SIR WILLIAM ROWAN HAMILTON

William Rowan Hamilton was born in Dublin's Dominick Street, at midnight on the 3rd of August 1805. When he died some sixty years later he was without question the most famous Irish scientist of his day. He has yet to relinquish that title. Graduate and professor of this College, he is the obvious subject for the Discourse in this his bicentennial year, or Hamilton Year, coinciding and co-equal with the celebrations of Einstein Year and the World Year of Physics. But to confine Hamilton within the timescale of a discourse is a challenge: there is just too much Hamilton material, all of it readily available in three biographies; namely, the 3-volume opus by Robert Graves published in 1882, 85 and 89 and the two more recent books of Thomas Hankins and Sean O'Donnell, published in 1980 and 1983, respectively; moreover, at this point in the year there may already be signs of Hamilton-fatigue; yet I am encouraged by (as well as indebted to) the many colleagues in College and beyond who have plied me with their favourite Hamilton anecdote. With all this in mind I have chosen to present Hamilton to you, in the first instance as a Trinity man of science, supplementing this with a broader extramural sketch that follows, if you will permit me, a mathematical pattern. While you, as it were, bask in the reflected glory of the former, you might grant me this little indulgence in the latter.

We may establish Hamilton's first connection with this College when, at the age of three, he is sent to live with his father's brother in Trim. The Reverend James Hamilton was a Classics graduate of the University of Dublin, having entered Trinity at the early age of fifteen. Uncle James was then a Church of Ireland curate and manager of the Meath Diocesan School, living and working in the manor house known as Talbot's Castle. His appointment was "to instruct the children in the said school in the Latin tongue and in all other tongues in which you are skilled". It is no great surprise then that Hamilton's mother can boast that William's "reciting is astonishing and his clear and accurate knowledge of geography beyond belief; but you will think this nothing when I tell you that he reads Latin, Greek and Hebrew!" Unsurprising except that William is then aged just five! Two years later the boast is of French and Italian, including the prodigy's desire to read Homer in French and three years later again there is added a litany of oriental languages. The pace of Hamilton's mathematical training in this same period is more modest.

In 1818, however, that changed dramatically, when the young Hamilton encountered Zerah Colburn, the American Calculating Boy. Hamilton was put up against Colburn as a local champion: he was well beaten but he had caught the mathematical bug. In an interesting parallel of role-reversal Colburn went on to become a professor of languages. By 1821, we find Hamilton immersed in this new interest of mathematics but, to Uncle James' alarm, in the wrong sort of mathematics, that is, material *not* on the course for entry to Trinity! Also, in this period, a second mentor appears. He is Charles Boyton, newly elected to Fellowship in Mathematics and Hamilton's future Tutor in College. But well before that, Boyton is advising the schoolboy Hamilton on the 'new' French mathematics and Hamilton is finding a small error in Laplace's celebrated book *Méchanique Céleste*. If Boyton was surprised, he was at least prepared for more of the same. In July of 1822, Hamilton writes to his sister:

> "I called on Charles Boyton, the Fellow, last week. He was trying to solve a problem in Analytic Geometry, which he showed me, and I had the pleasure of solving it before him; for, two days after, when I brought the solution, I found that he had not succeeded!"

A parable, perhaps, for today's new Fellows! Also in this period, Hamilton developed two further interests that would intersect with his mathematical career. One was the composition of poetry, an activity he shared with his sister Eliza; the other was astronomy, following the gift of a telescope from his father's first cousin, Arthur Hamilton. On the death of Hamilton's father in 1819, this same Cousin Arthur would provide Hamilton and his siblings with a Dublin 'home', as well as with considerable general support.

