

# INTERACTION OF LIGHT WITH MATTER

Already...the speed of light can be related to the **permittivity**,  $\epsilon$  and the **magnetic permeability**,  $\mu$  of the material by

$$v = \frac{1}{\sqrt{\epsilon\mu}}$$

Remember  $\epsilon = \epsilon_r \epsilon_0$  and  $\mu = \mu_r \mu_0$  where  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$  and  $\epsilon_r$  depends on the material  $\mu_0 = 4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{ m}^{-1}$  and  $\mu_r$  depends on the material

In free space  $\epsilon_r = \mu_r = 1$  and  $v = c = 1/\sqrt{(\epsilon_0\mu_0)} = 3 \times 10^8 \text{ ms}^{-1}$

$$v = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{1}{\sqrt{\epsilon_r \mu_r}} \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

But remember

$$v = \frac{c}{n}$$

so the refractive index,  $n$ , is given by

$$n = \sqrt{\epsilon_r \mu_r}$$

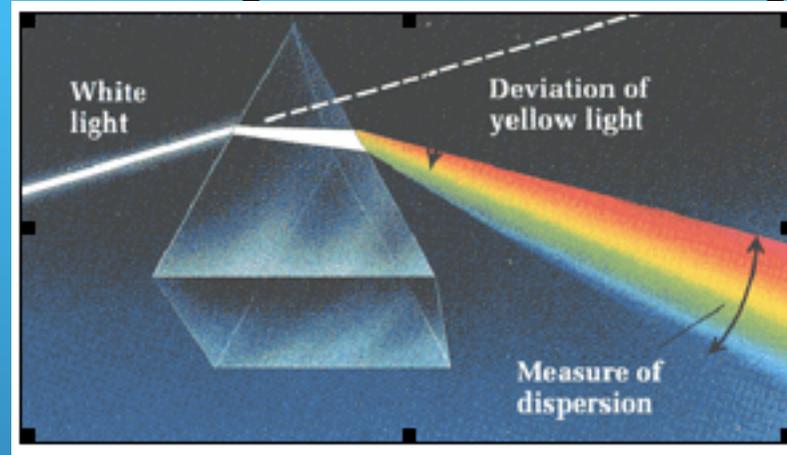
In most materials (ie non-magnetic materials),  $\mu_r \approx 1$  so

$$n \approx \sqrt{\epsilon_r}$$

These related quantities are obviously very important if we want to understand the interaction of light with matter.

**Where have we seen an example of how refractive index depends on wavelength?**

A prism breaks up white light into its constituent colours.  
This works because light of different wavelengths is refracted differently.



$n$  varies with wavelength!

This is known as **Dispersion**.

*We are interested in how  $n$  varies with wavelength.*

**How does light interact with matter?**

Light consists of oscillating electric and magnetic fields.

In non-magnetic materials (most materials) the **B** field has very little effect.

We will focus on the effects of the **E** field.

An electric field exerts a force on a charge such that  $F = qE$ .  
Initially we will deal with the interaction of light with **dielectric** materials.  
These are materials with no free electrons, ie non-metals.

When an E field is applied to a dipole, the **internal charge distribution** is distorted  
This results in the generation of **electric dipole moments**.

A dipole moment,  $p$ , consists of a positive and negative charges,  
 $q$  and  $-q$  separated by a distance,  $x$ , and is defined as:

$$p = qx$$

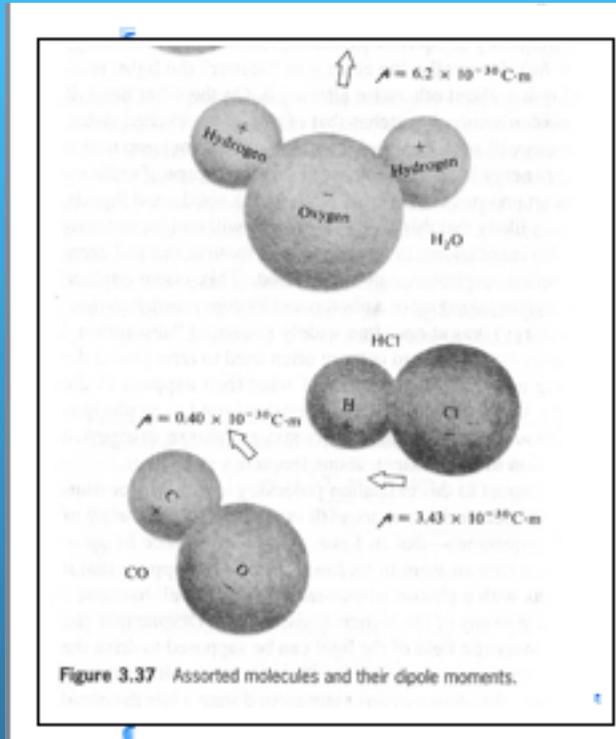
The total dipole moment per volume is called  
the **Electric Polarisation,  $P$**  and is given by:

$$P = Np$$

Where  $N$  is the number of dipoles per volume

There are 3 main types of dipoles.

1. *Some molecules have permanent dipole moments*

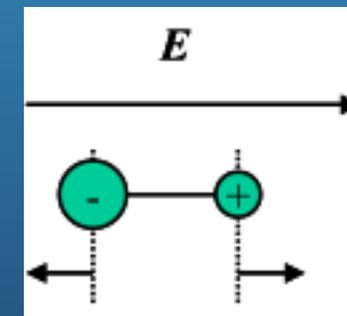


An  $E$  field causes the dipole moment to align with the field. Thus the dipoles oscillate in an oscillating  $E$  field.

This is known as **orientational polarisation.**

2. *Some materials are ionic in nature ie  $\text{Na}^+\text{Cl}^-$ .*

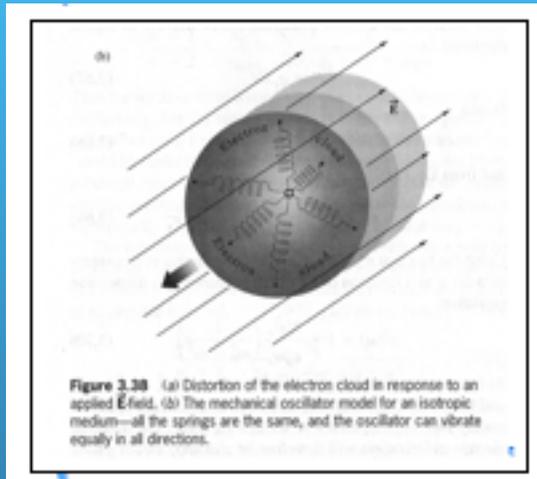
Here, in the presence of an electric field, the +ve and -ve ions move in opposite directions. This causes a dipole which oscillates in an oscillating  $E$  field.



3. *Some materials have neither permanent dipoles nor are ionic*

However all materials have **electrons and nuclei**.

Here, in the presence of an electric field, the *electrons (-)* and *nuclei (+)* move in opposite Directions. This causes a dipole which oscillates in an oscillating  $E$  field



Here, in the presence of an electric field, the *electrons (-)* and *nuclei (+)* move in opposite directions. This causes a dipole which oscillates in an oscillating  $E$  field

### The Dipole Oscillator model (Lorentz Model)

We know that an electric field exerts a force on a charge.

Lets look at all the forces that a charge in a material experiences when when an electric field is present.

The electric field exerts a force on the electron:

$$F_e = -eE$$

However the electron is attracted back to the nucleus because it is **bound**. This gives rise to a restoring force proportional to the displacement,  $\mathbf{x}$ :

$$\mathbf{F}_r = -k\mathbf{x}$$

The electron feels a **frictional** force proportional to its velocity,  $v = dx/dt$ :

$$\mathbf{F}_f = -m\gamma\mathbf{v} = -m\gamma d\mathbf{x}/dt$$

This is known as **damping**

Note that, in the absence of an electric field and if friction is negligible, the only force is the restoring force.

