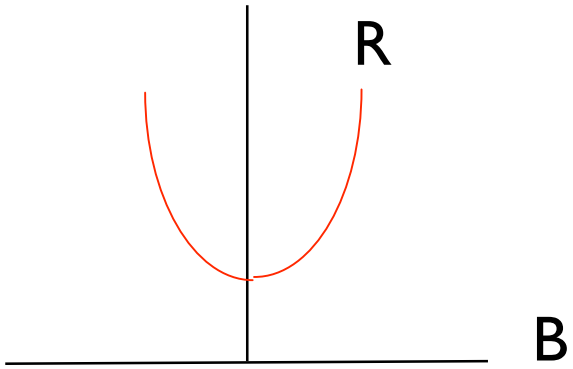
$E/j = \rho_{xy} = R_0 B$

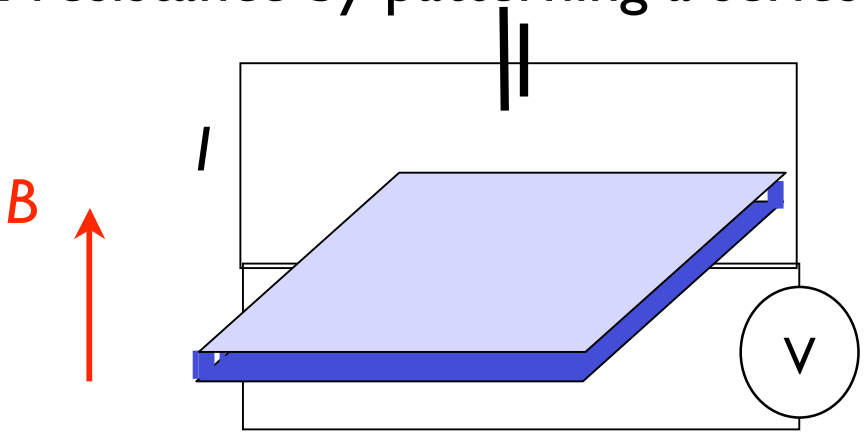
Hall voltages linear in field are produced in semiconductor plates, especially Si and in 2-deg GaAs/GaAlAs structures. These are four-terminal devices, and the current source and high-gain amplifier are often integrated on a chip.

Used for secondary field measurements – each probe must be calibrated — and as proximity sensors. About a billion are produced each year.

2.2.4 Classical magnetoresistance

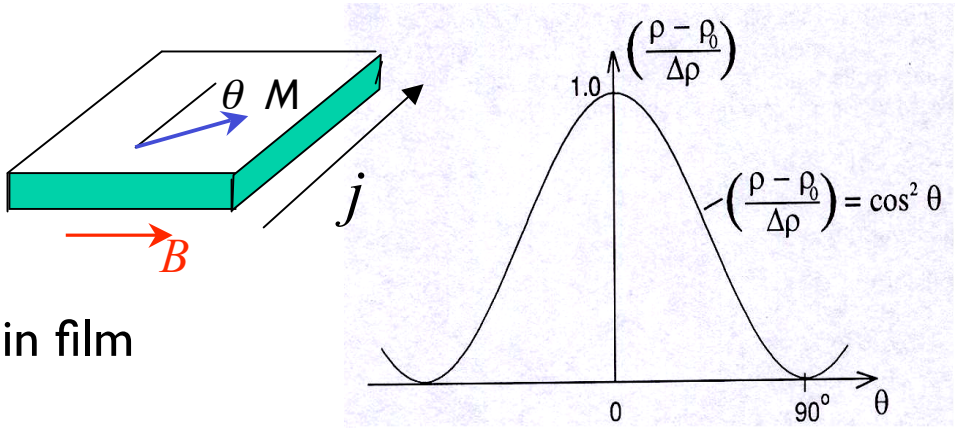
The simplest Lorentz force device is a semicoductor or semimetal which exhibits classical positive B^2 magnetoresiatance. High-mobility semiconductors such as InAs and InSb show large effects (~ 100 % T^{-1}). Field is applied perpendicular to the semiconductor slab, and it is possible to achieve a desired resistance by patterning a series of metallic contacts.

