## Chapter Three

## Varieties of voting systems

We argued in the previous chapter that the initial intuitive appeal of the simple majority rule principle is seriously undermined once we begin to take into account the numerous complexities that arise in the light of a deeper and more subtle analysis of political conflict and the possible structures of group preferences. It is self-evidently true, however, that binary-option majority rule is simply not applicable at all where a choice has to be made among three or more options. Given the normality, for example in the election of representatives to a legislative body, of the situation in which there are more than two options (candidates), no theory of voting can neglect the systematic analysis of different possible voting systems. We have, fairly informally, alluded to a few of these in passing, as well as pointing out the democratically anomalous consequences of some. In this chapter we turn explicitly to the task of identifying and explaining the main variants that need consideration and the criteria relevant to their assessment.

Let us begin with the criteria. As we said when discussing May's account of majority rule, anonymity and neutrality between options are obviously desirable and democratically justifiable from the perspective of basic prospective equality. If one person's vote was to have, because of whose vote it was, a greater determinative effect on the outcome than other votes, this would straightforwardly contravene initial institutional equality of opportunity. Lack of neutrality between options would have a similar, though more indirect effect. One quite common contravention of the neutrality between options condition occurs when the status quo is privileged by, for example, requiring a super-majority if it is to be changed. Privileging any option gives greater initial weight to those voters favouring that option. May's own decisiveness condition is very weak, amounting only to the claim that the procedure should assign $a$ value to each option, whether that value is positive, negative or "indifferent". Simply from the point of view of decision-making efficiency, however, it would be undesirable to have a decision rule that frequently resulted in an indeterminate result, necessitating a random selection of the option to be implemented. One could even criticise such a rule from a specifically democratic perspective, because in the situation where the result is determined by a random procedure, people's preferences are, obviously, not
determining the outcome; there is, in Lively's terminology, not retrospective equality because there is no retrospective effectiveness.

May's final condition, positive responsiveness, is more problematic. It was, of course, formulated explicitly to apply only to binary-option choices. Additionally, though there is no technical problem in minimally adapting the positive responsiveness criterion to more than two option choices, the modified criterion interpreted as supporting the plurality rule can produce extremely anomalous results. We formulated the basic argument in the last chapter, but it is worth analysing in more detail the kind of anomalous situations that can arise. Firstly, the reformulation of the criterion: if there is social indifference between any number of options, then if one voter's preference changes from option X to option Y , the option for which the total number of votes has increased, rather than staying the same or decreasing, should be selected as the winner. Problems arise in situations where the plurality winner's votes constitute a rather small percentage of the total number of votes and, in addition, the winning option would have been ranked very low by those not selecting it as their first choice. Suppose, to take a fairly extreme possibility, a group of people were selecting out of twelve applicants a single candidate for a single position, applying the plurality modified positive responsiveness criterion a particular candidate, A, could be the plurality winner with just over $8 \%$ of the total votes, while at the same time being thought the very worst candidate by the rest of the voters. Hence, at least $91 \%$ of the voters think A the worst candidate and, consequently, any and every other candidate would beat A with a $91 \%$ majority in a "straight fight". It can be concluded that this suggested modification of the positive responsiveness criterion generates serious problems with respect to democratic retrospective equality. The problems arise from two inter-related sources; the rank ordering of candidates is not elicited and, hence, not taken into account in selecting the winner. It would seem that what is needed is a criterion that refers to a richer set of relevant information. In his A Preface to Democratic Theory (1956) Dahl rather hastily concluded that if we wanted a voting rule to select the most preferred option we could identify that option as the option preferred by most. When a choice is being made between three or more options it is not at all obvious that "the most preferred" option is the option preferred by most, interpreted as the option with the most first preference votes. Such a criterion, as we noted in the previous chapter, would contravene the
principle cited by Lively as his basic rationale for selecting one decision-rule over another, namely, that the winning option should have more people in favour of it than against it.

When dealing with more than two options, the question of which is "the most preferred" option may not have a definitive, conclusively provable answer. What the above discussion illustrates, however, is that what is relevant is more than the bare first preference scores for each option. One criterion often suggested in this context is the requirement of monotonicity. A decision rule is monotonic if, given that X is ranked over Y by a particular voter, the reversal of that ranking increases the probability of Y being the winner. The criterion is meant to be quite general, applying not just to a binary option situation in which the preference reversal would be equivalent to promoting Y to the top preference position. A more explicit and stringent criterion relevant to this aspect of selecting a decision-rule is Michael Dummett's principle of "global sensitivity". The formulation of this principle also relates it directly to May's positive responsiveness rule. A decision procedure is globally sensitive if a tie between X and Y , whatever the total number of options being decided upon, would be broken of for even one voter the rank ordering of X and Y was revised anywhere in that voter's preference rankings. Suppose, for example, that there were five candidates for a position and, whatever the decision-rule being used, X and Y were tied for first place. Suppose further that some particular voter ranked X and Y fourth and fifth respectively. Global sensitivity requires that if that voter changes his/her mind, now ranking Y over X , that should be sufficient to break the tie. Social indifference between X and Y should be positively responsive to the reversal of the ordering of X and Y anywhere in anyone's preference ranking. The implications and the problems arising from the criterion of global sensitivity will be discussed later, but as can be appreciated from the above explanation, it is a quite plausible candidate for the replacement of May's criterion in situations of more than binary-option choice.

However we end up rating strict global sensitivity as a criterion for democratic decisionmaking, it does introduce a more subtle and nuanced way of thinking about responsiveness. What we want is a democratic decision-making system in which the outcome is a positive reflection of people's preferences over the various alternatives. Any reasonable approximation to this will involve a decision-making system which
elicits a rich informational base concerning the structure of people's preferences and which processes that information in a manner that guarantees to as many people as possible as much final effectiveness of their expressed preferences as possible. This, we could argue, is how we should interpret Lively's principle of maximising retrospective equality. (As Dummett also puts the matter, in Voting Procedures, pp. 173-4 and 255, we should want a system that is fair to options in being sensitive to their support and fair to voters by maximising equally the probability of final stage effectiveness of expressed preference). With these criteria in place, we will now systematically analyse and assess the main varieties of voting system.

We have already referred to the plurality system as the procedure seemingly suggested by the simple modification of May's responsiveness criterion. Given that it is a very familiar procedure, being used in United Kingdom national elections to parliament and in the United States presidential election, to name but two uses, we will begin our analysis with what is popularly called the "first-past-the-post" system. Any voting system consists essentially of two parts. The first part is constituted by a rule that specifies what preference information voters will have the opportunity to express. The second part identifies how that information is to be processed and, crucially, the criterion for determining the winning option. The plurality system requests a voter to select one and only one option as that voter's preferred, or top, option. The votes for each option are summed, each vote, obviously, being represented by an identical numerical quantity (usually 1). The option with the highest number of votes is deemed the winner, irrespective of whether that number is or is not more than $50 \%$ of the total valid vote. How, from a normative democratic perspective, should we assess the plurality system?

Firstly, it is administratively efficient, both in terms of the actual procedure of registering votes and in processing those votes. Secondly, it does not place any great informational burden on voters; as a consequence of which, apart from the presence sometimes of "tactical" voting ${ }^{1}$, the elicited information is likely to be reliable. If there

[^0]are numerous options in the field, one may not have enough relevant information about all of the options, but if a person has a preference at all it will be usually quite easy to identify one's top preference. Thirdly, as with most of the systems normally used, plurality meets the basic criteria of anonymity, neutrality and decisiveness (ties are possible but not likely). The really serious problems arise with regard to responsiveness and the informational basis of the decision-making.

Let us go back to the kind of situation that we were describing above, where the plurality winner has a rather small percentage of the total vote. To simplify the analysis, assume that a group is deciding amongst five options and that all voters do have preferences over the whole range of options, as outlined in Table 1 below:

| $22 \%$ | $20 \%$ | $20 \%$ | $19 \%$ | $19 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| A | B | C | D | E |
| B | C | B | E | B |
| C | D | D | B | C |
| D | E | E | C | D |
| E | A | A | A | A |

Table 1

Though the margin of $2 \%$ is narrow, A is the clear plurality winner. The first thing to notice is that with an actual plurality voting system, the only information elicited would be that contained in the first line. Hence the kind of situation illustrated in that table would never be revealed by the voting system. But what kind of situation is it? As we said above, not only is it a situation in which a mere $22 \%$ plurality determines the outcome, it is one in which a large majority of $78 \%$ rank A as the very worst. Further, that $78 \%$ would vote for any other options against A in a pairwise "head-to-head" vote between A and that option. This is an important point that needs emphasis. It might be thought that a "negative" majority ranking an option last is, for some reason, not as significant as a positive ranking of an option as most preferred. In our example, however, not only is the plurality winner ranked last by a $78 \%$ majority, but every other
preference has little chance of winning. The serious difficulties that the possibility of tactical voting can produce will be analysed later.
option would, as stated above, be ranked positively, as more favoured than the plurality winner. In a head-to-head choice between A and any other option, the other option would be ranked first by a $78 \%$ majority. Even if we had not thought beyond a simple majoritarian perspective, this could hardly be considered a democratically ideal result. Because of the paucity of the information elicited, what might be highly relevant second, third and fourth preferences and so on cannot be taken into account. Another way of putting this is to note that those voters unfortunate enough to have a minority first preference are immediately eliminated from the decision-making process, there being no opportunity for their other preferences to have any effect in determining the outcome. The plurality system elicits an extremely poor informational base and, hence, cannot even begin to process what will almost certainly be further relevant information in a manner that could lead to reasonable levels of final effectiveness or retrospective equality. There are various forms of run-off voting systems whose rationale is precisely the attempt to remedy these defects by eliciting more information and providing those whose first preference turns out to be in a minority with the opportunity to still have some effect in the decision-making process.