Newtons Law:  $\Sigma\mathbf{F} = m\mathbf{a} = m d^2\mathbf{x}/dt^2$

$$m \frac{d^2 x}{dt^2} = -kx$$

This is just the equation of SHM with natural frequency

$$\omega_0^2 = k / m$$

Thus even in the absence of a driving force, bound electrons have a **natural frequency of oscillation**,  $\omega_0$ .

We can write Newton's law for the case when light interacts with an atom. The oscillating electric field exerts a force on the electron causing it to oscillate.

Adding the restoring and frictional force, we get the equation of motion of the electron:

$m \frac{d^2 x}{dt^2} = \sum F$  Now inserting in the forces from above:

$$m \frac{d^2 x}{dt^2} = -eE - kx - m\gamma \frac{dx}{dt}$$

Now remember the E field of the light can be written as:

$$E = E_0 \cos(kz - \omega t) \rightarrow \rightarrow \text{or}$$

$$E = E_0 e^{i(kz - \omega t)}$$

We will find that the exponential version is easier to work with:

$$E = E_0 e^{i(kz - \omega t)} = E_0 e^{ikz} e^{-i\omega t} = E_1 e^{-i\omega t}$$

( $E_1$  is effectively constant as the wavelength of light is much larger than atomic sizes  $\rightarrow$  The  $E$  field is constant in space at any given instant in the vicinity of a given atom). This allows us to write the equation of motion as:

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = -\frac{e}{m} E_1 e^{-i\omega t}$$

$$\omega_0^2 = k/m$$

This is a differential equation that we must solve.

The easiest way is to guess a solution. If the  $E$  field of the light oscillates at an angular frequency,  $\omega$ , we can guess the electrons will oscillate at the same angular frequency. Then the electrons position as a function of time is:

$$x = A e^{-i\omega t} \longrightarrow \frac{dx}{dt} = -i\omega A e^{-i\omega t} \quad \frac{d^2 x}{dt^2} = -\omega^2 A e^{-i\omega t}$$

Putting this into the equation of motion:

$$-\omega^2 Ae^{-i\omega t} - i\gamma\omega Ae^{-i\omega t} + \omega_0^2 Ae^{-i\omega t} = -\frac{e}{m} E_1 e^{-i\omega t}$$

giving

$$A = \frac{-eE_1/m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

So that

$$x = \frac{-eE_1/m}{\omega_0^2 - \omega^2 - i\gamma\omega} e^{-i\omega t}$$

which can be rewritten as:

$$x = \frac{-e/m}{\omega_0^2 - \omega^2 - i\gamma\omega} E$$

Remember

$$E = E_1 e^{-i\omega t}$$

Suggesting that position is imaginary?

$$\frac{-e/m}{\omega_0^2 - \omega^2 - i\gamma\omega} = a + ib$$

Remember complex numbers.....we can write x as:

$$\begin{aligned} \frac{-e/m}{\omega_0^2 - \omega^2 - i\gamma\omega} &= \left( \frac{-e/m}{\omega_0^2 - \omega^2 - i\gamma\omega} \right) \left( \frac{\omega_0^2 - \omega^2 + i\gamma\omega}{\omega_0^2 - \omega^2 + i\gamma\omega} \right) = \\ &= \frac{(-e/m)(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} + i \frac{(-e/m)(\gamma\omega)}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} = a + ib \end{aligned}$$

We can write x as:

$$x = (a + ib)E_1 e^{i\omega t}$$

or

$$x = C e^{i\delta} E_1 e^{i\omega t} = C E_1 e^{i(\delta + \omega t)}$$

There are two ways of writing a complex number:

$$C e^{i\delta} = C \cos \delta + i C \sin \delta = a + ib$$
$$a^2 + b^2 = C^2, \tan \delta = b/a$$

So the imaginary part of x represents a phase shift,  $\delta$ , the position lags behind the electric field. Now back to x .....

$$x = \frac{-eE_1/m}{\omega_0^2 - \omega^2 - i\gamma\omega} e^{-i\omega t}$$

This oscillatory motion of the electron means we have an oscillating dipole,  $p$

$$p = -ex$$

and so an oscillating electric polarization,  $P$ :

$$P = -Nex$$

For most materials  $P \propto E$  and can be described by

$$P = (\epsilon_r - 1)\epsilon_0 E \quad \text{Thus}$$

$$P = -Nex = \frac{Ne^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} E_1 e^{-i\omega t}$$

By comparing these two equations we can write:

$$\epsilon_r - 1 = \frac{Ne^2}{\epsilon_0 m} \frac{1}{(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

NB: This has frequency dependence and is a complex number!