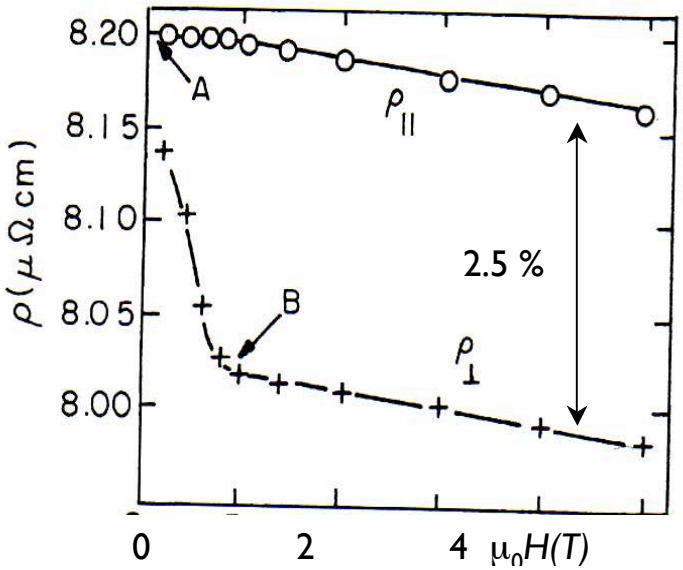




The sensors are nonlinear, two-terminal devices providing a good response in large fields. They are used as position sensors in brushless dc permanent magnet motors.

2.2.5 Anisotropic magnetoresistance (AMR)





AMR of permalloy

Discovered by W. Thompson in 1857

$$\rho = \rho_0 + \Delta\rho\cos^2\theta$$

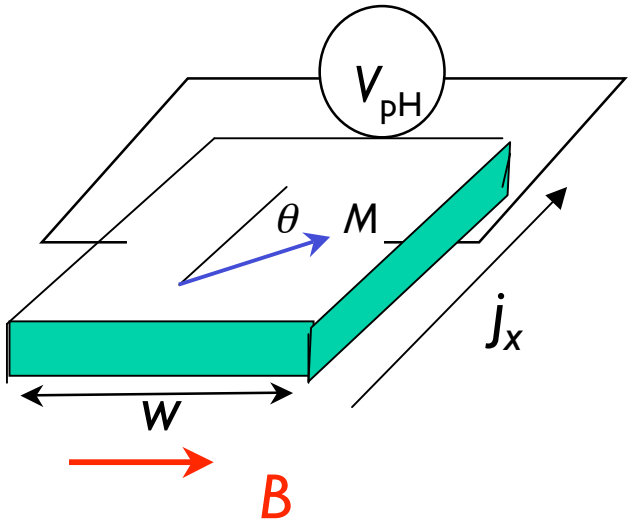
Magnitude of the effect $\Delta\rho/\rho < 3\%$
The effect is usually positive; $\rho_{\parallel} > \rho_{\perp}$

AMR is due to spin-orbit *s-d* scattering

High field sensitivity is achieved in thin films of soft ferromagnetic films such as permalloy ($Fe_{20}Ni_{80}$).

Planar Hall effect

Planar Hall effect is a variant of AMR; $\rho_{||} \neq \rho_{\perp}$ $\rho_{||}$ is when $j \,||\, M$ ….



$E_{||} = \rho_{||}\, j_x \cos\theta$ $E_{\perp} = \rho_{\perp} j_x \sin\theta$

Components of electric field parallel and perpendicular to the current are

$E_x = E_{||}\cos\theta + E_{\perp}\sin\theta$, $E_y = E_{||}\sin\theta - E_{\perp}\cos\theta$

$E_x = j(E_{||}\rho_{\perp} + \Delta\rho\cos^2\theta)$ $E_y = j\Delta\rho\sin\theta\cos\theta$

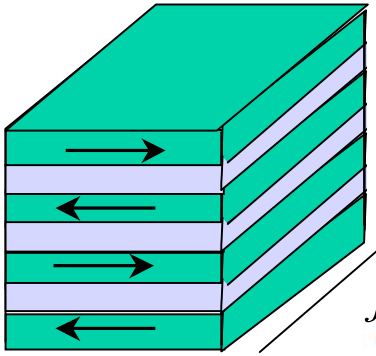
Hence

$$V_{pH} = j\, w\Delta\rho\, \sin\theta\, \cos\theta$$

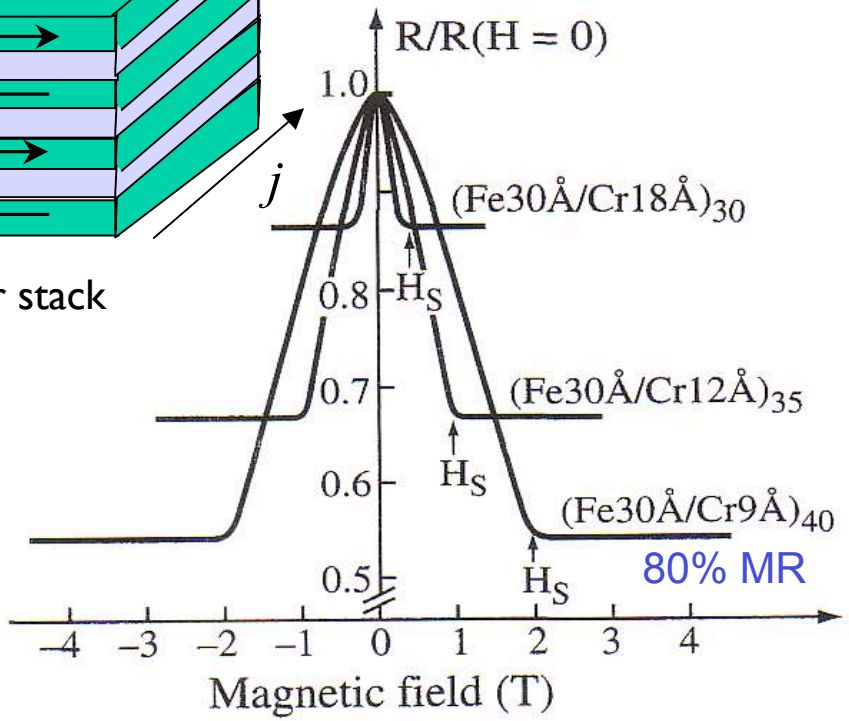
The biggest effect is when θ changes from 45 to 135 degrees.