The best-known run-off system is the two-stage run-off used in French presidential elections. Dennis Mueller provides a succinct account of what he calls the "Majority rule, run-off election":
"If one of the m candidates receives a majority of first place votes, this candidate is the winner. If not, a second election is held between the two candidates receiving the most first-place votes on the first ballot. The candidate receiving the most votes on the second ballot is the winner".
(Mueller, 1989)
As with the simple plurality system, voters are only asked to register a single top preference. However, unless one of the other options achieved an overall majority on the first ballot, those voters whose top preference turns out to have only minority support are given the opportunity to express their second preference, preserving into the second round the opportunity to contribute to the determination of the outcome. The system, of course, meets the simple anonymity, neutrality, decisiveness criteria as well as the plurality system. Furthermore, if an absolute majority winner emerges in the first round, the kind of situation discussed in the context of the plurality system cannot
occur. A first round majority winner cannot be ranked last by a majority and, being ranked first by a majority, would defeat each other option in a series of head-to-head votes. To illustrate these points, consider the following table describing a particular set of preferences over four options.

| $\mathbf{6 0 \%}$ | $25 \%$ | $10 \%$ | $5 \%$ |
| :---: | :---: | :---: | :---: |
| A | B | C | D |
| B | C | B | B |
| C | D | D | C |
| D | A | A | A |

Table 2

Although A is ranked last by the B, C and D supporters, when aggregated this obviously amounts to only $40 \%$. As a consequence, if A were to be run-off against each other option in turn, the evident result would be:

A (60\%) defeats B (40\%)
A (60\%) defeats C (40\%)
A (60\%) defeats D (40\%)
From a straight majoritarian perspective XXXX could challenge A's status as winner. Our overall assessment of the two-stage run-off system cannot, however, rest at this point. Things may not be that simple. Let us look at two kinds of case; one in which there is still an overall winner in the first round and one in which a second round run-off between the top two options is required.

To make our first point more starkly, we will preserve the rank-ordering of Table 2, but reduce the majority of A over the other options:

| $52 \%$ | $\mathbf{3 0 \%}$ | $\mathbf{1 2 \%}$ | $\mathbf{6 \%}$ |
| :---: | :---: | :---: | :---: |
| A | B | C | D |
| B | C | B | B |
| C | D | D | C |
| D | A | A | A |

Table 3

While A would still defeat each other option taken one by one, and while the percentage ranking A lowest is still a minority (48\%), it might be thought possibly significant to the determination of the outcome by the relevant popular preferences that literally $100 \%$ rank B either first or second, as compared with A being ranked first by $52 \%$ but last by $48 \%$. How this relative positional placing of options might be taken into account by a voting system will be discussed later in the chapter.

## Insert bit about Borda count:

$$
\begin{aligned}
& \text { A: } 52 \% 1^{\text {st }} \text { place } * 3=156 \\
& \text { B: } 30 \% 1^{\text {st }} \text { place } * 3=90 \\
& \quad 70 \% 2^{\text {nd }} \text { place }=140-\text { total } 230 .
\end{aligned}
$$

Meanwhile we will turn our attention to the second kind of case referred to above, the case where no option achieves an overall majority in the first round, necessitating a second round run-off between the two top-placed options. Again, to help us make our point clearly, we will consider a choice among five options, the following being the posited preferences over the options:

| $26 \%$ | $28 \%$ | $19 \%$ | $17 \%$ | $10 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| A | B | C | D | E |
| C | C | D | C | C |
| D | D | E | E | D |
| E | E | A | A | A |
| B | A | B | B | B |

Table 4

As can be easily ascertained, A and B are the clear front-runners, though neither has anywhere near an absolute majority over all other options combined. Hence, unlike in the plurality system where B would be deemed the winner, irrespective of how B was ranked by those not giving first place to B , a second round of voting takes place, the choice being between $A$ and $B$. the fact that $B$ is actually ranked last by everyone other than the $28 \%$ for whom B is the top preference is then reflected in the second ballot, the result being:

$$
\mathrm{A}=72 \%, \mathrm{~B}=28 \%
$$

Not only is the widespread low ranking of B taken into account, this is done primarily by allowing the $\mathrm{C}, \mathrm{D}$ and E supporters to retain a role in determining the final outcome by registering their preference between A and B .

A two-stage run-off system elicits a richer set of information and utilises that information in a way that increases the potential final effectiveness of those whose first preference gained only minority support, thus contributing to a generally higher level of retrospective equality as compared to the plurality system. The two-stage run-off can also be seen to be quite sensitive to the relative ranking of A and B , the front-runners, in the preference ordering of supporters of the minority options. Even though A and B are ranked fourth and fifth by $46 \%$ of the voters, it is that ranking that is crucial in determining the outcome. Had the preferences been such as to result in a tie between A and B , a change in the relative ranking of A and B by even one voter would result in a "positive" response, the option whose vote went up by one being selected as the winner. The reason for this sensitivity in the second stage is that the second stage is a binaryoption majority stage, sensitive only to the $\mathrm{C}, \mathrm{D}$ and E supporters' ranking of A and B . The relative ranking of $\mathrm{C}, \mathrm{D}$ and E across all voters is now irrelevant. This is a fact that creates consequences as democratically anomalous as those that emerge in the plurality system. The anomalies are all the more startling since in our example the plurality impasse between A and B seemed to have been resolved with a democratically decisive and unambiguous result favouring A over B precisely by giving the C, D and E supporters a role in determining the final result. There would seem no doubt that A ought to be the clear winner, with B the runner-up.

Suppose, however, re-examining Table 4, we look at B's support vis-à-vis the "minority" options. It emerges quite clearly that B would lose decisively (by $72 \%$ to $28 \%$ in fact) against either C or D or E. If B would lose against every other option, it seems hardly unambiguous that B was the runner-up. But, since B was defeated in the final round anyway, this might not be very significant. Suppose now, however, we look at $A$ 's support vis-à-vis C, D or E. the unambiguous $72 \%$ winner would be defeated by each option in turn by a $74 \%$ to $26 \%$ margin! The crucial point is that though C, D and E have only minority top preference support, they have far more second and third preference support than either A or B. That high second and third preference support is hidden by the fact that in the two-stage run-off they are eliminated on the basis of
having minority first preference support. Once again, the voting system as a whole does not elicit what might be relevant comparisons and though it confers some final effectiveness on those voters whose first preference is eliminated, a large range of their relevant preferences are rendered inoperative and ineffective. We can conclude that though two-stage run-off systems achieve in general higher levels of final effectiveness than plurality systems and though they block, in particular, the victory of an option that would be defeated by every other option, as the plurality system does not, they still score fairly badly in terms of the informational base elicited and retrospective equality. Before looking at systems that are much richer in informational terms, we are going to look at two voting procedures that, like plurality and run-off systems, can be considered to be very simple adaptations of the majority system for when there are more than two options. The first system, which we call majority head-to-head sudden death elimination, is of interest partly because it is the one actually used in electing the Speaker of the House of Commons in Westminster, and partly because it illustrates perfectly a XXXX commented on feature of voting situations, viz. that the mere order in which votes between options are taken can crucially determine the outcome. The system works as follows. From the set of options to be decided on, two are selected for a binary-option simple majority vote. The loser is eliminated once-and-for-all from the contest. Another option is selected to run against the winner of the previous round, with the loser of the second round being eliminated. The process continues until there are only two options, the majority winner of the final round being the overall winner. The order of the pairwise votes could be determined in a number of ways; for example, as is the case in the election of the House of Commons Speaker, the order is determined by a "chairman", in that case the Father of the House (explain). Or a simple rule like alphabetic order could be used or, perhaps more democratically, a random selection can be made. The basic theoretical point we want to make is, however, independent of the selection procedure. The specific anomalous consequence occurs when there is a voting cycle. Take a four-option race and the following overall ordering: of pairwise votes

$$
\mathrm{A} \rightarrow \mathrm{~B} ; \mathrm{B} \rightarrow \mathrm{C} ; \mathrm{C} \rightarrow \mathrm{D} ; \mathrm{D} \rightarrow \mathrm{~A} .
$$

Table 5 is a preference structure that would result in the above ordering.

| $28 \%$ | $26 \%$ | $24 \%$ | $22 \%$ |
| :---: | :---: | :---: | :---: |
| A | B | C | D |
| B | C | D | A |
| C | D | A | B |
| D | A | B | C |

Table 5

A would defeat B by $74 \%$ to $26 \%$.
B would defeat C by $76 \%$ to $24 \%$.
C would defeat D by $78 \%$ to $22 \%$.
D would defeat A by $72 \%$ to $28 \%$.
Now there are numerous orders in which A, B, C and D can be paired. But consider the following four scenarios.