This means the permittivity has **real** and **imaginary** parts.

$$\epsilon_r = \epsilon_1(\omega) + i\epsilon_2(\omega)$$

This can easily be worked out as

$$\epsilon_1(\omega) = 1 + \frac{Ne^2}{\epsilon_0 m} \frac{\omega_0^2 - \omega^2}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]}$$

Real

$$\epsilon_2(\omega) = \frac{Ne^2}{\epsilon_0 m} \frac{\gamma\omega}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]}$$

Imaginary

What does this mean?

Remember.....

So if  $\epsilon_r$  is complex,  $n$  is complex.

So..

$$n \approx \sqrt{\epsilon_r}$$

$$n = n_1 + in_2 \rightarrow \rightarrow \text{so} \rightarrow (n_1 + in_2)^2 = \epsilon_1 + i\epsilon_2$$

$$\varepsilon_1 = n_1^2 - n_2^2$$

$$\varepsilon_2 = 2n_1n_2$$

These can be rearranged to give  $n_1$  and  $n_2$ . Both  $n_1$  and  $n_2$  can be found from the equations for  $\varepsilon_1$  and  $\varepsilon_2$ .

Now consider a plane wave traveling in the  $z$  direction in some material:

$$E = E_0 e^{i(kz - \omega t)}$$

In a material  $k = 2\pi/\lambda = 2\pi n/\lambda_0 = nk_0$   
Therefore..

$$E = E_0 e^{i[nk_0 z - \omega t]}$$

But  $n = n_1 + in_2$  so

$$E = E_0 e^{i[(n_1 + in_2)k_0 z - \omega t]}$$

which can be rewritten as

$$E = E_0 e^{-n_2 k_0 z} e^{i[n_1 k_0 z - \omega t]}$$

This has an **oscillating** and a **decaying** component. In the oscillating component  $n_1$  is acting just like a normal refractive index, (note the  $n_1 k_0$  looks like  $nk_0 = k$ )

If we square the E-field (complex so  $E^2 = E^* E$ ), we get

$$E^2 = E_0^2 e^{-2n_2 k_0 z}$$

But remember that the light **Intensity** is proportional to  $E^2$ . Hence

$$I = I_0 e^{-2n_2 k_0 z}$$

$I_0$  is the light intensity  
when  $z=0$

$$I_0 = \frac{\epsilon c}{2} E_0^2$$

This says that the light intensity decays exponentially as it travels through the material if  $n_2 \neq 0$ .

This equation describes the absorption of light by a material and is called the **Lambert Beer Law**. It is usually written as  
where  $\alpha$  is the **absorption coefficient**.

This means

$$I = I_0 e^{-\alpha z}$$

where  $\alpha$  is the **absorption coefficient**.