2.2.6 Giant magnetoresistance.

Peter Grunberg and Albert Fert;



Fe/Cr stack



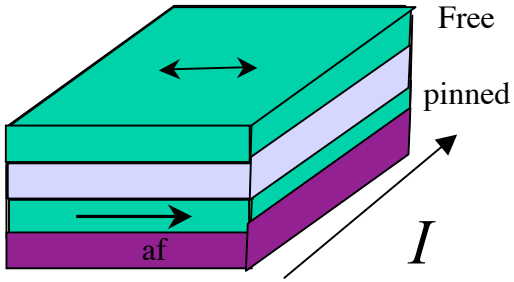
10^9 GMR sensors per year

Discovery of GMR 1988

Implementation in hard disk drives 1998

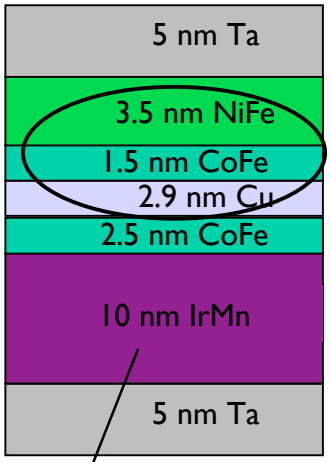
Nobel Prize 2007

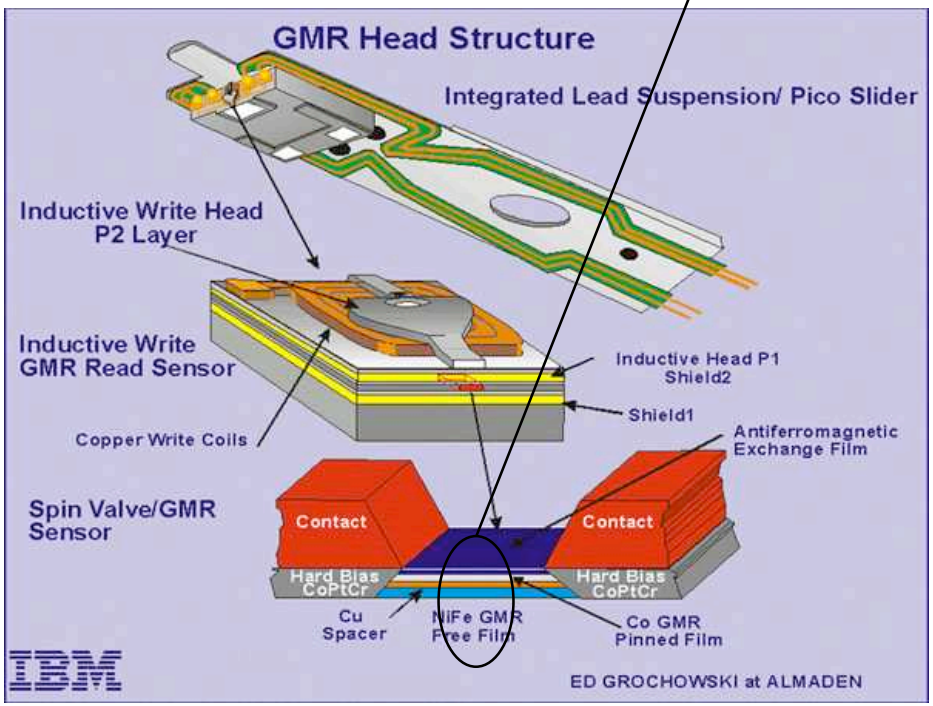
Exchange biased GMR spin valve

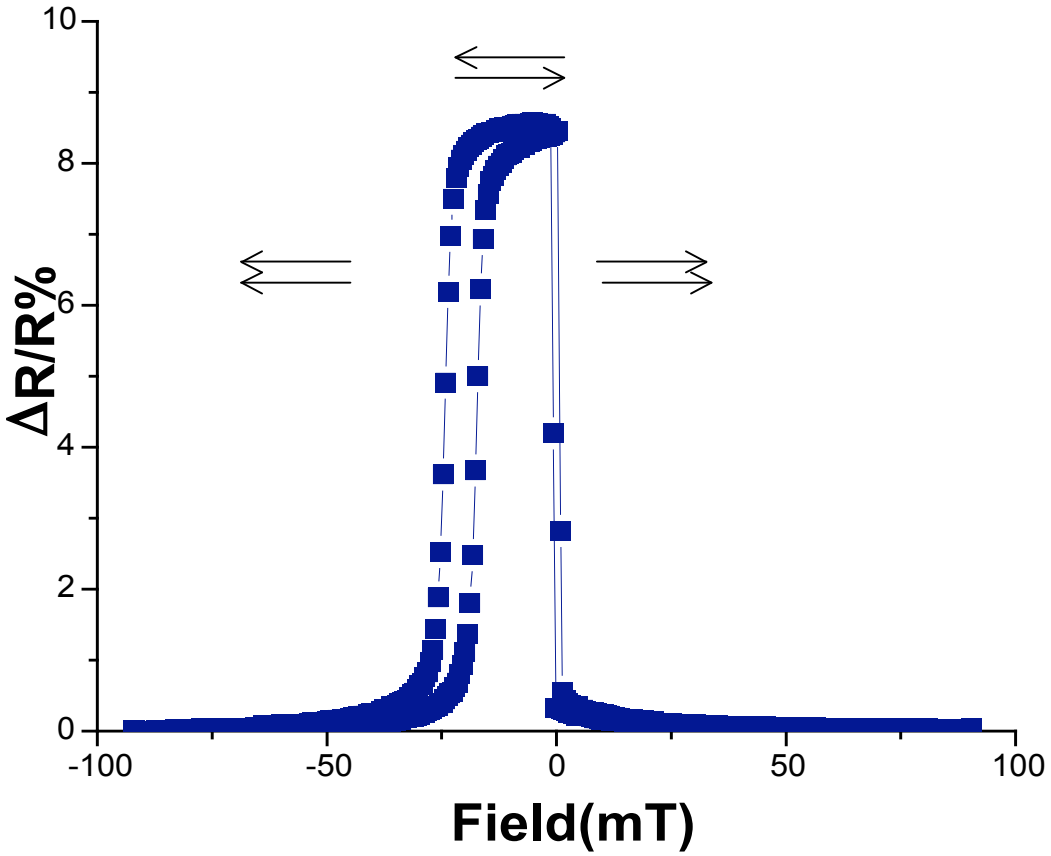


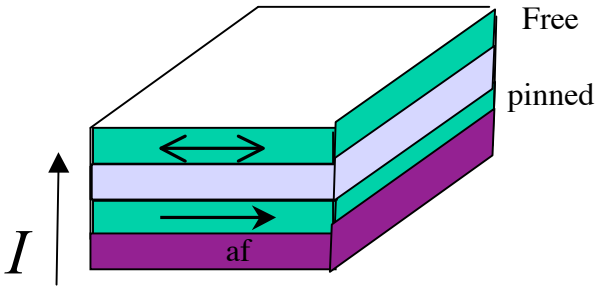
GMR spin valve

Exchange-biased stack



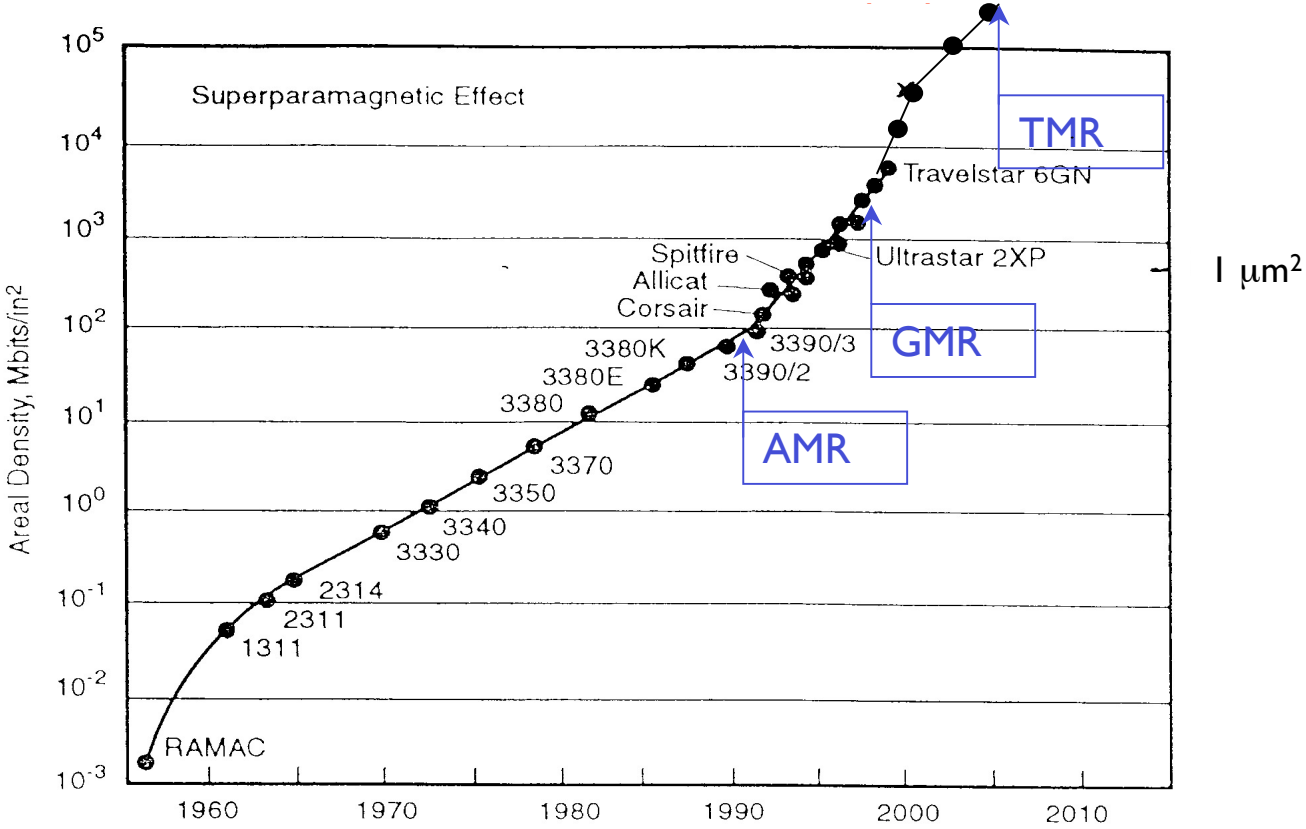




