



# Non-local Models of Solidification

Vaughan R. Voller  
Civil, Environmental, and Geo- Engineering, Professor  
University of Minnesota



## Event Details

**Date:** Thursday 18 December 2014

**Time:** 15:00

**Venue:** Crossland Lecture Theatre, Parsons Building, Trinity College Dublin

## Abstract

A common feature in solidification processes and phenomena is the occurrence of heterogeneities at various length scales, e.g., the morphologies of columnar and equiaxed crystals, and the distribution of solute components. With this in mind, it is noted that in the presence of heterogeneities, with a range of length scales, some transport process may exhibit non-local behaviours. By this we mean that the flux of a conserved quantity, at a point in space and time, may be controlled by features present at earlier times (memory) and/or features not located at the point of interest. In the case of diffusion like transport processes, the effects of non-locality can be modelled through using fractional order derivatives to replace the flux terms in the governing equations. Essentially such a device provides a phenomenological model that treats the non-locality in the flux as a weighted sum of gradient evaluations at points back through time and/or space. The aim of this talk is to explore if such non-local treatments may have relevance in modelling solidification processes and phenomena.

First we show how introducing fractional derivatives into the classical Stefan melting problem results in so called anomalous diffusion behaviour, where the time exponent for the movement of the melt interface differs from the expected value of  $\frac{1}{2}$ . We provide illustrative examples from phase change literature where such behaviour has been observed and then demonstrate, through a direct numerical simulation, how such behaviour is related to the nature and length scale distribution of heterogeneities. The talk is concluded by providing example non-local numerical solutions of various solidification systems along with a discussion of under what circumstance a non-local solidification model could be physically valid.

## Biographical Sketch

Vaughan Voller's principal research interest is developing numerical and mathematical techniques for computational models to describe and understand heat and mass transport phenomenon. A core theme has been constructing methodology for handling free and moving boundary value problems associated with phase-change systems. Key examples include: finite element-based modelling of melting and solidification phenomena, crystal growth, hydraulic fracturing, polymer mould filling, transport in porous media, and the formation of sedimentary deltas.

Voller and his group also have a strong interest in exact mathematical treatments such as studying analytical solutions of phase-change/moving-boundary problems related to crystal growth and sedimentary deltas, and investigating the applications of fractional calculus to describe non-local diffusive transport systems. In contrast to this more rigorous mathematical work, his group also has an emerging interest in developing reduced complexity rule-based models of mass transport systems with particular emphasis on creating channel resolving models of sedimentary deltas.