Frequency shifts of cantilevers vibrating in various media

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A simple model is presented for rectangular cantilevers when vibrating in various media. The mass of the surrounding medium affected by the motion of the lever is calculated. It depends on the dimensions of the lever, on the excited mode, and on the density of the medium. Although the viscosity of the media is not taken into account, the resulting predictions for the resonance frequencies agree well with experimental data obtained for levers in air and water up to the seventh harmonic. © *1996 American Institute of Physics.* [S0003-6951(96)02045-1]

Tapping-mode atomic force microscopy (TMAFM) and dynamic force microscopy (DFM) are preferred techniques to investigate particular soft samples.[1-3] Both are designated as ac imaging modes but they are based on different types of interaction forces. In TMAFM, the tip of a vibrating cantilever briefly touches the surface at the bottom of each swing. This imaging mode can be interpreted as a hybrid form of repulsive contact and attractive noncontact atomic force microscopy (AFM).[4] In contrast, DFM stands for purely noncontact mode measurements where the interaction between tip and sample is dominated by long-range attractive forces.[5]

In order to extend the applications of scanning probe methods, it is essential to operate in a liquid environment. This is necessary to study biological matter under native conditions and to investigate processes at the liquid–solid interface such as electrochemical reactions and self-assembly of organic molecules. The disruption of soft samples, particularly biological ones, is reduced significantly when imaging in liquids since van der Waals forces, capillary forces, and friction forces are reduced.[2,6] Much progress has been made in operating not only contact mode AFM but also TMAFM and DFM in fluids.[4,7-11]

It is reasonable to say that local probe methods have become an important and sensitive technique to investigate sample surfaces in direct space. However, the imaging processes are strongly influenced by distorting interactions between the probing tip and the sample.[12] A reliable interpretation of the measurements can only be obtained if the dominant interactions between the sensor and its environment are known. Therefore, it is essential to investigate the resonance response of AFM cantilevers oscillating in different media in order to control their influence on the imaging processes.

So far, the attempts to model the dynamic behavior of TMAFM or DFM in various environments were based on a phenomenological model, replacing the lever by a harmonically bound sphere.[13,14] A description of viscous damping effects according to Stokes law is possible in this approach but the considerable shift of the resonance frequency due to an increased effective mass of the cantilever has not been explained in a satisfactory manner.[15,16]

In this letter we present a model which describes the motion of a rectangular cantilever oscillating in a medium of known density. The additional mass of the cantilever depends on the ratio of length to thickness of the beam, L/T, the density ρ_M of the surrounding medium and on the number of modes. The resulting frequency shift relative to the lever in vacuum agrees well with experimental data obtained for two levers of different dimensions suspended into air and water.

We measured the frequency characteristics of rectangular Si cantilevers with integrated tips driven by a piezoelectric actuator in different media far from any sample surface. An HP Spectrum Analyzer No. 3589 A was used to sweep the piezo linearly through a frequency range of 0–3 MHz and to analyze the resulting frequency spectrum of the cantilever oscillation detected by optical beam deflection technique. We analyzed the frequency spectra of cantilevers with lengths of 224 and 445 μm corresponding to spring constants of 0.22 and 29 N/m, respectively. The amplitude of the cantilever vibration was chosen to be 10–50 nm which is typical for ac mode. Measurements were done in dry air (30% r.H.) at normal pressure and temperature and in nanopure water.[17] For operation in liquids we completely immersed both, the cantilever and the piezoelectric bimorph, into a drop of water. The laser beam was coupled into the water through a glass cover slip to avoid an uncontrolled change in diffraction of the deflected beam which might be caused by evaporating liquid. This setup is appropriate for nonvolatile liquids and it has the advantage that fluid cell resonances can be excluded.

The undamped motion of a homogeneous bar of length L in vacuum is described by the differential equation for its amplitude $Y(x,t)$ at position x and time t

$$EI\,\frac{\partial^4 Y(x,t)}{\partial x^4} + m_B\,\frac{\partial^2 Y(x,t)}{\partial t^2} = 0, \tag{1}$$

where E is the module of elasticity for the bar and m_B its mass per unit of length, the rectangular cross section of the bar has an area $A = WT$, given by its width W and its thickness T, and $I = WT^3/12$ is the centroidal moment of inertia of the cross section. If the bar is clamped at one end, its nth eigenmode vibrates with a frequency

$$\omega_n^{\mathrm{vac}} = \frac{\alpha_n^2}{L^2}\left(\frac{EI}{m_B}\right)^{1/2}, \quad n = 1,2,\ldots, \tag{2}$$

with tabulated numbers α_n.[18,19]

Assume that the bar moves in an incompressible medium M without friction. For vertical speeds of the cantilever in

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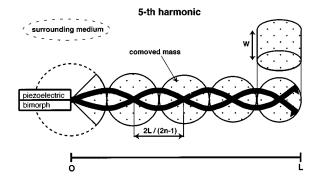[a)]Electronic mail: dreier@ubaclu.unibas.ch

FIG. 1. Scheme of a rectangular cantilever oscillating at it's fifth harmonic in a medium. The dotted areas represent the cylinder shaped comoved mass.

the order of 10^{-3} m/s, gases at normal pressure and room temperature will be considered as incompressible. In this case (1) is still valid, except for a comoved mass $m_{M,n}$ per unit length, adding to the mass of the bar: $m_B \rightarrow m_B + m_{M,n} = (1 + \mu_{M,n})m_B$. The dimensionless quantity $\mu_{M,n}$ accounts for the mass which has to be displaced by the bar when vibrating in the nth eigenmode.