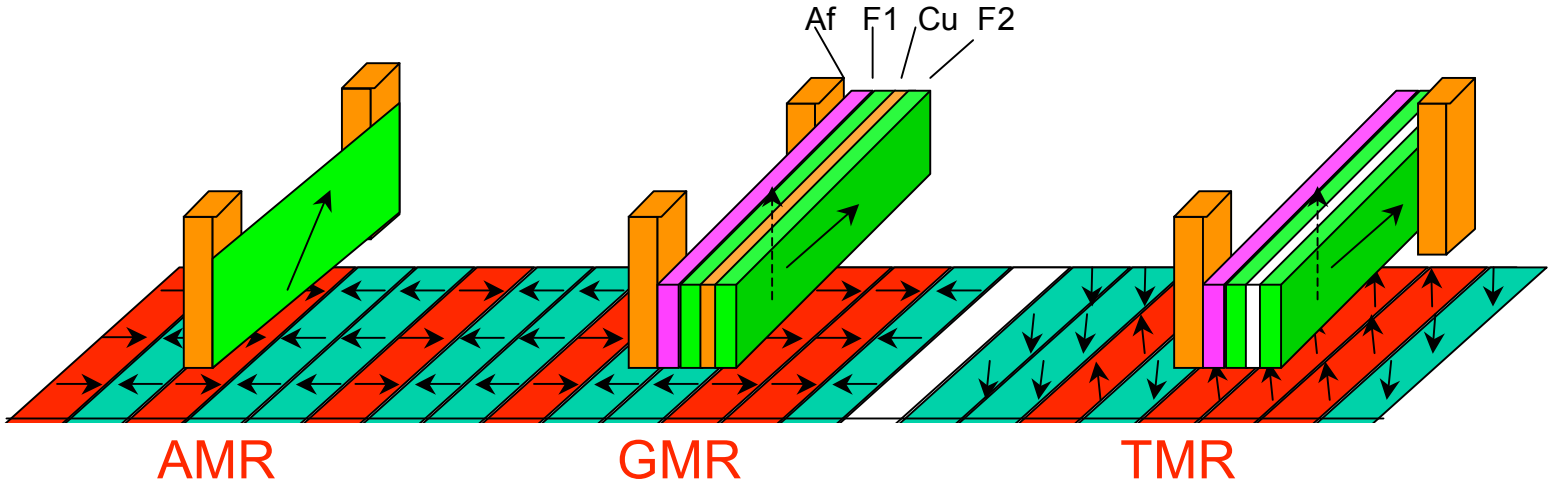


TMR spin valve

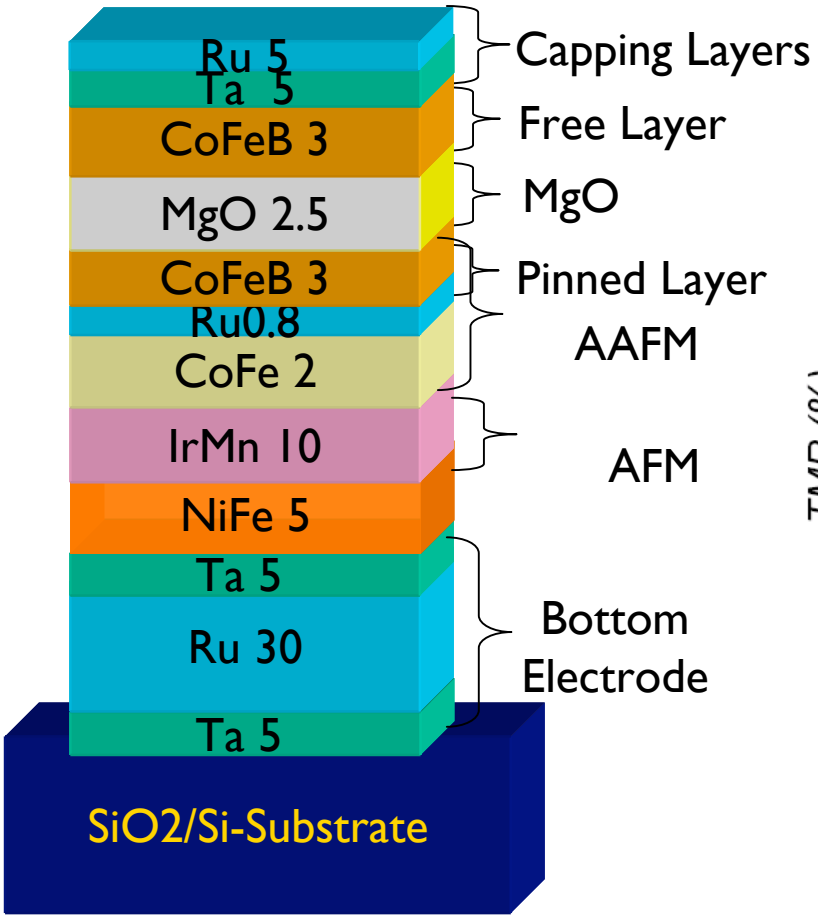
Exchange-biased stack



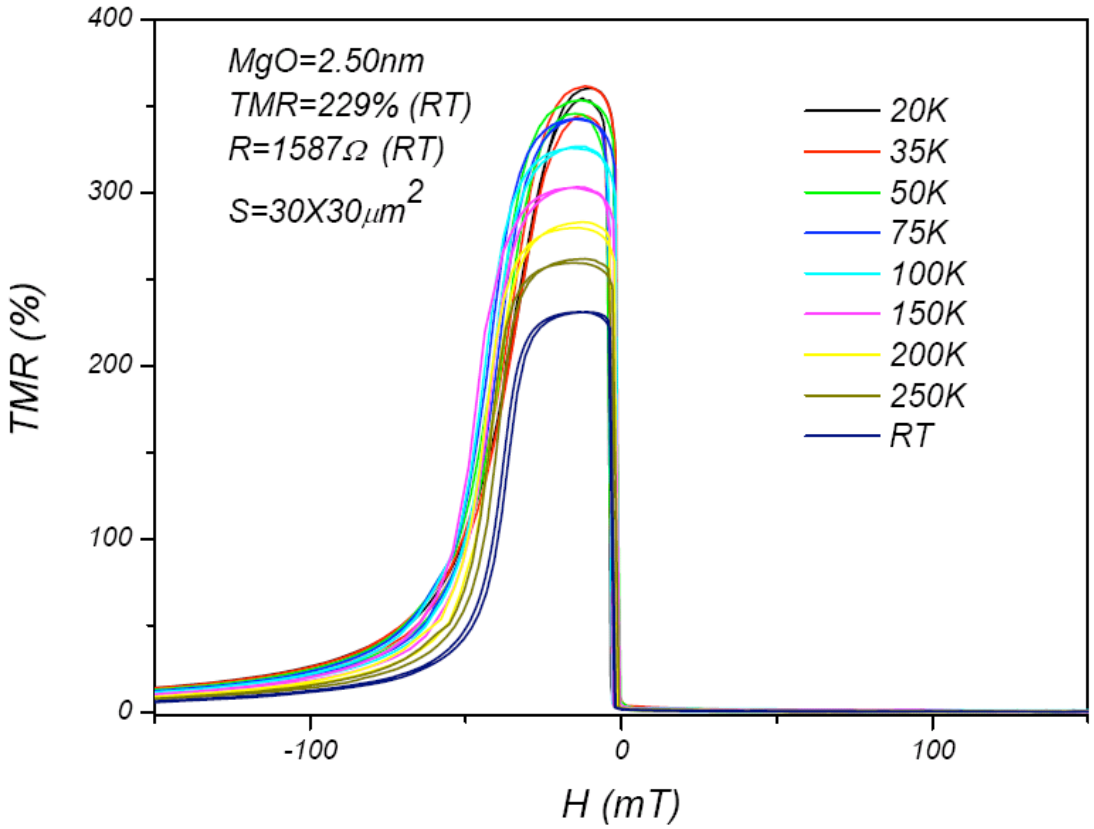
I μm²



2.2.7 Tunnel magnetoresistance.

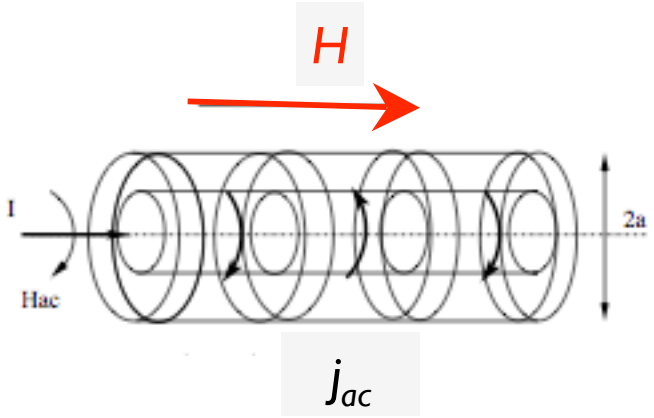


