Lecture 17.

Image formation
  Ray tracing
  Calculation

Lenses
  Convex
  Concave

Mirrors
  Convex
  Concave

Optical instruments
Laws of refraction and reflection can be used to explain how lenses and mirrors operate.

**Convex (converging) lens**
Parallel rays (e.g. from the Sun) passing through a convex lens.

Sunlight focused by magnifying glass may burn hole in paper placed at focal point F.
A farsighted person requires an eyeglass of strength 2.5 diopters. What is the focal length of the eyeglass lens?

\[ f = \frac{1}{D} \]  

(f is in metres)

Example

A farsighted person requires an eyeglass of strength 2.5 diopters. What is the focal length of the eyeglass lens?
Ray 1 entering lens parallel to optic axis will exit and pass through the focal point.

Ray 2 passing through the focal point will exit the lens and travel parallel to optic axis.

Ray 3 will undergo only a small deviation (not shown) (thin lens)

Real inverted image formed
Real image (may be projected and displayed on a screen)

Rays are reversible
**Image Formation (ray tracing)**

**Convex lens**

Object at a distance **less** than the focal length from the lens

![Diagram showing ray paths through a convex lens for different object positions](image)

**Simple magnifier**

- Image
  - virtual,
  - upright
  - magnified.
Concave (diverging) lens

Rays entering lens parallel to axis appear to originate at focal point.

Dashed lines indicate the direction from which the rays appear to come.
Concave (diverging) lens

Object outside the focal length

Virtual image always produced by concave lens

Cannot be viewed on screen since rays are diverging on the right of the lens

However can be viewed with the eye since the eye converges the rays onto the retina.
Ray diagrams are useful in sketching the relationship between object and image. Relationship may also be calculated.

Triangles AOB and DOC are similar:
\[ \frac{h}{s} = -\frac{h'}{s'} \]
\[ \frac{h'}{h} = -\frac{s'}{s} \]

Triangles EFO and DCF are similar:
\[ \frac{h}{f} = -\frac{h'}{s' - f} \]
\[ \frac{h'}{h} = -\frac{s' - f}{f} \]
\[ -\frac{s'}{s} = -\frac{s' - f}{f} \]
\[ \frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \]
Thin lens formula

\[ \frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \]

Object distance \(s\)
- positive if object is in front of lens
- negative if object is behind lens

Image distance \(s'\)
- positive if image is formed behind the lens (real)
- negative if is formed in front of the lens (virtual)

Focal length \(f\)
- positive -- convex lens
- negative -- concave lens
Magnification is defined as

\[ M = -\frac{s'}{s} \quad \text{or} \quad M = \frac{h'}{h} \]

M Negative:
- inverted image

M Positive:
- upright image
Object placed **inside** focal length of converging lens;

**image viewed**
- virtual,
- magnified
Example
An object 0.5 cm in height is placed 8 cm from a convex lens of focal length 10 cm. Determine the position, magnification, orientation and height of the image.
Example
An object is placed 45 cm from a lens of focal length -25 cm. Determine the position, magnification, and orientation of the image.
Combining Lenses

Effective focal length \( (f_{\text{eff}}) \) of combination of a number of thin lenses close together

\[
\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots
\]

Effective strength \( (S_{\text{eff}}) \) of combination of a number of thin lenses close together

\[
S_{\text{eff}} = S_1 + S_2 + \ldots
\]

Determine the combined strength of a thin convex lens and a thin concave lens placed close together if their respective focal lengths are 10cm and -20cm.
Mirrors

**Flat Mirror**
Concave Mirror
Convex Mirror

Curved mirrors are analogous to lenses
Ray tracing and thin lens equation also valid.
real and virtual images are also formed

**Flat Mirror**

Object and image distance equal
Object and image same size
Image, upright, virtual
Spherical Mirrors

Hollow sphere

Spherical mirror is a section hollow sphere

Principal or optic axis

Radius of curvature $R = 2f$
Curved Mirrors (Spherical)

**Concave** mirror
(converging)

Positive focal length

**Convex** mirror
(diverging)

Negative focal length

Thin lens formula may be used to determine object and image distances and focal lengths etc

**Lenses and mirrors**

Real image: inverted (h’ negative), positive image distance s’

Virtual image: upright (h’ positive), negative image distance s’
Mirrors

Concave shaving/makeup mirrors

Object placed at distance < f from mirror

Image is virtual, upright and enlarged.

Question: if object is placed at the focal point, where is image located?

Application: searchlight

C is the centre of curvature
Example
An object is positioned 5 cm in front of a concave mirror of focal length 10 cm. Determine the location of its image and its characteristics.
Optical instruments

Microscopes, telescopes, cameras etc

System may have many optical elements (example, lenses and mirrors)

Thin lens formula or ray tracing may be used to analyse behaviour of such systems

Simple compound microscope
two convex lenses

Object (height $h$)

image formed by objective lens is inside focal length of eyepiece lens.
Optical instruments

Dental loupes

Multiple lenses

Important Characteristics

• Resolution*
• Field width *
• Field depth
• Magnification

Resolution

• Ability to see fine detail

Field width

Size of operating site when viewed through loupe
Function of lens system diameter and magnification

Field depth

Depth or range of focus
Depends on, available light, optical design, magnification and accommodation

Magnification,

Important but not at the expense of resolution. Large fuzzy image of little use.
Optical instruments

Dental loupes

Typically Magnification
m ≈ 2.5 → 4.5

Optical design allows observer focus at infinity thereby relieving eyestrain

Galilean Design

Typical working distance
28-38 cm
Objective lens forms image (real, inverted) at focal point $F_0$ which is also the focal point $F_e$ of the eyepiece; virtual image is then formed at infinity.

Virtual image at infinity, Magnified and inverted

$$M = -\frac{f_0}{f_e}$$
Example
Effective focal length of the objective in the Hubble telescope is 57.8 m. What focal length eyepiece is required to give a magnification of $-8.0 \times 10^3$. 

$$m = -\frac{f_e}{f_0}$$

$$f_e = -m f_0 = -(-8.0 \times 10^3) \times 57.8 \text{ m} = 7.23 \times 10^3 \text{ m}$$
Applications

Image transport, Coherent fibre bundle

**Optical communications:**

Optical fibres used to transmit modulated laser beams; carrying information

**Telephone and internet communications**
Rate at which information can be transported proportional to frequency of light

Single fibre: many millions of phone conversations simultaneously.
Cable has many fibres
Endoscope

Endoscope for medical investigations—inserted through small incision or orifice to inspect and facilitate operation on interior parts of the body.

Flexible shaft includes:
- light source to illuminate area,
- image channel to view area under investigation,
- air or water conduit to clear debris,
- instrument conduit

**Typical endoscope**

- **Eyepiece**
- **Instrument entry**
- **Flexible shaft**
- Transmits light, air, water
Lens translated to change image distance $S'$ to adjust for different object distances $S$. Focal length of lens is fixed.

Real, inverted image formed on CCD array

\[
\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}
\]
Example:
An object of height 3 cm is positioned 40 cm from a concave lens with a focal length of -20 cm. Determine the position of the image, its magnification, height, and orientation.