1. Suppose we wanted A to be the winner; this can be achieved by ensuring the elimination of $D$, which can itself be ensured by first running $C$ against $D$. If $B$ is then run against $\mathrm{C}, \mathrm{C}$ will be eliminated. In the final ballot, A defeats B . So $A$ is the winner if the order is:
i. $\quad \mathrm{Cv} \mathrm{D}$,
ii. $\mathrm{B} v \mathrm{C}$,
iii. A v B.
2. If we wanted $B$ to win, we would have to get $A$ eliminated. The following order would suffice:
i. A v D: A eliminated
ii. $\quad \mathrm{C} v \mathrm{D}: \mathrm{D}$ eliminated,
iii. $\quad \mathrm{B} v \mathrm{C}: \mathrm{C}$ eliminated
leaving $B$ the overall winner.
3. To ensure that $C$ wins, we could run $A$ against $B$, thus eliminating $B$. If $A$ is then matched with D , A will be eliminated and D will go on to be beaten in the final round, leaving C the overall winner.
4. But we could get D to win by first running B against C , eliminating C , leaving A to then beat B . A would lose in the final round, leaving D the overall winner.

The central theoretical point is that though the system elicits a reasonable amount of comparative preference information and provides voters with a sequence of clear, simple choices producing almost always a decisive result, the end result can be determined at best by an arbitrary factor and at worst, by the machinations of a chairperson agenda-setter. This is because on an absolutely identical set of preferences which option wins can depend only on the sequence of ballots. If this is random, the outcome is determined randomly. If the sequence of ballots is chosen by a chairperson, a chairperson with a shrewd knowledge of preferences can actually ensure the victory of her/his own preference. Either way, the situation is seriously problematic from a democratic perspective.

The final system that is seen as a minimal improving modification of the plurality system or as an adaptation of the majority principle to the situations of multiple choice was labelled by its inventors S. Brams and O. Fisbaum "approval voting". For those unfamiliar with the system, it might at first seem a little baffling, though it is fairly simple to operate and has an attractive rationale. Voters are presented with a list of options and they are allowed to vote for as many or as few of these as they like; the initial idea being that one might vote for all options of which one approves, for which one has a positive preference. For example, with an option set of seven, the ballot form would be like a plurality ballot form:

## Approval Voting

| A | $\mathbf{x}$ |
| :--- | :--- |
| B |  |
| C |  |
| D | $\mathbf{x}$ |
| E | $\mathbf{x}$ |
| F |  |
| G |  |

## Table 6

As filled out above this would indicate a voter's approval of A, D and E, and each of these options would receive one vote each. The overall winner is simply the option with the most votes. The system, as can be seen, is extremely similar to the plurality system, but with the possibility of voters selecting more than one option to vote for. The normal way of illustrating the results of an approval ballot over a number of voters is to posit a rank-ordering of the options for each voter, indicating the options voted for by drawing a line under the last option approved of - with all the options under the line not being given a vote. The ballot form above might, for example, represent the following rankordering:

D

E
-
C

B

G

## F

Table 7

It is important to note that the ordering of the preferences is postulated; it could not be reconstructed from the ballot, which indicates, as it were, only that A, D and E are above the line, with B, C, F and G below. The result of a typical approval ballot with five voters and five options might be:

| (1) | (2) | (3) | (4) | (5) |
| :---: | :---: | :---: | :---: | :---: |
| B | B | C | C | A |
| C | $\underline{\text { C }}$ | B | $\underline{\mathbf{A}}$ | $\underline{\mathbf{D}}$ |
| D | A | $\underline{\mathbf{A}}$ | B | B |
| A | D | E | D | C |
| E | E | D | E | E |

Table 8

Each occurrence of an option above the line counts as a vote for, hence to compute the scores we count the number of columns in which a particular option appears above the line. (In an actual ballot we would simply count the number of ballot forms on which an option had been voted for). The score for Table 8 would be:

| C | 4 votes |
| :--- | :--- |
| B | $\mathbf{3}$ votes |
| A | $\mathbf{3}$ votes |
| D | $\mathbf{2}$ votes |
| E | $\mathbf{0}$ votes |

Table 9

Option C would be declared the winner with the largest number of positive votes.
The basic rationale behind approval voting is that it can elicit relevant strong preference information in addition to voters' first preferences. It has many positive characteristics. As we have just said, it gives voters the opportunity to signal positive support for more than their first preference, generating a relatively rich informational base. Furthermore, it achieves this without placing on voters the burden of deciding a precise rank-ordering of all options. Finally, it utilises all of each voter's positive preferences, independently of what that voter's first preference might be. The relevance of this last point may not be immediately obvious, but it can be explained by comparing approval voting to the two-stage run-off system described earlier. In the specific example given, the claim of A to be clearly the "strongest" candidate, with B the obvious runner-up was seriously challenged when we calculated that with the postulated preference structure C, D and E would each defeat both B and A in straight pairwise votes. The reason that the strengths of $\mathrm{C}, \mathrm{D}$ and E were hidden lies in the fact that their high ranking among A and B supporters is never taken into account (in fact never elicited) because A and B are first preference front-runners. Furthermore, given that C, D and E are eliminated as minority first-preference options, only the relative ranking of A and B in that group gets elicited and utilised. Suppose we were to go back to the postulated ranking in Table 4, adding this time an "approval line" cut-off for each group of voters:

| $26 \%$ | $28 \%$ | $19 \%$ | $17 \%$ | $\%$ |
| :---: | :---: | :---: | :---: | :---: |
| A | B | C | D | E |
| C | $\underline{\text { C }}$ | D | C | C |
| D | D | $\underline{\text { E }}$ | $\underline{\text { E }}$ | $\underline{\text { D }}$ |
| E | E | A | A | A |
| B | A | B | B | B |

Table 10

Summing the percentages for each option above the approval line produces the following result:

| C | $\mathbf{1 0 0 \%}$ |
| :--- | :--- |
| D | $46 \%$ |
| E | $46 \%$ |
| B | $28 \%$ |
| A | $26 \%$ |

The strength, in particular, of option C is not masked by the elimination of C, D and E as only having minority first preference support. The central point is that all positive preferences are given an actual role, contributing to the determination of the winner. The problems with the approval system stem from the fact that all positive preferences are given the same identical weight in the determination of the outcome, irrespective of the possibility that a positively approved option may, for a particular voter, be a very low third-ranked preference.

Equal weighting, irrespective of preference ranking, creates two main problems for voters. While voters do not face the "burden" of rank-ordering all their preferences, they are faced with the quite difficult task of deciding where the cut-off point should be. This difficulty is compounded by the very strong incentives to engage in strategic or tactical voting. The problems here are obvious. Suppose that a particular voter has a strong preference for A and a positive, though very much weaker, preference for B and C . Furthermore, this voter suspects that there is enough support for B and C among other voter to make B and C serious challengers to A, while the other options D and E are
perceived as non-starters. It would not be particularly sensible to register a positive vote for B and C. Such a "full vote" for B and C is not needed to defeat D and E and could only pose a threat to the success of A. The intent of the approval system to give weight to lower-level, though still positive preferences, would in such circumstances be seriously undermined. The equal weighting of all approved options creates its own problems even if tactical voting is avoided. An approval vote victory can come about by an option's receiving the largest number of positive votes, which derive from what is in fact a set of relatively low rankings. Consider the following case of five voters:

| A | A | B | B | A |
| :--- | :--- | :--- | :--- | :--- |
| B | D | D | A | B |
| C | $\underline{\text { C }}$ | $\underline{\text { C }}$ | C | $\underline{\text { C }}$ |
| D | B | A | D | D |

Table 11

C is the clear approval winner with five votes, A and B being tied in second place with four votes each; this is despite the fact that C never gets a better than next-to-last rating, while A , in particular, secures three out of five first rankings and one second preference. That situations even more anomalous can arise can be seen by examining Table 12:

| $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| :--- | :--- | :--- | :--- | :--- |
| A | B | D | E | D |
| B | A | B | D | E |
| C | $\underline{\text { C }}$ | $\underline{\text { E }}$ | C | C |
| D | D | A | A | A |
| E | E | C | B | B |

Table 12

Again, C is the clear winner with four approval votes, though if ignoring the approval lines we count the number of columns in which each option in turn is ranked above C , we can easily ascertain that C would lose in a pairwise vote to every other alternative. A defeats C in columns 1, 2 and 3; B defeats C in columns 1, 2 and 3; D defeats C in columns 3 , 4 and 5 , as does E. While not wishing to claim, of course, that majority
victory is the only significant factor, the fact that C would be in a minority to every other option is surely an important consideration. The approval system can elicit and use more information in a manner that guarantees relatively high levels of final stage effectiveness; that there is a strong motivation towards not revealing the relevant information and that the information is not used in a manner that matches what might be very significant orders of preference means that it does suffer from important weaknesses. A system that might claim to overcome these weaknesses by giving voters the opportunity to rank-order preferences and hence to give greater weight to higherranked preferences is the Alternative Vote system.