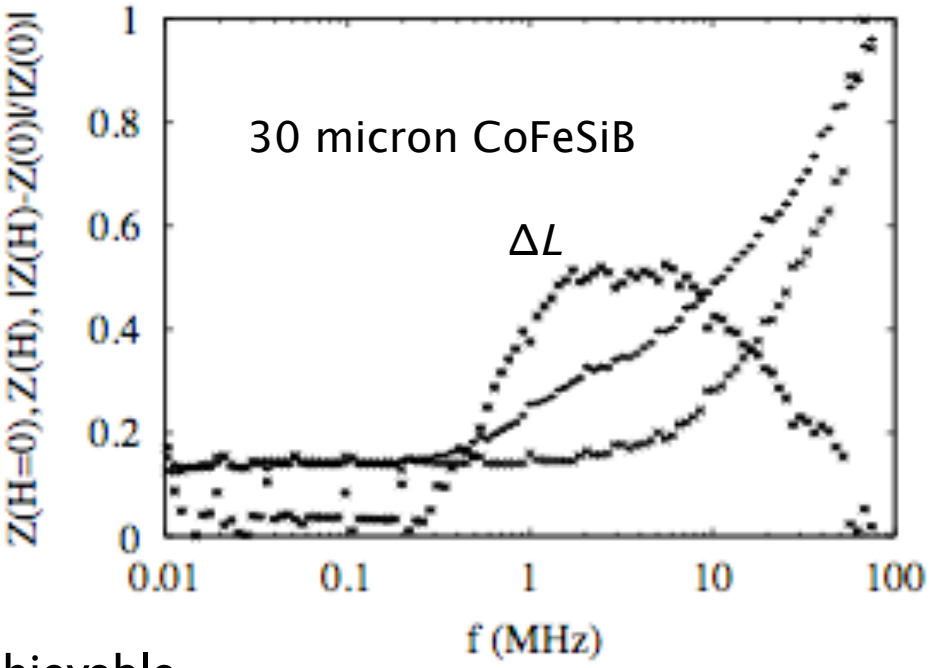
Magnetoresistance is > 100%, 10 times as great as for GMR spin valves



2.2.8 Giant magnetoimpedance.

GMI sensors are soft ferromagnetic wires (sometimes permalloy-plated copper wires} or films. An ac current is passed along the wire, and *L* is measured as a function of applied field. At high frequency, the skin depth < wire diameter. Permeability depends on *f* and *H*.





Very high field sensitivity, 10^4 % mT^{-1} is achievable
Used in Wii games, three-axis compasses …..