In the following, a simple expression for $\mu_{M,n}$ will be derived. For moderate and large n, the nth eigenmode resembles a sinusoidal function having n nodes, including the one at the clamped end. The distance between two neighboring nodes is roughly $2L/(2n-1) \simeq \lambda_n/2$, where λ_n is the wavelength associated with mode n. Consider $(n-1)$ cylinders of height W and radius $L/(2n-1)$ with their axes located at the node lines across the bar (Fig. 1). The cylinders can be visualized by letting rotate a rectangle of size $WL/(2n-1)$ along its side with length W about each of the nodes. The mass of fluid contained in these cylinders is given by $Lm_{M,n}$, and even for *small* amplitudes it is this amount of fluid which has to be accelerated in order that the lever can

perform its vibration. Summing up the contributions of all the cylinders, the correction factor for the nth eigenmode is found to be given by

$$\mu_{M,n} = \frac{m_{M,n}}{m_B} = \frac{WL\rho_M \pi}{A\rho_B} \frac{(n-1)+1/4}{3} \equiv \frac{L}{T} \frac{\rho_M}{\rho_B} g_n, \quad (3)$$

where m_B has been expressed in terms of the area, A, and of the density of the bar, ρ_B. An nth cylinder associated with the clamped end contributes roughly a quarter of its volume to the comoved mass; hence the factor 1/4 in the numerator of the factor g_n. The additive mass is reduced by an overall factor of 1/3 to account for fluid motion from above the lever to below. Numerically, this factor follows from replacing the cylinders by a ''double cone'' obtained by rotating an isosceles triangle with base W and height $L/(2n-1)$ instead of a rectangle. Consequently, the factor $\mu_{M,n}$ is seen to depend on the ratio (L/T), on the ratio of the densities involved, (ρ_M/ρ_B), and—through the factor g_n—on the mode under consideration. Since, for increasing n, the volumes of the cylinders get quadratically smaller while their number increases only linearly with n, the comoved mass will decrease for eigenmodes with higher frequencies.

As a result, the frequencies of the bar in a medium of density ρ_M are given by

$$\omega_n^M = \frac{\omega_n^{vac}}{\sqrt{1+(L/T)(\rho_M/\rho_B)g_n}}, \quad n=1,2,\ldots, \quad (4)$$

if the derivation of $\mu_{M,n}$ is assumed to hold for *all* values of n. To sum up, the motion of the lever affects a layer of fluid which is approximately *half a wavelength* λ_n thick. Thus, its thickness may easily exceed the thickness of the cantilever by a factor of 10–50.

Figure 2 shows experimental data and theory for two different cantilevers in air and in water. Their resonance fre-
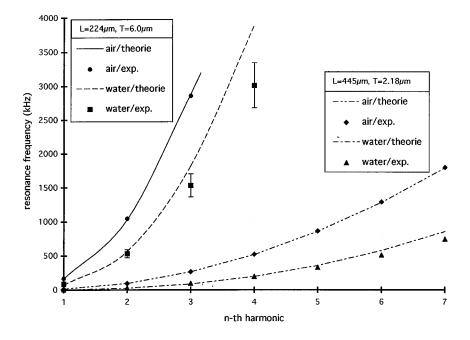

FIG. 2. Resonance frequencies obtained from theory and experiment plotted vs the nth harmonic oscillation of rectangular cantilevers in gaseous and liquid environments. Error bars for results in air and for the long cantilever ($L=445$ μm) in water are of the size of the symbols representing the experimental data. The errors of the calculated values for water ($L=224$ μm) are comparable to those of the experimental results.

quencies are plotted versus the nth harmonic oscillation. Experimental data are represented by discrete symbols. The full and the dashed lines represent numerical values calculated from Eq. (4) for a lever with $L/T \sim 40$, whereas the dash–dotted and the dash–dot–dotted line corresponds to $L/T \sim 200$. To guide the reader's eye, the calculated frequencies have been connected by smooth curves.

To calculate the frequency of the nth eigenmode using Eq. (4) the thickness T of the cantilever must be known. On the data sheet attached to the cantilever wafer,[20] only point probes of the geometric parameters are listed. To get a more precise value for the thickness of an individual cantilever the relation[18] $T = 2\pi\sqrt{12\rho_B/E}L^2\omega_0/\alpha_n^2$ is used, where ω_0 is the fundamental frequency of the level *measured* in air. The accuracy of theory and experimental data is mainly limited by the errors made in determining the resonance frequency.

The overall trend of the calculated resonance frequencies agrees well with the measured values. Note that in Fig. 1 the dependence of the correction $\mu_{M,n} = (L/T)(\rho_M/\rho_B)g_n$ on the dimensions of the lever ($L/T = 40$ and 200), on the densities (air and water), and on the mode (n ranges from 1 to 7) is checked. The agreement is better for levers in air than in water. This does not come as a surprise since the motion has been assumed to be frictionless: when changing from gaseous to liquid environment, the Q value gets reduced by a factor of 20–30 due to viscous damping effects. The Q factor has been determined from the relation $Q = \omega_0/\Delta\nu$, where ω_0 is the frequency corresponding to amplitude maximum and $\Delta\nu$ is the full width at $1/\sqrt{2}$nd maximum of the amplitude curve. In addition, the model predicts that the comoved mass vanishes for large values of n since $\mu_{M,n} \to 0$. However, it seems reasonable to think of a boundary layer being attached permanently to the vibrating cantilever. Including this layer in the model would reduce the discrepancy between the observed and calculated resonance frequencies for increasing n.

Finally, we would like to mention a possible influence of the comoved mass on a sample surface. The radius of the cylinders is about $L/(2n-1)$ and, thus, exceeds the length of the probing tip for small n. Consequently, an indirect interaction between the cantilever and the sample surface mediated by the comoved mass is likely to occur. This effect might be important when monitoring objects loosely attached to the surface.

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