Physics of the Interstellar and Intergalactic Medium

Lecture 8: Nebulae

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How quickly does the H II region form?

- Ionization front is radius where the number of Lyman continuum photons = number of inflowing hydrogen atoms
  
  \[ V_{IF}(t) = \frac{dR_{IF}}{dt} = \frac{1}{4\pi n_H R_{IF}^2} \left( N_{Lyc}^* - \frac{4\pi}{3} R_{IF}^3 \alpha_B n_H^2 \right) \]

- Just difference between those initially available and those already absorbed. Integrate to get radius of \( R_{IF} \)

\[
R_{IF}^3(t) = R_S^3 \left[ 1 - \exp \left( -t/\tau_{rec} \right) \right] \quad \tau_{rec} = 1/n\alpha_B
\]

- Where \( \tau_{rec} \) is the recombination time:
  - \( \tau_{rec} \sim 100 \) years for \( n=1000 \) cm\(^{-3}\)
  - Velocity of front for O4 star reaches \( V_{IF}=4000 \) kms\(^{-1}\)
  - Expression valid until front slows down and shock forms.
Common transitions:

- [SII] 6717/6731 Å
- [NII] 6748/6784 Å
- Hα 6563 Å
- [OI] 6300/6363 Å
- [OIII] 5007 Å
- [OII] 3729/3726 Å
Recombination lines, 2 photon-continuum
Sir William Huggins: "On the evening of the 29th of August, 1864, I directed the telescope for the first time to a planetary nebula in Draco (NGC 6543). The reader may now be able to picture to himself to some extent the feeling of excited suspense, mingled with a degree of awe, with which, after a few moments of hesitation, I put my eye to the spectroscope. Was I not about to look into a secret place of creation? I looked into the spectroscope. No spectrum such as I expected! A single bright line only!"
Nebulium identified ~ 5000 Angstroms

- Resolved into 2 separate lines
  - 5006.9 Ang.
  - 4958.9 Ang.
- Identified in 1925 as doubly ionized oxygen [O III] – 75 years to identify
- [Coronium 5303 Å Fe XIV (1930s)]
- Very strong coolant for H II regions
Temperature for pure H II region \( \Gamma = \Lambda(T_\text{e}) \)

Mean thermal energy per photoionization (lecture 4)

\[
\Delta E = \frac{4\pi \int_{\nu_0}^{\infty} h(\nu - \nu_0) \frac{\sigma_\nu J_\nu}{h \nu} d\nu}{4\pi \int_{\nu_0}^{\infty} \frac{\sigma_\nu J_\nu}{h \nu} d\nu}
\]

If we use the \( \nu^{-3} \) photoionzation \( \sigma \) then if \( T_\text{eff} < 150,000 \text{ K} \) and the spectrum can be described by a blackbody in the Lyman continuum then

\( \Delta E \approx kT_\text{eff} \)

And the heating rate:

\( \Gamma_H = x^2 n^2 \alpha_B(T_\text{e}) kT_\text{eff} \)

Mean cooling rate

\( \Lambda_{fb} = x^2 n^2 \beta_B(T_\text{e}) kT_\text{e} \)

\( + \Lambda_{ff} = x^2 n^2 \beta_{ff} kT_\text{e}^{1/2} \)
H Lyman continuum heating

$T_{\text{star}} = 30000 \text{ K}$

$J_\nu$

Hydrogenic approx

$\sigma \sim \left(\frac{v_0}{v}\right)^3$
A Transcendental Equation

- Equating heating to cooling can re-write simply as
  \[ \Gamma_H = \Lambda_{bf} + \Lambda_{ff} \]
  \[ \alpha_B k T_{eff} = \beta_{fb} k T_e + \beta_{ff} k T_e^{1/2} \]
- A transcendental equation that can be solved numerically
- Solution: \( T_e \sim 0.7-0.9 T_{eff} \) (depending on shape of ionizing rad.)
- H II regions are typically observed around B2 – O3 stars
- \( T_{eff} = 25,000-50,000 \) K
- Pure hydrogen nebulae have higher \( T_e \)'s than we observe
- But the FIRST H II regions were this hot!
- Metals make all the difference – why?