The Alternative Vote system (which when used to elect a number of candidates in multi-seat constituencies is the same as the Single Transferable Vote system, see pp. $\mathbf{x x x x})$ operates in the following way. Firstly, voters are given the opportunity of rankordering all the options on the ballot form, indicating the most preferred option by placing a " 1 " against $i t$, the next by a " 2 " and so on. If one has any preference between options it is sensible to rank-order all the options. An option may be so lowly preferred that one might not wish to give it a positive vote, but if, in the last analysis, it would be preferable to some other option that one liked even less, it would be worth registering this. As Dummett puts it: we may detest A, but if we abhor B even more it is sensible to rank A over B , so as to decrease the chances of the abhorred option being selected. The first stage of the processing consists in counting each option's number of first preferences. Secondly, the option receiving the fewest first preferences is eliminated, with the votes of the eliminated option being transferred to the options listed second on the ballot forms. A simple example will illustrate the procedure for those unfamiliar with it.

| 40 | 35 | 15 | 10 |
| :--- | :--- | :--- | :--- |
| A | B | C | D |
| C | C | B | B |
| B | D | D | A |
| D | A | A | C |

Table 13
The first count of first preferences results in:

| A | 40 |
| :--- | :--- |
| B | $\mathbf{3 5}$ |
| C | $\mathbf{1 5}$ |
| D | $\mathbf{1 0}$ |

D, having the fewest first preferences, is eliminated. The 10 votes in question all have B in second place. ${ }^{2}$ When these votes are transferred, the second count has the following results:

A $\quad 40$
B 45
C $\quad 15$

C, now being the lowest, is eliminated. Since all of C's second preferences are for B, the final result is 40 votes for A and 60 for B , so B is declared the winner.

One final, slightly technical point needs to be made. After the first count elimination, an option may be eliminated whose second preference has already been eliminated. So if D is eliminated and a particular ballot has preferences as follows:

| $1^{\text {st }}$ | D |
| :--- | :--- |
| $2^{\text {nd }}$ | B |
| $3^{\text {rd }}$ | A |
| $4^{\text {th }}$ | C |

and B has already been eliminated, the vote is transferred to A. In summary, the procedure starts with counting first preferences, eliminating the lowest and transferring those votes to the remaining "live" options, recounting the totals after the transfers, eliminating the then lowest until only two options remain. At that stage the option with a simple majority is the final winner.

[^1]A moment's reflection will reveal that the alternative vote system is, in logical structure, similar to the two-stage run-off. This allows a greater range of relevant information to come into play in determining the outcome as compared with the two-stage run-off. Because in the two-stage run-off, C, D and E are eliminated simultaneously, their relative ranking, and in particular the very strong support for $C$, is masked. If we referred to Table 4 on page $\mathbf{~ x x x x}$ and computed the result of sequential elimination with alternative vote transfer, that result would be:

| $1^{\text {st }}$ count | (i) | $\mathrm{A}=26 \%$ | (ii) | eliminate E and transfer second preferences |
| :---: | :---: | :---: | :---: | :---: |
|  |  | B $=28 \%$ |  |  |
|  |  | C $=19 \%$ |  |  |
|  |  | $\mathrm{D}=17 \%$ |  |  |
|  |  | $\mathrm{E}=10 \%$ |  |  |
| $2^{\text {nd }}$ count | (i) | $\mathrm{A}=26 \%$ | (ii) | eliminate D and transfer second preferences |
|  |  | $\mathrm{B}=28 \%$ |  |  |
|  |  | $\mathrm{C}=29 \%$ |  |  |
|  |  | $\mathrm{D}=17 \%$ |  |  |
| $3^{\text {rd }}$ count | (i) | A $=26 \%$ | (ii) | eliminate A and transfer second preferences |
|  |  | $\mathrm{B}=28 \%$ |  |  |
|  |  | $\mathrm{C}=46 \%$ |  |  |
| $4^{\text {th }}$ count | (i) | $\mathrm{B}=28 \%$ | (ii) | C declared the winner |
|  |  | $\mathrm{C}=72 \%$ |  |  |

The main advantages of the alternative vote lie in the fact that, compared to all systems examined so far, it elicits the full range of relevant comparative preference information. Secondly, it utilises that information in a way that gives some probability of final stage effectiveness to those whose first preferences (and, perhaps even second preferences) turn out to have only minority support. Thirdly, it allows the registration of lower order preferences without a voter thereby weakening the chances of his/her higher preferred options; lower preferences only come into play if higher level preferences have already been eliminated. These are significant advantages, though the system does have serious weaknesses that derive from the under-utilisation of the preference information elicited. That the full range of preference information may not be actually used is easily demonstrated. Suppose some option is eliminated, the supporters of which indicate a high level of support as second preference for an option that has already been eliminated
in an earlier count. Although the voters' next preferred option comes into play, their strong support for the already eliminated option remains totally ineffective. The same is true of, say, second preference options of those voters whose first preference stays alive until the end. This is particularly worrying for the eventual runner-up; a very large number of second preferences remain "locked into" that option, never becoming effective. And even though the voters whose first preference is eventually victorious may have no grounds for complaint that their lower level preferences did not register in determining the outcome, it is still anomalous that possibly very strong second preference support for an option remains completely inoperative. This general point, that the utilisation of the preference information can be both radically incomplete and haphazard, ${ }^{3}$ can be shown to result in certain extremely anomalous consequences. Take, for example, the very simple Table 14, with four voters ranking five options:

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| E | E | E | E |
| B | A | D | C |
| C | D | A | B |
| D | C | B | A |

Table 14

Examination of the first two lines across will show that E has no first preferences and hence would be eliminated in the first round of an alternative vote. Further examination will demonstrate, however, that E would defeat every other option by four votes to three in straight votes. ${ }^{4}$ The alternative vote system, by prioritising first preferences, fails to detect the strength of E's support.

Furthermore, the incomplete and haphazard utilisation of the preference information, results in the system not being monotonic. To be monotonic, a voting procedure must be such that if support for an option increases, its chances of being selected must also increase. In fact, with the AV system an option's first preference support can increase

[^2]dramatically at the expense of other options' proportionate decrease, but the result will be that whereas the option would have won with the lower level of first preference support, it is now defeated by an option for which the first preferences have gone down. Table 15 shows the original preferences over four options:

| 70 | $\mathbf{6 0}$ | $\mathbf{5 0}$ | $\mathbf{4 0}$ |
| :--- | :--- | :--- | :--- |
| A | B | C | D |
| D | C | D | B |
| B | A | B | C |
| C | D | A | A |

Table 15

The AV procedure in the three counts would go as follows:
$1^{\text {st }}$ count
(i) $\mathrm{A}=70$
$\mathrm{B}=60$
$\mathrm{C}=50$
$\mathrm{D}=40$
$2^{\text {nd }}$ count
(i) $\mathrm{A}=70$
$B=100$
$\mathrm{C}=50$
(ii) eliminate D and transfer 40 second preferences to B
(ii) eliminate C and transfer 50 third preferences to B (D having already been eliminated cannot benefit from second preferences)

$$
\begin{aligned}
& 3^{\text {rd }} \text { count } \quad \text { (i) } \quad \text { (i) } \mathrm{A}
\end{aligned}=70
$$

(ii) B declared the winner

Suppose, however, that B's first preferences increase dramatically at the expense of C and D , with the resulting figures:

| 70 | 73 | 38 | 39 |
| :---: | :---: | :---: | :---: |
| A | B | C | D |
| D | C | D | B |
| B | A | B | C |
| C | D | A | A |

Table 16

Support has migrated from C (12 votes) and D (1 vote) to B; what would the AV result be with increased support for B and decreased support for C and D ?

| $1^{\text {st }}$ count | (i) | $\mathrm{A}=70$ | (ii) | eliminate $C$ and transfer 38 second preferences to D |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{B}=73$ |  |  |
|  |  | $\mathrm{C}=38$ |  |  |
|  |  | $\mathrm{D}=39$ |  |  |
| $2^{\text {nd }}$ count | (i) | $\mathrm{A}=70$ | (ii) | eliminate A and transfer 70 second preferences to D |
|  |  | $B=73$ |  |  |
|  |  | $\mathrm{D}=77$ |  |  |
| $3^{\text {rd }}$ count | (i) | $\mathrm{B}=73$ | (ii) | D declared the winner |
|  |  | $\mathrm{D}=147$ |  |  |

While B's first preferences have gone up and D's have gone down, D ends up victorious, whereas $B$ could have won with much lower first preference support. Of course the crucial factor is not just that B's first preference support has gone up; it derives from the fact that with C being eliminated first D benefits from C 's transfers and then benefits from A's transfers which were originally locked into A, given that A was still live until the final round.