Meanwhile Uncle James maintained academic pressure on Hamilton, directed towards the College Entrance examination, which he finally took in July 1823; he should have taken the examination the previous year but there had been some illness and the watchful James held him back. The result was a vindication of James' stewardship; Hamilton came first out of one hundred candidates and received a premium or book prize for excellence in Hebrew. In his first year in College Hamilton came first in his division in each subject, gaining the Chancellor's Prize for poetic composition *twice* and receiving an *optime* in Classics, an indication of ability that is beyond examination. Hankins remarks that Hamilton's College career lacks interest because of the sheer inevitability of it all. But in his second year in College Hamilton did not receive the expected premium and he missed the January examinations of the following year through illness. When he did take the examinations in April he was awarded his second *optime*, this time in Science. For his final year Hamilton set himself the considerable task of winning a double gold medal in Classics and Science.

Hamilton's College success was noised abroad, with rumours reaching Cambridge as early as 1824. But Hamilton was not to be measured just by examination. In December of his second year he submitted an original research paper to the Royal Irish Academy; the paper, titled *On Caustics*, was judged "novel and highly interesting" but was rejected on various grounds, including lack of clarity. The paper was re-worked and presented two years later as *Theory of Systems of Rays*, a novel application of Cartesian methods to describe light rays. The paper was accepted.

Hamilton was clearly destined for Fellowship after graduation but fate intervened, in the guise of the Established Church. In late 1826 the Andrews Professor of Astronomy, The Reverend John Brinkley, was elevated to the bishopric of Cloyne. The Board delayed in appointing a successor until the following June. Charles Boyton vigorously canvassed his colleagues on behalf of his star pupil, claiming that Hamilton "had been in the habit of staying days and nights at the Observatory employed at the instruments and [had] made himself acquainted with their use in detail". This was a considerable exaggeration! At the same time Boyton urged Hamilton to apply for the post, emphasising the financial advantages of the appointment over Fellowship and adding "I would be glad to see you provided for, and I would be glad to produce to the world *one* creditable act of the Board!"

On June 16th, 1827, the Board duly obliged and unanimously elected Hamilton Andrews Professor of Astronomy; the position also included the titles of Royal Astronomer and Director of Dunsink Observatory. He was not quite 22 years old and two quarterly examinations short of his degree. And he was no astronomer. After an initial enthusiasm for the tedious round of observation, Hamilton spent little time in the meridian room, leaving these duties to his assistant

Thompson and to his sisters Sydney and Grace. In 1831 he confessed to Romney Robinson, his opposite number at Armagh Observatory, that: “My tastes, as you know, are decidedly mathematical rather than physical, and I dislike observing”. In that same year, there was an opportunity to redeem the situation when Bartholomew Lloyd was elected Provost, vacating the Chair of Natural and Experimental Philosophy. In the event Lloyd’s chair was filled by his son, Humphrey, and Hamilton remained at Dunsink.

Humphrey Lloyd would have an important role in the next chapter of Hamilton’s career. Following on from the publication of *Theory of Systems of Rays* Hamilton developed his ideas in three further *Supplements*. In the preparation of the last of these he made a novel discovery. When a ray of light is incident on certain (biaxial) crystals, the emergent light separates into two rays. This double refraction was well known but Hamilton predicted that under very specific conditions of alignment the emergent light would instead be a hollow cone. No such conical refraction had ever been experimentally observed. Hamilton announced his discovery to an evening meeting of the Royal Irish Academy on 22nd October, 1832. The following day he asked Lloyd to try to verify the prediction. Lloyd’s initial attempts were unsuccessful due to the poor quality of his biaxial crystal sample. After a matter of days Hamilton became impatient and pressed Lloyd into agreeing to an approach to George Airy, the British Astronomer Royal, in hopes that *he* would have more success. But Lloyd also persisted with his own experiments and with a good crystal he finally verified the prediction in December.

Modern physics is not much excited by Conical Refraction but its announcement electrified the scientific community of the day. Airy called it “ perhaps the most remarkable prediction that has ever been made” and it would later be commonly compared to the subsequent prediction and discovery of the planet Neptune by Adams and Leverrier. I hope you will forgive now some blatant advertising but two days hence in the Burke Theatre my colleague, Professor James Lunney, will give a striking demonstration of this phenomenon and a distinguished visitor, Professor Sir Michael Berry FRS, will leave you in no doubt as to the complexity of the mathematics behind it. Other contributions to this Trinity Week Symposium will place Hamilton in his social and scientific times while Professor Brendan Kennelly will speak about creativity in poetry and science.