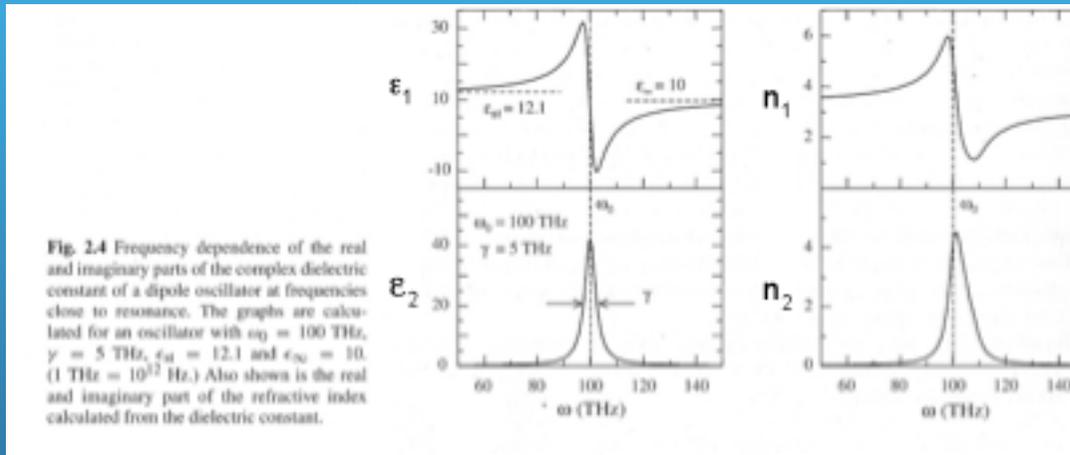
This means

$$\alpha = 2n_2 k_0 = \frac{4\pi n_2}{\lambda_0}$$

All this means that the real and imaginary components of the refractive index have physical meanings.

$$E = E_0 e^{-n_2 k_0 z} e^{i[n_1 k_0 z - \omega t]}$$

The real part of  $n$ ,  $n_1$ , then controls the velocity of light in the medium while the imaginary part,  $n_2$ , controls the absorption of light. Remember both  $\epsilon_1$ ,  $\epsilon_2$ ,  $n_1$  and  $n_2$  vary with frequency.



The peaks in  $\epsilon_2$  and  $n_2$  always occur at  $\omega = \omega_0$  and are known as *resonances* and represent the *absorption of light*.

In transparent materials like glass, it seems that there is no absorption. This just means that in such materials  $n_2$  is very small at visible frequencies.

In such a transparent dielectric, when  $n_2$  is very small, it means

$$\begin{array}{l} \varepsilon_1 \approx n_1^2 \qquad \varepsilon_1 = n_1^2 - n_2^2 \\ \text{that} \rightarrow \varepsilon_2 \approx 0 \quad \rightarrow \text{because} \rightarrow \rightarrow \varepsilon_2 = 2n_1 n_2 \end{array}$$

Hence

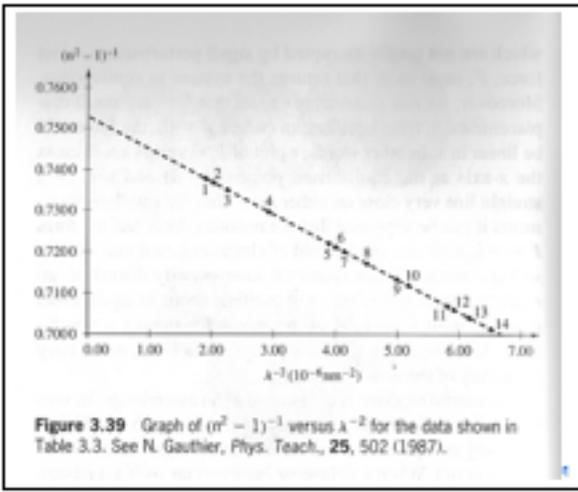
$$\varepsilon_1(\omega) = n_1^2 = 1 + \frac{N e^2}{\varepsilon_0 m} \frac{\omega_0^2 - \omega^2}{[(\omega_0^2 - \omega^2)^2 + (\gamma \omega)^2]}$$

In materials where electrons are **tightly bound** such as glass, the restoring force constant,  $k$ , is very high. ( $\omega_0^2 = k/m$ )  
This means  $\omega_0^2$  is very large ( $\omega_0^2 = k/m$ ).

