Module Code	**EEU33E03**
Module Name	**PROBABILITY AND STATISTICS**
ECTS Weighting	5 ECTS
Semester taught	Semester 2
Module Coordinator/s	Associate Professor ANTHONY QUINN
Module Learning Outcomes with reference to the Graduate Attributes and how they are developed in discipline	On successful completion of this module, students should be able to: **LO1** Quantify and manipulate beliefs in uncertain propositions related to key engineering contexts, such as traffic phenomena (times and counts), device reliability, bio-imaging, radioactive decay and detection phenomena, and noisy communication **LO2** Distinguish between the vital notions of independence and dependence, and relate the latter to ideas of conditioning, association, causation and prediction **LO3** Represent discrete stochastic systems with probability graphs, and compute key inferences, both operationally and in Matlab, minimizing the computational load using Markov chains **LO4** Apply and analyze the standard discrete and continuous parametric probability models (distributions) governing uncertainty in engineering: Bernoulli, geometric, binomial, multinomial, Poisson, uniform, exponential, m-Erlang, Gaussian (univariate, multivariate, and mixtures) **LO5** Use expectation to evaluate measures of location, spread and dependence for random variables with these distributions **LO6** Construct regression functions for dependent random variables (uniform or Gaussian), equipping these with standard uncertainty intervals **LO7** Convert information-bearing experimental data into quantified beliefs, summarize these data via sampling statistics, assess dependence between data and exploit this in prediction, linearly regress bivariate Gaussian data, and test competing hypotheses for data **Graduate Attributes: levels of attainment** To act responsibly - Enhanced To think independently - Enhanced To develop continuously - Enhanced To communicate effectively - Enhanced

Module Content

OBJECTIVES

The primary objective of 3E3 is to provide a secure and accessible grounding for all sophister engineering students in probability and statistics. The module equips them with consistent methods for reasoning amid the uncertainties they encounter in their professional practice. In this way, the module supports decision-making in the uncertain contexts of real engineering practice.

The most important context in which engineers cope with uncertainty is when they gather and process information-bearing data, and use them to plan future actions. Therefore, statistical design is the main extended context for the application of probability methods in this module. Being a module for all sophister engineers, examples are drawn liberally from their universal human experience (games of chance, social analyses, COVID-19 statistics, etc.), but also from accessible contexts in engineering, notably vehicular traffic phenomena, reliability and lifetime of devices, causal inference in bioengineering, low-count and high-count sensor outputs in imaging systems, quantization and observation noise, and telecommunications.

The keystone of the module is a philosophical one. The fundamental relationship between uncertainty and information (learning) is explored from the start. An initial review of propositional logic is provided, being a familiar context for the engineer, so that the student can be confident in formulating propositions associated with an uncertain experiment, and in understanding logical relationships between propositions (sufficiency, necessity, equivalence). The probability calculus is then developed as a consistent means of quantifying and manipulating the engineer's beliefs in these uncertain propositions (i.e. the Bayesian perspective), opening a path beyond merely logical relationships. In this way, the foundation of the module is a unitary one, with such notions as logic, uncertainty, information, observation (data) and imprecision (noise) all embraced within a Bayesian notion of probability.

The module explores engineering contexts that induce the canonical probability models, in both the discrete case (Bernoulli, geometric, binomial, multinomial, Poisson) and the continuous case (uniform, exponential, *m*-Erlang, Gaussian). Their mixtures and transformations are developed as a response to practical modelling needs. There is a special emphasis on the concept of dependence (conditioning), and its relationship to the key notions of correlation, association, causation and prediction. In this context, probability graphs are introduced for systems of discrete dependent phenomena, and key inferences are computed. Markov chains are introduced as a key, tractable, and widely applicable relaxation into dependence. A main learning outcome for the student is the ability to choose the appropriate probability model to apply in a range of engineering contexts, such as those listed above, guided by an understanding of the assumptions that justify the deployment of each model.