In 1835 Hamilton was awarded the Royal Medal of the Royal Society – the other recipient that year was the English Physicist, Faraday. In the same year he was knighted by the Lord Lieutenant in the Long Room, where his bust normally now resides; I am grateful to the Librarian Robin Adams for facilitating its temporary relocation here during this discourse. But well before the honours for Conical Refraction materialised, Hamilton had begun to work on a much less-regarded topic, one that would nevertheless guarantee his fame in modern times. He had extended his thinking to mechanics, making an analogy between the path of a light ray and the path of a moving body. His two papers in 1834 entitled *On a General Method in Dynamics* were well received but mainly where they impinged on the broader topic of the nature of the aether, the mysterious medium that was then believed to support light. But today the core of Hamilton's method, the so-called Hamiltonian, is central to quantum mechanics, as a 'google' computer search will dramatically demonstrate.

In that busy year of royal honours, Hamilton published a further paper with a most poetic title: *Algebra as the Science of Pure Time*. As the word 'pure' might suggest, the genesis of this paper was more metaphysical and it would fix the path of the rest of Hamilton's research. Hamilton used time as a series of 'steps' forward or backward to reflect the positive and negative numbers, respectively; in this way negative numbers acquire, for the first time, a status equal to that of positive numbers. More significantly, Hamilton was drawn into the algebra of complex numbers. Applying his time step methods, Hamilton replaced the complex number combination of 'real and 'imaginary' parts with the concept of number pair or couple. The consensus at the time was that while all this clarified the foundations of algebra, it did not lead to anything new; couples and complex numbers being of equivalent application. But if the concept could be extended to triplets or higher order groups, that would be novel.

For the next eight years Hamilton laboured on triplet numbers. Their triadic appeal was clear; where number couples could correspond to two-dimensional or planar geometry, triplets would describe the full three-dimensional geometry of space. Fundamental problems arose, however, in the multiplication of triplets, problems that did not occur in the multiplication of couples. Hamilton became increasingly engaged by the philosophy of triads, going on to construct a personal triad of Will, Mind and Life. In 1843, however, he returned to the mathematical aspect of triplets. Hankins offers a number of reasons for this renewed interest, including the prospect of the triadic re-shuffle of chairs, following Humphrey Lloyd's resignation to become Senior Fellow. In this scheme Professor James MacCullagh, would take up Lloyd's chair, vacating his own chair in Mathematics in favour of Hamilton. But issues of Fellowship arose (Hamilton was unwilling to take the examination) and despite the Board's concern about the management of Dunsink, Hamilton remained outside the College walls, and on the lower salary.

Whatever the reason for the renewed interest in triplets, it finally bore fruit in the autumn of that year. On the 16th of October, while walking with his wife along the Royal Canal, en route to the Academy, a possible solution appeared in the shape of not three but four- member numbers or quaternions: Hamilton would later recall the event thus:

> "An undercurrent of thought was going on in my mind, which gave at last a result, whereof it is not too much to say that I felt at once the importance. An electric circuit seemed to close; and a spark flashed forth, the herald (as I foresaw, immediately) of many long years to come of definitely directed thought and work……nor could I resist the impulse –unphilosophical as it may have been – to cut with a knife on a stone of Brougham Bridge, as we passed it, the fundamental formula."

Historians argue over the veracity of this latter act. Yet, it seems not wholly out of character, and is worthy of such a *Eureka* moment. What is certain is that the fundamental formula $i^2 = j^2 = k^2 = ijk = -1$ appears in Hamilton's pocket notebook entry of the day.