Then for optical frequencies  $(\omega_0^2 - \omega^2) \gg (\gamma \omega)^2$

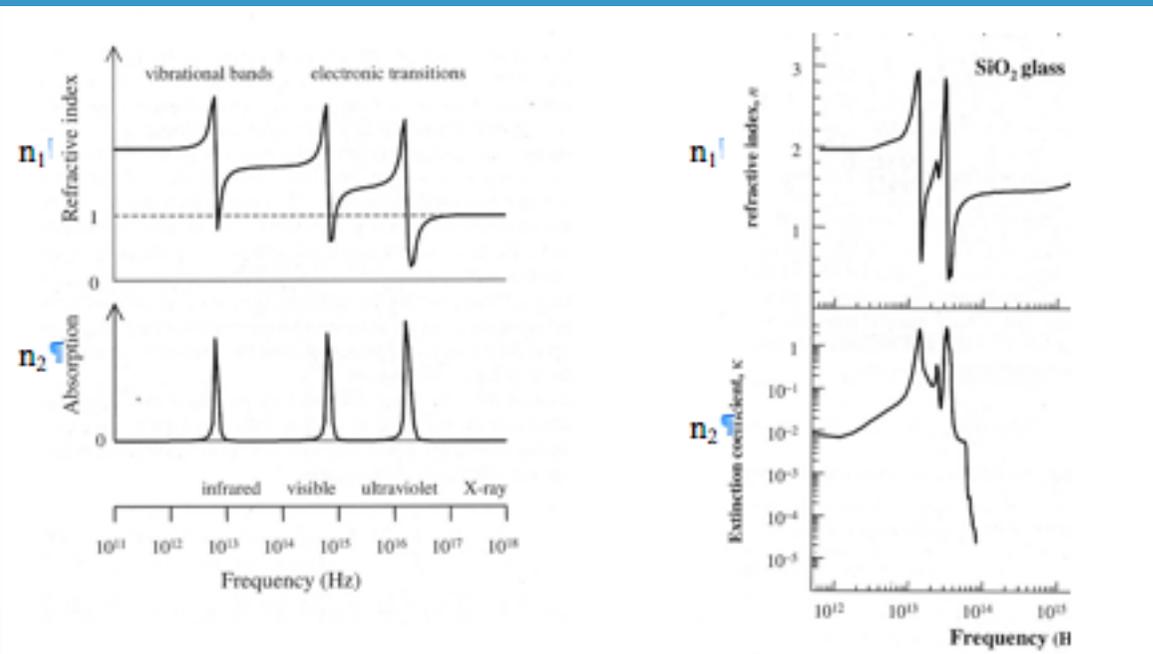
In this regime  $(\gamma \omega)^2$  can be neglected and:

$$n_1^2 \approx$$



Refractive index versus wavelength data for Crown glass

In reality, a given atom may have a number of different  $\omega_0$



Calculated from theory → → → → → Measured

## INTERACTION OF LIGHT WITH METALS

The same analysis applies to metals as to dielectrics.

$$\begin{aligned}\epsilon_1(\omega) &= 1 + \frac{Ne^2}{\epsilon_0 m} \frac{\omega_0^2 - \omega^2}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]} \\ \epsilon_2(\omega) &= \frac{Ne^2}{\epsilon_0 m} \frac{\gamma\omega}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]}\end{aligned}$$

However there are some notable differences. In metals the interaction with light is dominated by the **free electrons**. These are completely unbound and so the restoring force constant,  $k$ , is zero. This means that  $\omega_0^2 = 0$  ( $\omega_0^2 = k/m$ ).

Furthermore, the damping (frictional) forces tend to be negligible in some metals. This means that in many cases  $\gamma \approx 0$ .

Putting these conditions into the equations above give us:

$$\begin{aligned}\epsilon_1(\omega) &= 1 - \frac{Ne^2}{\epsilon_0 m} \frac{1}{\omega^2} \\ \epsilon_2(\omega) &\approx 0\end{aligned}$$

It is convenient to write  $Ne^2/\epsilon_0 m = \omega_p^2$  where  $\omega_p$  is the **Plasma Frequency**

So we can write:

$$n^2 \approx \epsilon_1$$

So

$$n^2 \approx 1 - \frac{\omega_p^2}{\omega^2}$$

The thing to notice here is that  $n < 1$ !

This appears to defy Einstein's Special theory of relativity.

However the velocity  $n$  applies to the **phase velocity**.

The velocity that optical signals travel at in reality is the **group velocity**. It turns out that this can never exceed  $c$ .