2.2.9 Magneto-optic sensors.

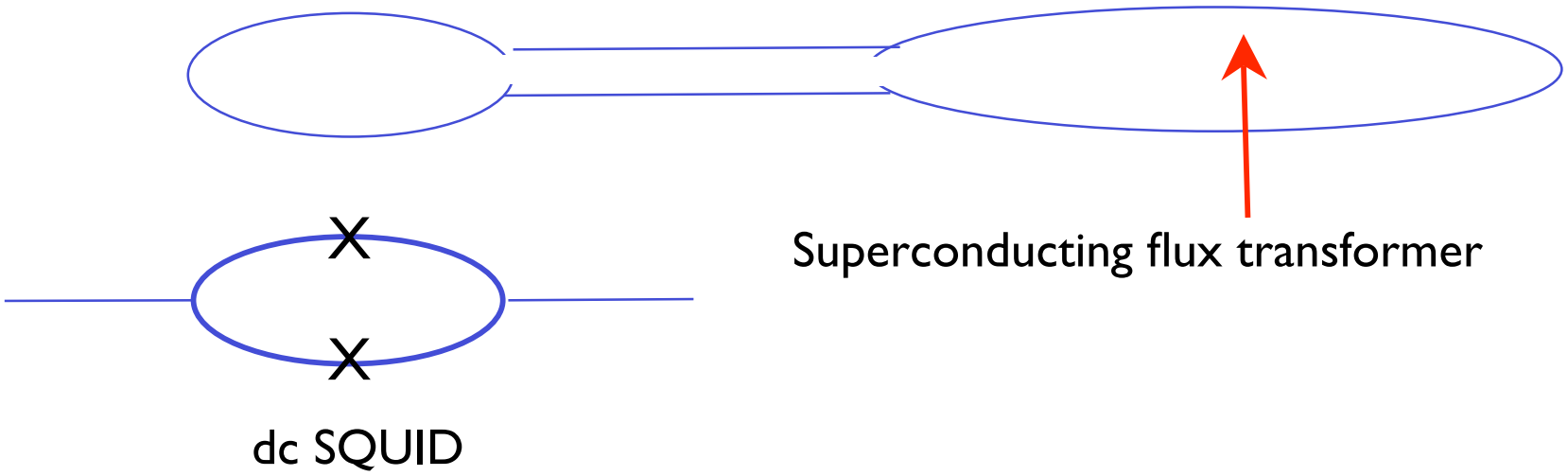
Optical fibre field sensors, based on magneto-optic Faraday effect. They are bulky, and used for large fields.

Rotation sensors based on Sagnac effect.

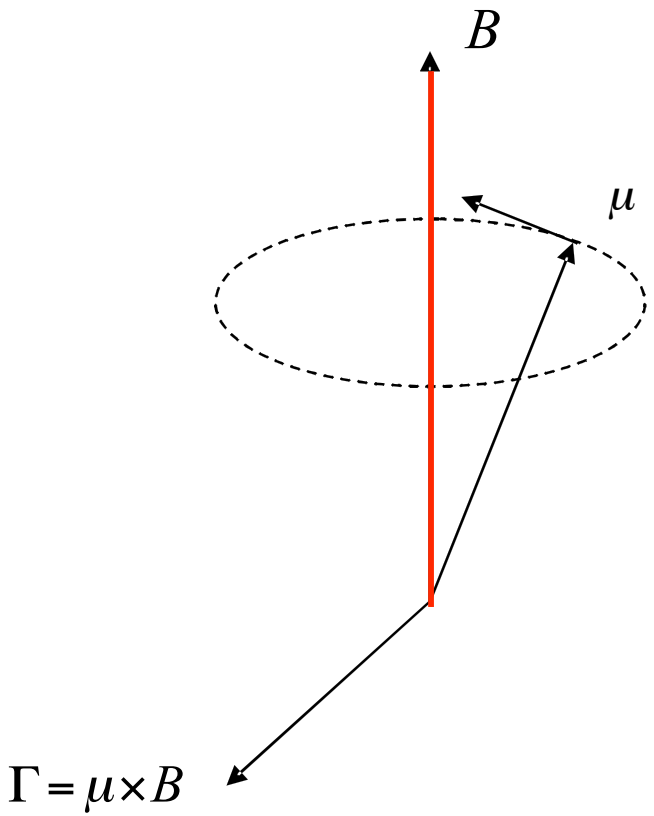
2.2.10 Superconducting quantum interference devices.

SQUIDs detect the change of flux threading a flux-locked loop. The flux is generally coupled to the SQUID via a superconducting flux transformer. The device is sensitive to a small fraction of a flux quantum. SQUIDSs offer ultimate field sensitivity. They generally operate with a flux-locked loop.

$$\Phi_0 = 2\ 10^{-15}\ \mathrm{T\ m^2}$$



2.2.11 Nuclear magnetic resonance.



Torque creates precession at the Larmor frequency $f_L = \gamma B/2\pi$

Protons (in water for example) can be polarized by a field pulse, and then allowed to precess freely (fid) at the Larmor frequency in the field to be measured. f_L in the Earth's field is ~ 2 kHz.

Rb or Cs vapor can be magnetized by optical pumping with circularly-polarized light, and the nuclear precession measured. The Rb-vapour magnetometer provides an extremely precise, absolute value of the magnitude of the field.

These magnetometers have been packaged on a chip.