The announcement of quaternions created a mixed response. There existed a considerable number of mathematicians who, like Hamilton, were trying to solve the triplet problem, and would continue to try to do so. Others moved ahead to hypercomplex groups, such as 8-member octaves. Hamilton remained true to quaternions, absolutely convinced that they represented a completely new method of mathematical notation, one capable of displacing all previous methods, including Cartesian geometry. Over the next ten years Hamilton produced a large number of papers on the topic, including the lengthy *On Quaternions: Or on a New System of Imaginaries in Algebra* which appeared in eighteen installments in the scientific journal

Philosophical Magazine. This period culminated in the publication of his book, *Lectures on Quaternions*, based on the lectures given in this College. The book was long and difficult to read; it was not a success.

In response to the urgings of fellow mathematicians Hamilton now began work on a simple introductory manual. However, like its precursor, the manual quickly expanded into an even longer book. *Elements of Quaternions* was not published until after Hamilton's death and the Board was obliged to pay the printer's considerable bill. Of the 500 copies printed many were given away as presentation copies.

Quaternions did however open the way to several fruitful areas, such as the relaxing of the commutative law of multiplication, whereby A times B must equal B times A. Furthermore, just as a complex number was the sum of a number and an imaginary, a quarternion is the sum of a number and a line in space. Hamilton was the first to distinguish these as the familiar scalar and vector, respectively, of modern vector analysis. In addition, the invention of the *calculus* of quaternions would be later described by James Clerk Maxwell as comparable to the invention of triple coordinates by Descartes. At the end of this week The Central Bank will launch a 10 euro Hamilton commemorative coin, incorporating the so-called "nabla" symbol of that calculus; This suggestion by our colleague Professor Denis Weaire creates a pleasing symmetry with the coin's obverse harp of Ireland.

After Hamilton's death a great battle was joined between the quaternion camp, notably Tait, and the inventors of vector analysis, Josiah Gibbs and Oliver Heaviside. There was much intemperate language on both sides with Heaviside writing dismissively that "quaternions furnishes a uniquely simple and natural way of treating quaternions"! Yet the appeal of quaternions persisted well into the 20th century, and in some unusual corners. Perhaps the most unusual is *The Insect*, the in-house journal produced by post-1916 rebel prisoners in Lincoln Jail. In Volume 2 dated 15th September 1918, among the entries to a poetry competition, there is a sonnet-length verse addressed to someone called *Quaternia*. Here are the closing couplets

And in the ages yet to rise and roll
Until annihilation's awful knell shall toll
Shall thou and I beloved find the means
To knock Algebra into smithereens

A suitably explosive ending for a future Taoiseach and President? And surely some head-scratching at British counter-intelligence for the title of the poem (as the competition required) was "My best Girl".

It seems likely that Dev had some involvement in the 1943 decision to issue a commemorative stamp, marking the centenary of Hamilton's discovery. Fortunately, Erwin Schroedinger was on hand to redress the balance as to Hamilton's achievements. He is quoted in the Irish Times thus:

> "I daresay not a day passes – and seldom an hour – without somebody, somewhere on this globe, pronouncing or reading or writing or printing Hamilton's name. That is due to his fundamental discoveries in general dynamics"

Yet, this year's commemorative stamp also foregrounds the quaternion formulation; there is an undeniable iconic appeal in an equation of such simplicity. Therefore, I am emboldened to employ a rough quaternion structure to discuss the wider extramural life of Hamilton and his times. That is, I mean to place Hamilton in tow with some significant triplets. The earliest (and best founded) example of this is the Honourable Society of Four devised by the teenage Hamilton in 1819, comprising himself and his sisters, Grace, Eliza and Sydney, the fourth sister Archiana being too young to be involved. The importance of these three women in Hamilton's life cannot be overstated, with Grace as the household manager, Eliza as fellow poet and close confidante and Sydney as astronomical assistant. The need to provide a home for his sisters after their parents' deaths played some part in Hamilton's decision to go to Dunsink. The same sisters' abrupt departure from Dunsink on Hamilton's marriage must have caused considerable upheaval on many fronts.