A feature of the module is that it develops statistics consistently, using the same inductive inference principles described above. Often a blind spot in the formation of the engineering student, statistical inference is correctly re-cast in this module as an application of probability modelling, with the consistent statistical transformation being prescribed by the chosen model. In this way, the student gains the skill to *design* data transformations, dispensing with the traditional cook-book of statistical recipes which has little appeal to engineers with design interests. The simple nonparametric case is emphasized, using the elegant device of the empirical distribution. This allows students to derive appropriate descriptive statistics for their data, to estimate probabilities, and to describe association and regression phenomena quantitatively. An accessible introduction to parametric estimation is also provided, via moment matching techniques, and linear regression is developed via estimation of the bivariate Gaussian parameters. Finally, some of the standard statistical hypothesis tests are motivated and applied. Together, these topics introduce the engineer to the culture, nomenclature and practice of statistics as commonly encountered in engineering practice.

The module is an invitation to the student to confront uncertainty as a fundamental phenomenon – and resource – in engineering systems, to appreciate probability as a consistent framework for the design, analysis and optimization of such systems, and to explore and exploit data in a principled way.

SYLLABUS

Review of Propositional Logic

- Uncertain experiments in engineering
- Sample space, propositions and events
- Propositional logic: equivalence, necessity, sufficiency

The Foundation of Probability Modelling

- The axioms of probability and the probability triple
- Conditional probability; dependence vs. independence
- Key probability calculus: chain rule, Bayes' rule, theorem of total probability
- Conditional independence

Sequential Experiments

- Independent sequential experiments
- Sequential Bernoulli trials: geometric, binomial and multinomial probability laws
- Introduction to computations on probability graphs, and with Markov chains

	Univariate Random variables Probability functions for random variables (cdf, pdf, pmf) Key discrete probability models: Bernoulli→geometric→binomial→multinomial→Poisson Key continuous probability models: uniform, exponential, *m*-Erlang, Gaussian Functions of random variables Expectation, and the moments of random variables **Multiple random variables** Marginal and conditional distributions Discrete-continuous case: finite mixture models Correlation and regression The bivariate normal (Gaussian) distribution, and linear regression **Statistics and Data Analysis** Random sampling: the empirical distribution and its moments nonparametric sampling statistics nonparametric probability estimation Parameter estimation by moment-matching Analysis of errors Design of statistics for description, regression and hypothesis testing
Teaching and Learning Methods	The module will be delivered via about 44 real-time, live sessions, delivered in a hybrid manner via Blackboard Collaborate Ultra (BbCU), and also from College lecture rooms, consistent with College COVID-19 regulations at the time. A 3:1 ratio is maintained between theory and tutorial content during the contact sessions. Problem-solving experience is vital, and is gained primarily via the tutorial exercises and the in-session discussions, and also via about 8 homework sheets distributed uniformly across the semester. Attendance at all the module contact sessions is compulsory, and is necessary for the acquisition of the learning outcomes. All audio-visual (AV) and textual record captured during the sessions will be available for consultation by students in BbCU, following each session.

Assessment Details

Assessment Component	Assessment Description	LO Addressed	% of total	Week due
Mid-semester in-class test	Real-time, online test (1 hour)	1-5	15	8
End-of-semester in-class test	Real-time, online test (1 hour)	1-7	15	12
Final examination	Real-time, online examination (2 hour)	1-7	70	Semester-2 assessment period

Reassessment Requirements

Examination only (2 hours)

Contact Hours and Indicative Student Workload

Contact hours:
44

Independent Study (preparation for course and review of materials):
56

Independent Study (preparation for assessment, incl. completion of assessment):
25

Recommended Reading List

The primary learning resource is the full set of lecture notes and self-study materials developed during the contact sessions, and captured as 44 AV recordings in BbCU.

The recommended text for the module is:

1. Bertsekas, D.P. and Tsitsiklis, J.N., *Introduction to Probability,* 2nd ed., Athena Scientific Press, 2008.

Secondary recommended texts are as follows:

2. Ross, S.M., *Introduction to Probability and Statistics for Engineers and Scientists*, 5th ed., Academic Press, 2014.

3. Pishro-Nik, H., *Introduction to Probability, Statistics, and Random Processes*, Kappa Research LLC, 2014. (Available online at www.probabilitycourse.com)

	4. Applebaum, D., *Probability and Information,* 2nd ed., Cambridge University Press, 2008.
Module Pre-requisite	Engineering Mathematics (up to Year 2 incl.)
Module Co-requisite	None
Module Website	See module in Blackboard
Are other Schools/Departments involved in the delivery of this module? If yes, please provide details.	No
Module Approval Date	4th October 2021
Approved by	Professor Anthony Quinn