O4 star \( T_e = 27,000 \) K which is much higher than observed
\( T_e = 7,200 \) K with other elements present
Temperature for H II region with metals

Forbidden line cooling from metals (elements heavier than helium) is important because there are atomic energy levels accessible to thermal electrons, representative example, e.g., O$^+$ (O II) 3728 Ang.

\[ \Gamma_H = \Lambda_{OII} \]

Eq 5.35 p. 76 (Dyson and Williams) numerically has the solution

\[ T_e^{1/4} \exp \left( -3.89 \times 10^4 / T_e \right) = 2.5 \times 10^{-6} T_{\text{eff}} \]

<table>
<thead>
<tr>
<th>$T_{\text{eff}}$ (K)</th>
<th>Equilibrium $T_e$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000</td>
<td>7450</td>
</tr>
<tr>
<td>40,000</td>
<td>8500</td>
</tr>
<tr>
<td>60,000</td>
<td>9300</td>
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</tbody>
</table>

More realistic values
Electron densities and $T_e$

- For $n_e$ consider the ratio of emission fluxes from two lines with different critical densities
- $10^2 \text{ cm}^{-3} < n_{\text{crit}} < 10^6 \text{ cm}^{-3}$
- same element and ion helps reduce inherent uncertainties
- Not all ratios are useful at a given $n_e$
Thermal Bremsstrahlung (radio)

Bremsstrahlung radiation (free-free): acceleration from Coulomb interactions

Thermal (Planck) source function result of a Maxwellian distribution of electrons

Opacity corrected for stimulated emission (important when $h\nu \ll kT$) at radio wavelengths is

$$\tau \approx C \nu^{-2.1} T_e^{-1.35} n_e^2 L$$

Note: synchrotron radiation: acceleration is a result of a magnetic field.
Rayleigh-Jeans Approximation

- At radio wavelengths and nebula temperatures
  - Planck Function can be described by the Rayleigh-Jeans Approximation

\[ S_v = B_v(T_e) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1} \approx \frac{2\nu^2kT}{c^2} \]

Equation of radiative transfer for slab geometry (and no background)

\[ I_v = B_v(T_e)(1 - e^{-\tau_v}) = \frac{2\nu^2kT_e}{c^2}(1 - e^{-\tau_v}) \]

Also written in terms of brightness temperature, \( T_B \)

\[ I_v \equiv \frac{2\nu^2kT_b}{c^2} \]

Equation of radiative transfer becomes

\[ T_B = T_e(1 - e^{-\tau_v}) \]
Radio continuum spectrum

\[ I_v = \frac{2v^2 k T_e}{c^2} \left( 1 - e^{-\tau_v} \right) \]

- In the limits of very small and very large optical depths (high and low frequencies, respectively) we approach

\[ [\tau << 1] \quad I_v \propto v^{-0.1} T_e^{-0.35} \]
\[ [\tau >> 1] \quad I_v \propto v^{+2} T_e \]

- Spectral turn-over

![Graph](image-url)
Radio Recombination Lines

Large principal quantum numbers semi-classical~Bohr model

\[ \nu_{n,m} = R_M \left( \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right) \]

\[ R_M = R_\infty \left[ 1 + \frac{m_e}{M} \right]^{-1} \]
Observed Recombination Lines

Shows effect of reduced mass

\[ \alpha \quad \Delta n = 1 \]

\[ \beta \quad \Delta n = 2 \]

\[ \gamma \quad \Delta n = 3 \]

\[ \delta \quad \Delta n = 4 \]

\[ \varepsilon \quad \Delta n = 5 \]

Pulsar dispersion measures

- EM waves propagate through free electron gas with refractive index $m$
  $$\frac{c}{v_{\text{phase}}} = m = \sqrt{1 - \frac{v_{\text{plasma}}^2}{v^2}}$$

- $v_p$ is the plasma frequency ($v > v_p$)
  $$v_{\text{plasma}}(\text{Hz}) = \sqrt{\frac{n_e e^2}{\pi m_e}} = 8.97 \times 10^3 \sqrt{n_e (\text{cm}^{-3})}$$

- Signal propagates at the group velocity
  $$\frac{1}{v_{\text{group}}} = \frac{d}{dv} \left( \frac{v}{v_{\text{phase}}} \right)$$

- Some algebra …
  $$v_{\text{group}} = \frac{c^2}{v_{\text{phase}}} = c \sqrt{1 - \frac{v_{\text{plasma}}^2}{v^2}}$$
Pulsar dispersion measures

- For high frequencies expand expression for $\nu_{\text{group}}$ in terms of $\nu$.