A related quaternion comprises Hamilton and his own three children, William Edwin, Helen Eliza and Archibald. None of these showed any particular interest in science or scholarship generally. Perhaps this in not unconnected with the ambience of Dunsink, as caught in the anecdotal breakfast query “Well, Papa, can you multiply triplets yet?”. William Edwin emigrated to Canada where he worked in the newspaper business, Archibald followed Uncle James into the church and Helen Eliza married another clergyman, John O’Regan and died young of complications following the birth of a son. It is descendants of that family who preserved much of Hamilton’s correspondence and papers.

A rather different quaternion comprises Hamilton and three younger Irishmen of wealth and influence, Viscount Adare, Aubrey de Vere and Francis Edgeworth, step-brother of the novelist. Hamilton initially tutored the young Adare, and it was he who later would make the suggestion that Hamilton move to the Chair of Mathematics in this college. Edgeworth and de Vere were poets and all four shared a passion for the idealist philosophy that informed Hamilton’s mathematics. There were later discussions about the Oxford Movement; both Edgeworth and de Vere converted to Rome but Hamilton remained in the Church of Ireland, becoming churchwarden at Castleknock in 1851. Overall, the group provided Hamilton with important entrées to society, as well as access to female relatives of marriageable age.

Hamilton’s intention to marry was an important factor in his choosing Dunsink over the celibate life of a Fellow. In this regard, Hamilton forms a quaternion with three very different women, Catherine Disney, Ellen de Vere and Helen Bayly. Hamilton would later write that towards Catherine he had felt himself a lover, towards Ellen he had been a brother and to Helen he had been a husband. Of the three, his interaction with Ellen was the least complicated. He chose to interpret her declaration that she could “not live happily anywhere but at Curragh” as a rejection of his as yet undeclared suit; in any case her brother Aubrey was opposed to the union ‘on philosophical grounds’. Hamilton was more persistent with Helen Bayly, whom he eventually married in 1833. However, the future Lady Hamilton was semi-invalid and for much of the marriage lived with her relations. So much so that the Armagh astronomer Romney Robinson could remark that she was ‘but an abstract idea’.

Catherine Disney was an abstract idea of a different kind. Hamilton met and fell in love with her during his second year in College but her parents arranged for her to marry an older, more established man, the Reverend William Barlow. Hamilton was deeply upset and this probably occasioned the illness at the beginning of his third year in College; his attachment to Catherine, or to the ideal of her, permeated his life. During an encounter with the now Mrs Barlow at Armagh Observatory some five years later, Hamilton became so agitated that he broke the crosswires on the eyepiece of the telescope. Their paths crossed again when Catherine’s son James entered Trinity, leading to an exchange of letters of an increasingly passionate nature and culminating in Catherine’s attempted suicide in 1848. Hamilton was greatly affected by this turn of events. Although Catherine would no longer reside with her husband, the only outward expression of Hamilton’s feelings for her was his steering of her son though the Fellowship Examinations; that he was simultaneously tutoring Barlow Jnr in quaternions while supplying the same material to the Examiner might seem less than proper. But there is no denying the propriety and the pathos of the scene in which Hamilton presents the dying Catherine with a copy of his *Lectures on Quaternions*, describing this later as:

> “Kneeling, I offered her the Book which represented the scientific labours of my life. Rising, I received, or took as my reward…a kiss, nay, many kisses”.

And he continues…

> Yet I dare to affirm that our affectionate transport in those few permitted moments, was pure as that of those who in the resurrection neither marry nor are given in marriage, but are as the Angels of God in Heaven”

The final act was Hamilton’s hurried retrieval of his letters from within Catherine’s deathbed.