To repeat, all of the problems arise because AV, though it elicits the full range of preferences, it never uses all the relevant information, and the information it does use is used in a non-systematic way which sometimes results in completely counter-intuitive results. To determine whether any other system can utilise and process preference information less anomalously and in a way that produces a better chance that more people's expressed preferences will be finally effective, we need to move on to analyse the Condorcet and Borda Count voting procedures. The system are named after their inventors, the Marquis de Condorcet and Jean-Charles de Borda respectively. Both were $18^{\text {th }}$-century members of the French Academy of Science; not a great deal is known about de Borda, but the Marquis de Condorcet was a prominent Enlightenment "philosophe", writing extensively on mathematics, science, social theory and philosophy. Despite his "minor" nobility background he was an enthusiastic supporter of the French Revolution, becoming an elected representative in the Legislative and Constituent Assemblies. His public criticism of Robespierre for the latter's cavalier reaction to Condorcet's draft constitution put him in acute danger. He went into hiding for several months in Paris, but when he attempted to flee Paris he was recognised and
arrested. He died in prison in 1794. The Condorcet system of voting was only a small part of his logico-mathematical analysis of collective decision-making and though he accepted that the system had certain difficulties, he believed it to be the most thorough and systematic method of aggregating preference information.

The basic principle of the Condorcet system is that out of a list of options every possible pair of options is (effectively) voted on, the strength of each option against every other option being thereby tested, with all of this information being used to determine the outcome. If we have five options, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E , the Condorcet system, in its most explicit form, involves a vote on (1) A v B, (2) A v C, (3) A v D, (4) A $\vee \mathrm{E}$, (5) B v C, (6) $\mathrm{B} v \mathrm{D},(7) \mathrm{B} v \mathrm{E},(8) \mathrm{C} v \mathrm{D},(9) \mathrm{C} v \mathrm{E},(10) \mathrm{D} v \mathrm{E}$, i.e. every possible pairing from the set A-E. The result of each pairwise "head-to-head" is recorded, and the option that defeats all the others is declared the Condorcet winner. We have implicitly alluded several times to this idea of a Condorcet winner, as when we demonstrated above that the Alternative Vote system can result in the elimination in the first round of the option that would defeat every other option in a set of pairwise votes. We can go back to the example we used in that context to illustrate more fully the operation of the Condorcet system in practice. Table 14 is repeated here:

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| E | E | E | E |
| B | A | D | C |
| C | D | A | B |
| D | C | B | A |

Table 14(a)

Assuming that each column represents the preference ranking of a single (or equal number of) vote(s), if we want to compute the result of a head-to-head between any two options, we simply count the number of columns in which the first option is ranked above the second, and the number in which the second is ranked above the first, and apply the simple majority rule to determine the winner of that head-to-head. The result of the ten pairwise votes on the basis of Table 14 would be:
(1) A v B: a tie at A (2) $=\mathrm{B}(2)$
(2) A v C: a tie at $\mathrm{A}(2)=\mathrm{C}(2)$
(3) A v D: a tie at A (2) $=\mathrm{D}(2)$
(4) A v E: E defeats A: E (3) > A (1)
(5) $\mathrm{B} v \mathrm{C}$ : a tie at $\mathrm{B}(2)=\mathrm{C}(2)$
(6) B v D: a tie at $\mathrm{B}(2)=\mathrm{D}(2)$
(7) $\mathrm{B} v \mathrm{E}$ : E defeats B: E (3) > B (1)
(8) C v D: a tie at $\mathrm{A}(2)=\mathrm{C}(2)$
(9) $\mathrm{C} v \mathrm{E}$ : E defeats C: $\mathrm{E}(3)>\mathrm{C}$ (1)
(10) D v E: E defeats D: E (3) > D (1)
$\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D all tie with each other in the pairwise contest, but E defeats each one in turn and hence is the Condorcet winner.

Before proceeding with our analysis it is worth noting a technical point that emerges from the above illustration. When describing the Condorcet system, it is said that each option is voted on against each other option in turn. This might seem from a practical perspective a somewhat unattractive feature of the system. Even with only five options, ten rounds of voting would be needed. If we increased the numbers of options to ten (a not unusual number of candidates in an election) the number of separate rounds of voting would rise dramatically to forty-five. ${ }^{5}$ The above illustration shows that this is easily avoided in practice. If voters rank-order their preferences on a single ballot form, as in AV, the results of pairwise comparisons can be computed, as we did above. Why should the result of a Condorcet vote be considered particularly democratic? The crucial point here is the thorough and systematic way in which the very extensive range of comparative preference information is processed. Let us analyse this claim into its component parts and assess the outcome determination procedure in terms of its democratic credentials.

Firstly, each bit of information represents a simple binary comparison between one option and another. Providing each bit of information is not too difficult for a voter. The

[^3]information itself is simple and unambiguous: A is preferred to B , or B to A , or they are thought equal. Secondly, the strength of support for each option against every other option is registered. But, thirdly, and unlike the situation with AV, all of this information is computed and utilised in the measurement of the aggregate strength of support for each option. In AV, a lower level preference, say a third preference, is only ever computed if and when higher preferences have been eliminated. Whereas, with the Condorcet system, the stated preference for one option over another on each ballot form is computed for every option, the total aggregate strength of support for each option being calculated. As we put it above, the information is extensive and it is processed thoroughly and systematically. The democratic advantage is threefold in terms of fairness to options, global sensitivity and consequent approximation to retrospective equality. The fairness to options derives from the fact that all support for each option is computed and utilised, unlike AV where the information concerning E's strength is registered by each voter singly but is never computed because of E's elimination in the first round. The thorough computation and simultaneous utilisation of strength of support for each option results in a high level of positive sensitivity. To illustrate this point, imagine that the preferences of Table 14 remained identical, except that for some reason option E was withdrawn. The remaining calculations of the Condorcet outcomes would be unaffected, with each pairing resulting in a tie. Suppose now that the ranking of B over A at the very bottom of the fourth column were to be reversed, the procedure would be immediately sensitive to this, registering the winner of $A v B$ as $A(3)>B(1)$, though A would not be the overall Condorcet winner, since A still only ties with C and D. (The implications of this will be investigated shortly). That each voter's total preference ranking is guaranteed to go into the determination of the measure of aggregate strength of support for each option contributes to an approximation to equality of final stage effectiveness. Finally, partly deriving from the system's sensitivity and its fairness to options and voters and partly deriving from a strong intuitive interpretation of "strength of support", many theorists believe that a Condorcet winner is clearly both the "most preferred" option and selected in a maximally democratic fashion.

There are, however, weaknesses in the system, and this should begin to bring home the point hinted at above that there is no single perfectly democratic system. What are these
weaknesses? There are three main problems deriving from the possibility (in fact, the quantifiable probability) of a Condorcet cycle occurring and from the fact that the Condorcet system is not sensitive to where in a preference-ranking an option is located, only whether it is ranked above or below some other specified option. We have encountered Condorcet cycles earlier, the simplest occurring when three equal groups of voters rank the three options A, B and C as follows:

| A | B | C |
| :--- | :--- | :--- |
| B | C | A |
| C | A | B |

Table 16

The Condorcet pairwise comparisons can be readily calculated.

$$
\begin{aligned}
& \mathrm{A}(2)>\mathrm{B}(1) \\
& \mathrm{B}(2)>\mathrm{A}(1)
\end{aligned}
$$

But A does not defeat C ; in the second and third columns C is ranked above A , hence C (2) $>\mathrm{A}$ (1)

Not only is there simply no Condorcet winner, but whichever option might be selected, a strong argument against its being implemented would be that some other option would clearly defeat it. Our own example of the preferences mapped in the modified Table 14 illustrates the same point. We withdrew E and reversed the B:A ranking in the fourth column, producing the following table:

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| B | A | D | C |
| C | D | A | A |
| D | C | B | B |

Table 14(b)

As we noted above, $\mathrm{A}(3)>B(1)$, and though B ties with C and D , so too does A . This is the kind of situation in which, it is argued, though there is no Condorcet winner, there is not complete indeterminacy. A ties with C and D , as they do with each other and with B, but A, unlike any other option, actually defeats at least one other option, namely B.