- Derive travel time of pulsar signal:

$$t = \int_0^L \frac{ds}{\nu_g} = \frac{L}{c} + \frac{e^2}{2\pi m_e c} \int_0^L n_e ds$$

Integral = Dispersion Measure

$$D = \int n_e dl$$

- Practice measure the change in delay ($\Delta t = t-L/c$) function of frequency:

$$\frac{d\Delta t}{dv} = \frac{e^2}{\pi m_e c} \frac{D}{v^3}$$
A measure of inhomogeneity

- Testing our assumptions!
- Compare emission from diagnostics that are dependent on density, e.g., line optical depths, pulsar dispersion measures.
- With diagnostics that are dependent on density², e.g., H-alpha emission, continuum (free-free) radio optical depths.
- If \( \frac{\langle n_e \rangle^2}{\langle n_e^2 \rangle} \neq 1 \) then clumped.

\[ \propto \int n_e \, dl = \langle n_e \rangle L \]
\[ \propto \int n_e n_e \, dl = \langle n_e^2 \rangle L \]
Uniformity of nebulae and the ISM

- Biggest limitation in simplified analysis is that plasmas are not uniform – instabilities within single component, or multiple components

- ISM observations show that 90% of ionized ISM fills only 25% of its volume

- H II Regions and nebulae are also clumped
If the gas pressure ($P$) can be written in terms of the density ($\rho$) then

$$P = K\rho^\gamma$$

- where, $K$ constant
- $\gamma$ is the ratio of specific heats $c_P/c_V$
- $\gamma=5/3$ - adiabatic perfect mono-atomic gas
- $\gamma=1$ - isothermal

Consider small amplitude acoustic perturbations in 1-D then the speed of sound $a_s$ is given by

$$\frac{dP}{d\rho} = a_s^2 = \gamma K\rho^\gamma \Rightarrow a_{s0}^2 \equiv \gamma \frac{P_0}{\rho_0}$$
In absence of sinks and sources, the rate of change of mass in volume \( V \) equals the mass flux through elements of area \( ndA \) (where \( n \) is the normal)

\[
\frac{d}{dt} \int_V \rho \, dV = -\int_A \rho u \cdot \hat{n} \, dA = -\int_V \nabla \cdot (\rho u) \, dV
\]

Differentiate inside the integral on LHS and bring across

\[
\int_V \left[ \frac{d\rho}{dt} + \nabla \cdot (\rho u) \right] \, dV = 0
\]

Since \( V \) is arbitrary, we require the integrand to vanish

\[
\frac{d\rho}{dt} + \nabla \cdot (\rho u)
\]
Propagation of disturbances

Assume a uniform gas of constant pressure and density at rest, then perturb

\[ P = P_0 + P_1 \]
\[ \rho = \rho_0 + \rho_1 \quad P_1 = \gamma K \rho_0^{\gamma - 1} \rho_1 \quad [0] \]
\[ u = u_0 + u_1 \]

- Where 0 indicate rest values, and 1 perturbed values.
- Linearize the Equation of Conservation of Mass

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \]
\[ \frac{\partial (\rho_0 + \rho_1)}{\partial t} + (u_0 + u_1) \frac{\partial (\rho_0 + \rho_1)}{\partial x} + (\rho_0 + \rho_1) \frac{\partial (u_0 + u_1)}{\partial x} = 0 \]
\[ \frac{\partial \rho_1}{\partial t} + u_1 \frac{\partial \rho_1}{\partial x} + \rho_0 \frac{\partial u_1}{\partial x} = 0 \quad \Rightarrow \quad \frac{1}{\rho_0} \frac{\partial \rho_1}{\partial t} + \frac{\partial u_1}{\partial x} = 0 \quad [1] \]
Propagation of disturbances

- Linearize the Equation of Conservation of Momentum (Eulers Equation)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x}
\]

\[
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = - \frac{1}{\rho_0} \frac{\partial P_1}{\partial x}
\]

\[
\Rightarrow \frac{\partial u_1}{\partial t} = - \frac{1}{\rho_0} \frac{\partial P_1}{\partial x} \quad [2]
\]

- Differentiate [1] w.r.t Time and [2] w.r.t. Space, substitute [0] for \( P_1 \) then subtract the two

\[
\frac{\partial^2 \rho_1}{\partial t^2} - a_{s0}^2 \frac{\partial^2 \rho_1}{\partial x^2} = 0
\]

- Standard wave equation: 2 opposite waves travelling with speed \( a_{s0} \)
Conclusions

- If material is moving with uniform velocity $v$ then by a change of reference frame there are now 2 solutions travelling with

$$u_0 + a_{s0} \quad \text{and} \quad u_0 - a_{s0}$$

- The velocity is constant because we have ASSUMED tiny perturbations in our derivation. Recall

$$a_s^2 = \gamma K \rho^{\gamma - 1} \Rightarrow a_{s0}^2 \equiv \gamma \frac{P_0}{\rho_0}$$

- So if $\gamma > 1$ then the sound speed is greater when the density is higher and this is where shocks come in...