Catherine Disney was the subject of much of Hamilton’s poetry and a quaternion of poets has many candidates. Leaving aside those already mentioned, that is, Eliza, Edgeworth and de Vere, the choice falls on William Wordsworth, Samuel Taylor Coleridge and Speranza, the mother of Oscar Wilde. Hamilton met Speranza late in life and although he turned down her invitation to be the young Oscar’s godparent, they remained friends. In his book *Oscar Wilde: The*

Importance of Being Irish, Professor Davis Coakley describes the 'Feast of Poets', an evening of poetry at the Observatory in April of 1858, attended by Speranza, among others. Hamilton later wrote to Lady Wilde:

> "I am unable to recall – so much of human music was there in the poetical party at which you were so kind as lately to assist – whether the birds were singing at that time. This morning I have unlocked the hall-door, that I might listen more freely to the storm, the tempest, the whirlwind of delight, and of music with which the birds are now surrounding this house and me."

The Wordsworth connection was of an earlier vintage: Hamilton met Wordsworth on holiday in England a few months after his appointment at Dunsink; the elder poet also visited the observatory two years later, where they engaged in animated discussions about Science and Poetry. Hamilton then still entertained hopes of becoming a great poet but Wordsworth firmly advised against this, stressing the great effort of craftmanship required in poetry, just as in mathematics. If Hamilton was disappointed at Wordsworth's rejection, he had the consolation of an introduction to Coleridge. Hamilton's sonnet in praise of the great telescope at Birr closes with the following couplet

> gleamed to the west, far seen, the Lake below;
> and through the trees was heard the river's flow

which has more than an echo of Kubla Khan. But it is Coleridge's philosophy that is of real importance and, in this regard, Speranza should give up her place in favour of Kant, in whose writings Hamilton found ready resonance for his own ideas about science.

There are two further triplets that are worthy of more detailed discussion but which are only briefly noted here. The first is the public fora in which Hamilton presented his work, namely this College, the Royal Irish Academy and the British Association. Hamilton's main duties in college were his lectures on Astronomy and his oratory on these occasions was greatly appreciated; the introductory lectures were attended by the public, including, to the Fellows' dismay, a number of women! Hamilton was a more significant presence in the Academy, being an effective President for many years. He was also a major figure in the fledgling British Association for the Advancement of Science, where both his oratory and his counsel were much in demand; Hamilton and Humphrey Lloyd co- hosted a Dublin meeting of the association in 1835, which was the occasion of Hamilton's knighthood. It is appropriate to mention in this company the National Academy of Sciences, then newly formed in America, which shortly before Hamilton's death elected him as the first on the list of foreign associates.

The other triplet might be labelled 'antagonists'. Hamilton was generally well-liked, describing himself as possessing "a great fondness for an argument but not a disposition to quarrel". One possible antagonist might be The Reverend William Barlow, Catherine Disney's husband, but on balance he was the more injured party. A much better candidate is James MacCullagh, the mercurial professor of mathematics at Trinity. On more than one occasion MacCullagh claimed priority of discovery in relation to Hamilton's research, most notably in conical refraction and in quaternions; in the latter case he claimed to have anticipated quaternions in a theorem on the ellipse that he had placed on the Fellowship examination the previous year. But MacCullagh was becoming increasingly unstable and took his own life in 1847. That Hamilton was much distressed at this tragedy is well-founded; that he should empathize with MacCullagh is understandable for he too seems to have suffered on and off from depression and his latter years were disfigured by a degree of alcohol abuse. It might be argued then that the only real antagonist was Hamilton himself!

But to close on a more positive note, I would like to select one from a quaternion of altogether gentler images. It might be the chop of indeterminate age, discovered as a bookmark in Hamilton's study; or it might be the pillowcase full of books that Hamilton carried around with him, spilling its contents off the top of a Birmingham omnibus; it might even be the Icosian board game that Hamilton invented and marketed, but that turned out to be too easy for children. My own preference is the string passed through the bedroom wall at Trim and attached to his nightshirt; it is an appropriate choice given that my time is up. So, I will leave it to that other Trinity man, Uncle James, to awaken again that splendid mix of mathematics and philosophy, poetry and passion that is Sir William Rowan Hamilton.