This is formally defined in terms of an option's "majority number", the number of other options that a specific option defeats. It is then suggested that when there is no Condorcet win, the respective majority number should be the relevant measure of the comparative strength of support, with the implication that the option with the largest majority number should be deemed the most strongly supported. Appealing as this suggestion might be, it is not without its difficulties. Firstly, supporters of D, for example, might complain that though they accept that their option only ties with B, the crucial comparison for them is between A and D ; and in that comparison, A cannot claim any superiority over D. How would things look, they might add, if B was withdrawn? We are left only with parity of support for A and D!! The following table increases the ambiguity in the measure of comparative support in the context of majority numbers.

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| B | D | D | A |
| C | A | A | C |
| D | C | B | B |

Table 16

Without going into all of the details, an inspection of Table 16 would show that A has a majority number of 2 , $D$ of 1 , while B and C tie with each other and with D . But supporters of D might well argue that how many other options A defeats is not as relevant in choosing between A and D as is the fact that D would clearly defeat A by 3:1. This is where and how the possibility of voting cycles really bites. More generally, what we are beginning to encounter here is perhaps a more general problem that has no unique solution. What we are looking for is a criterion that will measure "strength of support" in a manner, as we put it before, that is fair to outcomes, that equally empowers voters to the maximal extent and that is sensitive to all the relevant preference information in determining the outcome. The really fundamental difficulty derives from the fact that our pre-theoretic concept of "strength of support" is multidimensional and indeterminate.

There are at least six dimensions to our concept of "strength of support": the number of first preferences won, the number of options an option defeats, the number of votes by
which one option defeats other options, whether one specific option defeats or is defeated by some other specific option, where precisely in the preference rankings the options are when one defeats the other and, finally, what might be called the postulated underlying quasi-conditional measure of degree of preference. Before going on to comment on the dimensions that need further explication, the general point is that there is no a priori universally imperative rule determining how we should produce comparative scores on each dimension, nor, especially, how we should weight one dimension against another. Take, for example, the fifth dimension: where in the preference rankings two specific options occur, and not just whether one is ranked above the other. Suppose A and B are ranked one above the other and vice versa in the same number of preference ranking. A beats $B$ the same number of times that $B$ beats $A$. But surely it is relevant in determining strength of support to note that when A defeats $B, A$ is at the top of the preferences, with B the very last out of, for example, ten options. Whereas when B defeats A, A is always not only high up, but second in the preference rankings. While we might not be prepared to say by exactly how much a second placing to a first should count over a tenth to a first placing, we might be prepared, fairly unequivocally, to agree that it should count, say as a tie-breaker. But what if we bring in one of the other dimensions, such as the number of votes by which one option defeats another and what if the scores on the two dimensions are running counter to each other? Assume that B actually defeats A, $52 \%$ to $48 \%$, but that the placings in the preference rankings are still as above, i.e. in the $48 \%$ of cases that B is defeated it is the last of ten, whereas in the other $52 \%$ of cases A is second to B's first place. Should the $52 \%$ of second placings outweigh the mere $4 \%$ difference in first place scores? The question is not posed in the expectation of an answer, but rather to make the point that it is highly unlikely that we would be able even to begin specifying the criteria we would use to judge one answer to the question better than another.

The situation increases in complexity when we bring in the other relevant dimensions. Take for example the fifth dimension, what we called quasi-cardinal degree of preference. In very simple quantitative comparisons such as the length of two sticks, we can determine cardinal measures; stick X being 1 metre, Y being 10 centimetres long. We can then compare the measures in precise quantitative terms; X is ten times the length of Y. Ordinal measures simply rank the things compared as $1^{\text {st }}, 2^{\text {nd }}, 3^{\text {rd }}$ and so on.

Some ordinal rankings, like the order in which racehorses cross the finishing line, are in fact underpinned by strict cardinal measure; though, obviously, the mere ordinal ranking implies nothing about the underlying cardinal measures. Degrees of preference are almost certainly not realistically comparable on the basis of cardinal measures. Someone may judge a month's holiday in a villa in France to be far preferable to a weekend in Blackpool, but it is not likely that we can realistically derive a quantitative measure of the degree of difference, at say 625 times more preferable. But this is where quasi-cardinality is relevant. We say about our potential holidaymaker that they prefer the villa in France much more; we might add that the preference for the villa over the weekend in Blackpool is itself much greater than the preference for the weekend over the day-trip to the same resort, without being able to say by how much more. We might speculate, for example, that differences in higher order rankings are more significant, in general, than differences in lower-order rankings. The point of all this being that it is obviously invalid to infer that if someone ranked the villa first, the weekend second and the day-trip third, that the weekend in Blackpool was midway in degree of preference between the day-trip and the villa. Even when there is strict cardinal underpinning, inference from ordinal to cardinal is strictly invalid. The difficulties for preference aggregation derive from the facts that quasi-cardinally comparable degrees of preference will underpin pure ordinal ranking and, secondly, we can plausibly, though not conclusively, speculate about these quasi-cardinal degrees, without being able to say by how much more. Consider an example that explicitly brings into play our first dimension, the number of first preferences, the fifth dimension, where, generally, options are placed in preference rankings and the sixth dimension, the possible quasicardinal underpinnings of the ordinal ranking. To construct our argument we will return to Table 14(a), which had a clear Condorcet winner that had received no first preference votes. This time we will modify the assumption that each column represents an equal vote, introducing different percentages of the total votes represented by each column.

| $\mathbf{4 7 \%}$ | $\mathbf{2 3 \%}$ | $\mathbf{2 0 \%}$ | $\mathbf{1 0 \%}$ |
| :---: | :---: | :---: | :---: |
| A | B | C | D |
| E | E | E | E |
| B | A | D | C |
| C | D | A | B |
| D | C | B | A |

Table 17
Despite the fact that A's first preference vote stands at $47 \%$, E is still the Condorcet winner, defeating A with $53 \%>47 \%$, B by $77 \%>23 \%$, C by $80 \%>20 \%$ and $D$ by $90 \%>10 \%$. Although we have consistently argued against accepting number of first preferences as the single criterion of strength of support, it is not obvious that that dimension should be completely over-ridden by the Condorcet criterion. The general point is that there is a multiplicity of dimensions but no determinate rule dictating any specific weighting. The specific point vis-à-vis the Condorcet criterion is that it only takes into account whether one option is ranked above another option, not where the options are ranked, which latter dimension, it is argued, must to some extent be relevant in measuring strength of support. It is precisely this argument that lay behind JeanClaude de Borda's challenge to the Condorcet system.

Almost everyone, at least almost everyone in Europe, will be familiar with a modified version of the Borda Count voting procedure; it is the system used to select the winner of the Eurovision song contest. The first stage of the unmodified procedure is identical to the first stage of the Alternative Vote and the first stage of the Condorcet system when, in the latter case, the information on which the pairwise comparisons are to be made is elicited by a once-off rank-ordering of the options by each voter. The second stage, however, is very different and in many ways simpler. Each option is assigned a number of points depending on where the option is ranked on a ballot. The simplest points rule is to assign $n-1$ points for a top rank, $n-2$ for second place, down to $n-n$ for the lowest ranking, where n is the number of options. If there are, say, five options and a particular voter ranks them in the order A, C, D, B, E; n is equal to 5 , so the top-ranked

A is awarded 4 points, C is awarded 3 and so on down to E which is given zero. The points for each option are summed, the option with the highest number of point being declared the winner. In addition to being simpler than the computation of the results of pairwise comparisons, the Borda Count system has several distinct advantages. Firstly, it shares all of the Condorcet system's advantages over the earlier alternatives, it elicits the full range of preference ranking information, and it processes all that information thoroughly and systematically, everyone's lower level preference rankings are effective in contributing to the determination of the outcome independent of eliminations and non-eliminations. The strength of support for each preference is taken into account and the expression of a preference ranking by each voter plays a positive role; in Dummett's terminology the system is fair to options and fair to voters. The crucial novel element of the Borda Count system is that, in awarding decreasing points depending on the order of preferences, it is sensitive, not just to whether one option is ranked above another, but also to where it is ranked, and full comparative ranking contributes to the determination of the outcome. This results in quite definite advantages as compared to Condorcet.

Firstly, the Borda Count system does not produce cycles. Like any other system, in the unlikely event of certain precise specific numbers of votes being cast in specific ways, ties are possible. But almost all Condorcet cycles are avoided. This can be illustrated very easily. The simplest Condorcet cycle occurs with three options and three voters:

| A | B | C |
| :--- | :--- | :--- |
| B | C | A |
| C | A | B |

Table 18
Suppose the columns represented blocks of voters rather than each a single voter. A cycle would occur if each group was of equal size, one-third of the electorate. For simplicity, assume we have 99 voters, so the above case would produce the table:

| 33 | 33 | $\mathbf{3 3}$ |
| :--- | :---: | :---: |
| A | B | C |
| B | C | A |
| C | A | B |

Table 19
This would obviously produce a cycle; in this precise instance where the groups were of identical size a Borda Count would produce a three-way tie, the respective scores being $\mathrm{A}=99, \mathrm{~B}=99, \mathrm{C}=99$. But in all other situations that would produce Condorcet cycles, the Borda Count would produce different scores. For example:

| $\mathbf{3 8}$ | $\mathbf{3 1}$ | $\mathbf{3 0}$ |
| :---: | :---: | :---: |
| A | B | C |
| B | C | A |
| C | A | B |

Table 20
This is still a cycle. A has a majority over B, B over C and C over A; however the fact that A is ranked first by 38 voters, as distinct from B's 31 and C's 30 , is registered and utilised in the Borda Count: A scores 106, B scores 100 and C scores 91 . The simple reason for this is that, in terms of our six dimensions of strength of support, the first dimension, the number of first preferences and the third dimension, the number of votes by which each option defeats each other option, are systematically utilised in determining the outcome.

The second positive aspect of the Borda Count is its sensitivity to where a ranking of one option over another occurs. To refer back to the sort of cases discussed above, A being ranked first to B's tenth out of ten, would be counted differently from B's being ranked first to A's second out of ten. The point is that though we may not be able to give a precise numerical answer to the question of how much weight we should give to this dimension of strength of support, it seems clear that some weight should be given to
it. And this is what a Borda Count does as compared to Condorcet (or, for that matter, to AV or the two-stage run-off).

Registering and utilising where in each voter's rank ordering a particular option is placed is what lies behind the Borda Count system's claim to be globally sensitive. Once we understand the simple mechanism by which points are assigned and summed, it is evident that an option's Borda score is sensitive to where precisely that option is placed in each voter's ranking. The most minimal change, moving an option even one place up or down, would affect the option's score positively or negatively. Hence if two options had the same score and even one voter who ranked the options one immediately above the other minimally changed the ranking by reversing the order, the Borda score of the option whose ranking was minimally raised would respond positively. If the options had been tied in the first place, the one whose Borda score had been minimally raised would be selected as the winner. This global positive responsiveness is what lies behind the Borda Count's superiority as a social welfare maximiser. The following table adapted from Mueller (1989) and Merrill (1984) calculates the utilitarian efficiency of different voting systems. While the precise numbers assume the possibility of full cardinal measures of utility and meaningful interpersonal comparison, they can also be read as a measure of the differential effectiveness of expressed preference, since a precondition of preference satisfaction is the effectiveness of the expressed preference in contributing to an outcome.

Utilitarian Efficiency for a random set of 25 voters

|  | Number of Options |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Voting system | 3 | 4 | 5 | 7 | 10 |
| Plurality | 83.0 | 75.0 | 69.2 | 62.8 | 53.3 |
| Two-stage run-off | 89.5 | 83.8 | 80.5 | 75.6 | 67.6 |
| Alternative Vote | 89.5 | 84.7 | 82.4 | 80.5 | 74.9 |
| Approval | 95.4 | 91.1 | 89.1 | 87.8 | 87.0 |
| Condorcet | 93.1 | 91.9 | 92.0 | 93.1 | 94.3 |
| Borda Count | 94.8 | 94.1 | 94.4 | 95.4 | 95.9 |

Table 21

Although we are here primarily concerned to identify the superiority of the Borda Count in terms of securing the effectiveness of voters' expressed preferences, the table provides an extremely telling synoptic comparison, the one noteworthy point being the very poor showing of the simplest modifications of the majority principle, namely Plurality and Run-off.

Although in abstract "idealised" situations such as those that these calculations are based on, the Borda Count system proves itself superior, it cannot, in the real world, be considered to be the perfect system. Most of the problems with the system derive from the possible "unreliability" of the information that the expressed preference rankings are taken as giving. This possible unreliability has three rather different sources. Firstly, we might wonder whether it is too much of a burden for real-life (rather than axiomatically postulated) voters to be asked to rank-order all the options in the field, when the number of options begins to get reasonably large. The first point that should be noted here is that although the major contender to the Borda Count, the Condorcet system, seems to present voters with a series of simple choices between A and B, B and C and so on, all theorists accept that it is logically equivalent to rank-ordering. If people cannot produce reasonably reliable Condorcet comparisons, then the Condorcet system can xxxx be used. If they can, then the informational base for a Borda Count is available. This somewhat undermines the second part of this argument, which claims that while people's judgments concerning high-level preferences might be reasonably reliable, discrimination between options low down on people's scales of preference may not be so. People might tend to order lower-level preferences somewhat randomly and unpredictably. The problem with this is that such ordering has definitely a positive impact on the Borda score and hence random unsystematic lower-level ordering could determine the actual outcome. It should be noted that this is even more relevant to the Condorcet system, since in that system a ranking of, say, A over B at the bottom of the scale is simply registered as a win for A ; as would a similar ranking with A at the top of the scale and B somewhere down the middle. The Borda Count system would discriminate between the two cases and give much less weight to the first. Furthermore, although, as in AV, voters are allowed to produce a complete ranking, it is not necessary to require them to do so; the normal method of treating unranked options being to award an equal number of points determined by finding the average of the set of points
assigned to the placings not used. It is, of course, an empirical question whether lowerorder rankings are relatively random, but two counter-arguments do carry some weight Firstly, although a tie between a number of strongly supported options that is broken by low-level rankings might be being broken randomly, it might also be the case that it is being broken by genuine "considered" preference judgments. That fact has to be taken into account when considering closing off that possibility. Secondly, some simple technical adjustments, such as assigning greater weight to higher-level ranking differences, would also mitigate this difficulty.

The same technical adjustments could also be used to address the second "reliability" problem. This problem derives from the almost inevitable mismatch between the equal interval in the Borda score of two adjacently ranked options and the underlying quasicardinal preference degree. This might be thought to be a problem, as above, in the intervals at the top of the scale as compared to those at the bottom. That the quasicardinal intervals at the upper end are likely to be larger than those at the bottom end is not, however, necessarily true. Take the example of the villa in France that we used when defining quasi-cardinal preference degree. Suppose the options were (A) a villa for a month in Tuscany (B) a villa for a month in France (C) a weekend in Blackpool and (D) a month in a Siberian labour camp. A person may rank A and B at the top with very little difference, C somewhere down the scale, with D a long, long way away at the bottom. Furthermore, if for a group of voters the agenda is set externally, a top preference vote does not necessarily indicate a particularly positive endorsement, far ahead of the other options. The top ranking might signify only that this option was thought to be the best of a very bad lot. However, if voters can determine their own agenda (as a democratic procedure should allow if it is to be maximally democratic) then the options to be decided on would represent some people's actual best outcomes and a top ranking would then indicate high positive endorsement. If it were thought that this possibility should be institutionally recognised, a greater numerical interval could be introduced between, for example, the top ranked option and the other rankings. This, in fact, is one of the modifications to the simple Borda Count, that has been introduced for the Eurovision Song Contest. The considerations do not seem to be conclusive one way or the other and are, perhaps, of no great import in a comparative assessment of
voting systems, since no other practically feasible system can pretend to elicit anything about quasi-cardinal measures in an inter-personally comparable manner.

The third source of possible unreliability on the informational base is the motivation towards strategic voting. Some theorists such as William Riker take the consequences of the possibility of strategic voting so seriously that they reject the very idea that voting can be thought to fairly reflect a fair aggregation of popular preferences. If there is any weight to this argument, it applies particularly to the Borda Count system, since it can be argued that it readily invites strategic voting, and invites strategic voting of a kind that produces collectively "irrational" choices. As we said earlier, strategic voting consists in the intentional manifestation of a preference ranking different from one's true preference ranking, in the attempt to secure an outcome that would be better in terms of one's true preferences than what might result if one expressed one's true preferences. The simplest kind of scenario in which a motivation to vote strategically can occur is the following. A parliamentary seat in a single-seat constituency is to be decided by plurality vote. Supporters of candidate C believe that C's support is no greater than $10 \%$ whereas A and B are thought to be running neck-and-neck at about $45 \%$ each. Some C supporters might think that, given that they much prefer A to B, whom they abhor, it would be sensible for them not to vote for C , who is obviously their first choice, but for A, so as to attempt to block the election of B. Not only does this strategy seem sensible, it seems both acceptable and "harmless", in the sense that if A were to be elected this would represent the actual fact that A had strong first and second preference support.

When voting systems are more complex, as with the Borda Count, difficulties and anomalies can arise. Consider again an election of a candidate to a single post, with first preference support much like that given in our first chapter. A, let us say, has 48\%, B $46 \%$ and C a mere $6 \%$. Suppose now, however, that a Borda Count system was to be used and the real preferences of the voters over the three candidates is as follows:

| $\mathbf{4 8 \%}$ | $\mathbf{4 6 \%}$ | $\mathbf{4 \%}$ | $\mathbf{2 \%}$ |
| :---: | :---: | :---: | :---: |
| A | B | C | C |
| B | A | A | B |
| C | C | B | A |

Table 21(a)
The Borda scores would be: $\mathrm{A}=146, \mathrm{~B}=142, \mathrm{C}=6$. Suppose, however, that some B supporters, suspecting that C's second preferences would favour A, reasoned that giving their own second preferences to A was decreasing the chances of their first preference, B , getting elected. To make the example simple and dramatic, suppose each and every B supporter decided to promote C to second place, so as to block A's election, with the following result:

| $\mathbf{4 8 \%}$ | $\mathbf{4 6 \%}$ | $\mathbf{4 \%}$ | $\mathbf{2 \%}$ |
| :---: | :---: | :---: | :---: |
| A | B | C | C |
| B | C | A | B |
| C | A | B | A |

Table 21(b)

The result now would be a decisive win for B, still with 148, whereas A now has only 104 , with C still trailing at 52 . This is very different from the plurality example; in terms of outcomes, A's true support is not being reflected, hence it is unfair to the (options/outcomes???). Secondly, B's supporters are giving themselves an advantage by exploiting the integrity of A's supporters. What might happen if each A supporter also realised that their second preference for B might go to elect B , and might do this more surely if B's supporters started to vote strategically, it might seem that the only reasonable response was to block B's election by demoting B to the bottom of the scale. Again to posit the extreme scenario, each A supporter reasons the same way, the result being:

| $\mathbf{4 8 \%}$ | $\mathbf{4 6 \%}$ | $\mathbf{4 \%}$ | $\mathbf{2 \%}$ |
| :---: | :---: | :---: | :---: |
| A | B | C | C |
| C | C | A | B |
| B | A | B | A |

Table 22

C is now the winner with $102, \mathrm{~A}=100$ and $\mathrm{B}=96$. The result, in not nearly reflecting the true support of the candidates, could be considered collectively irrational. Of course, to
make our point dramatically clear we have constructed an extreme scenario; C , who is thought the worst candidate by $94 \%$ of the voters, is elected because A and B supporters are reluctant to decrease the chances of their own candidate winning by voting for their real second preference in second place. Furthermore, if C's support had been significantly higher to begin with, a less extreme vote switch would have produced the anomalous result. The dilemma for A and B supporters is not easily avoided. Suppose there is a general recognition that the promotion of C by enough A and B supporters might result in C's election. This would introduce serious second, third, and fourthguessing uncertainty. Some A supporters might think B supporters would not risk electing C, so they might vote sincerely. This would allow A supporters to vote strategically. But if the B supporters suspected this, more uncertainty would be introduced. Riker's main point is not that the result of manifested preferences can never approximate to the true preferences result, but that we can never know. If even some strategic manipulation occurs frequently, we must conclude, according to Riker:

That the meaning of social choices is quite obscure. They may consist of the amalgamation of the true tastes of the majority (however "majority" and "amalgamation" are defined) or they may consist of the true taste of some people (whether a majority or not) who are skillful or lucky manipulators. If we assume that social choices are often the latter, they may consist of what the manipulators truly want, or they may be an accidental amalgamation of what the manipulators (perhaps unintentionally) happened to produce. Furthermore, since we can by observation know only expressed values (never true values), we can never be sure, for any particular social choice, which of these possible interpretations is correct.
(Riker, 1982, pp. 167-8)
What kind of threat do these arguments pose to a putatively positive assessment of the Borda Count system or, for that matter, to our whole enterprise? This latter question we alluded to in Chapter One, and it is a very serious question. If no voting system can provide anything approximating to reliable knowledge of people's preferences, values or judgments, how expressed preferences are aggregated can be of little concern. Riker's case is not, however, as strong as it looks. His argument is an example of what we might call worst-case scenario epistemology, where the worst that might happen in
terms of error is taken to justify total cognitive paralysis. A possibility is taken as a probability which is more or less "assumed" to be a likelihood which is then interpreted as an insurmountable barrier to the formation of "reasonably grounded" belief. Applying this general point to the specific case of the strategic manipulability of the Borda Count system, we should begin by identifying more precisely what it means to say that a system is manipulable; it means that situations can arise in which there might be a strong incentive to vote strategically. But how probable is it that a single vote would be pivotal, in the sense of crucially determining the outcome? The best estimates (see Brennan and Lomasky pp. xxxxx) put the probability very low. In addition, in many of these possible situations the preferred outcome will be the one achieved by straight voting anyway. Furthermore, the initial incentive will be weakened by uncertainty concerning one's beliefs about other persons' intentions; strategic voting can be extremely risky in the absence of solid information about how others will vote. Opinions differ, but theorists such as Mueller (p. 120) and Dummett (p. 212) conclude that the tendency in practice will not be great. On the general point of total pessimism concerning the interpretation of voting results in the light of the possibility that some people might have voted strategically, Riker's conclusion is a serious exaggeration. Such pessimism, if justified at all, would depend on the impossibility of determining people's actual, as distinct from expressed, preferences. In almost all of his own examples of strategic voting, he assumes that we can identify people's "true and well known tastes" (Riker p. 152); and, of course, the reasonable reliability of opinion poll information gathering provides evidence against which we can cross-check our interpretation of voting results. The possibility of strategic voting should not, then, lead to total scepticism concerning the meaning of collective choices, nor should it lead us to abandon the task of assessing the democratic credentials of different forms of preference amalgamation.

The final problem facing the Borda Count system is rather technical-sounding and concerns "independence from irrelevant alternatives". One of Kenneth Arrow's conditions for an acceptable method of aggregating preferences was that if the system ranked, say, A above B, this should be unaffected by how voters ranked some other, completely different, hence "irrelevant" alternative. In particular, suppose that when ranking three options $\mathrm{A}, \mathrm{B}$ and C , the aggregate ranking is $\mathrm{A}>\mathrm{B}>\mathrm{C}$, the ranking of A
above B, for example, should not be affected by how voters rank another option D. That the Borda Count might contravene this condition can be seen by examining the following example:
$60 \quad 40$
A
B A
C $\quad$ C

B

Table 23

The Borda Count scores are: $\mathrm{A}=120+40=160, \mathrm{~B}=80+60=140, \mathrm{C}=0$.
If the agenda is widened to include the option D , the following might happen.


Table 24

The Borda Count scores would then be: $\mathrm{A}=180+40=220, \mathrm{~B}=120+120=240, \mathrm{C}=60$, $\mathrm{D}=80$.

While B retains second place in the rankings of the first group, the group of 40 who rank B first now rank D above A , and though not a single voter reverses the order in which A and B are ranked, the introduction of D reverses the aggregate ranking of A and B, with B now defeating A by 240 points to 220 !

The air of actual paradox is somewhat illusory; the result comes about because with four places to use to express strength of support, 60 voters still rank $B$ second, but 40 now demote A to third place. The fact that there is distortion in the measure of underlying strength of support is undeniable. It is interesting to note, though, that the results of the vote which had the larger number of places was more discriminating. If with only three options, voters had been given four places, the vote could have been as follows:

|  | $\mathbf{6 0}$ | $\mathbf{4 0}$ |
| :---: | :---: | :---: |
| $1^{\text {st }}$ | A | B |
| $2^{\text {nd }}$ | B | - |
| $3^{\text {rd }}$ | C | A |
| $4^{\text {th }}$ | - | C |

Table 25
In that case, B would have defeated A even in the absence if D . What this discussion, along with the problems arising from the possibility of strategic voting, demonstrates is that no system of aggregating expressed preferences can purport to provide an absolutely accurate measure of the multi-dimensional "strength of support" and, hence, unerringly select the most preferred option. This does not mean that we cannot make highly significant comparisons. Some systems elicit richer information and utilise that information in a way that is likely to respond to the strength of support for options and confers on voters a greater likelihood of their preferences being effective in determining outcomes. Systems such as Plurality, Two-stage run-off and, to some extent, the Alternative Vote are clearly much worse in this regard, as our final diagram graphically illustrates. As S. Merrill III (who produced the original on which our diagram is based) said, it is ironic, and we could add of the highest democratic significance, that the systems most frequently used are clearly the worst.


[^0]:    ${ }^{1}$ Tactical, sometimes called strategic, sometimes insincere, voting occurs when a person does not vote in accordance with "true" preferences. This might in fact be quite a rational thing to do, as when using a plurality voting system one votes for a second preference because of a strong belief that one's first

[^1]:    ${ }^{2}$ Things can, of course, be more complicated than this; of those who support D first, some may prefer A second, and so on. No principle is at stake in over-simplification.

[^2]:    ${ }^{3}$ See Dummett (1983) pp. 172 and ff.
    ${ }^{4}$ The table is extremely symmetrical, but many other complicated structures would produce similar results.

[^3]:    ${ }^{5}$ The formula for calculating the number of pairs is $(n-1)+(n-2)+\ldots . .+(n-n)$ where $n$ is the number of options.