However, an important point is that if

$$\omega < \omega_p \quad \text{then} \quad n^2 < 0$$

This means that  $n$  is **imaginary** and as we saw, this will result in the wave being completely absorbed.

However in a metal, when the wave is absorbed, most of it tends to be **re-emitted** immediately. Thus when  $\omega < \omega_p$  we tend to see what we term **reflection**.

In general for materials we can define the **reflectivity**,  $R$ , as the fraction of the incident light intensity that is reflected. This is given by:

$$R = \frac{(n_{\text{Re}} - 1)^2 + n_{\text{Im}}^2}{(n_{\text{Re}} + 1)^2 + n_{\text{Im}}^2}$$

$$n^2 \approx 1 - \frac{\omega_p^2}{\omega^2}$$

For a metal with very little damping, as we saw when  $\omega < \omega_p$   
Then  $n^2 < 0$  and so  $n$  is purely imaginary and so has **no real part**.  
In this situation if  $\omega < \omega_p$  then  $R = 1$ .

However if  $\omega \gg \omega_p$  then  $\omega_p^2/\omega^2 \ll 1$   
Then the refractive index is real and  
 $n_{\text{Re}} \approx 1$  and  $n_{\text{Im}} \approx 0$  In this situation then,

If  $\omega \gg \omega_p$  then  $R \approx 0$

**This means that at low frequencies metals are very reflective, while for high frequencies they are very transparent.**

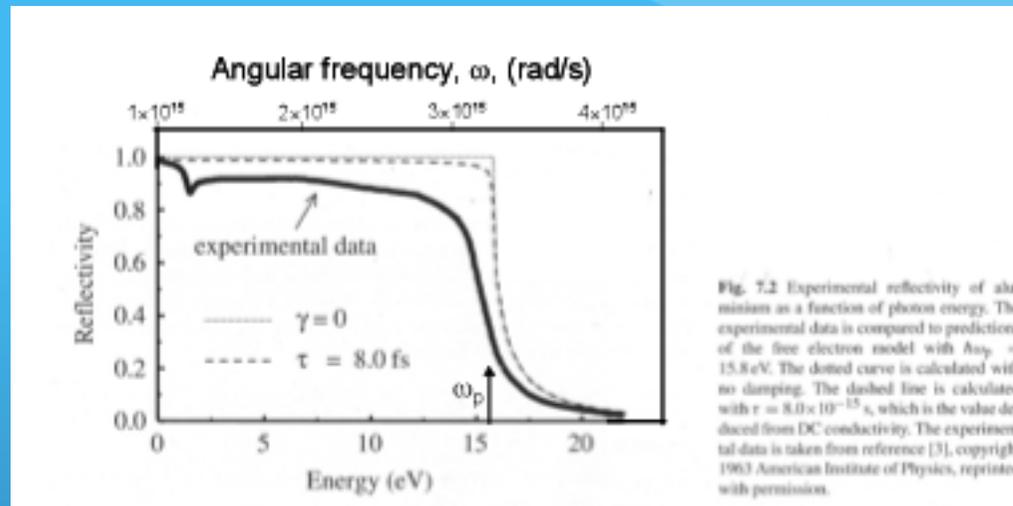


Fig. 7.2 Experimental reflectivity of aluminium as a function of photon energy. The experimental data is compared to predictions of the free electron model with  $\hbar\omega_p = 15.8$  eV. The dotted curve is calculated with no damping. The dashed line is calculated with  $\tau = 8.0 \times 10^{-15}$  s, which is the value deduced from DC conductivity. The experimental data is taken from reference [3], copyright 1963 American Institute of Physics, reprinted with permission.

This is used in communications.

The upper atmosphere, the **ionosphere**, is an **ionized plasma** with large numbers of **free electrons**.

This acts like a metal with very little damping.

The ionosphere is reflective for electromagnetic waves with frequencies below its plasma frequency.

Thus it can be used to bounce signals “around” the curvature of the Earth.

However if you want to send signals through the ionosphere, ie to a spacecraft, you need to use an electromagnetic wave with frequency higher than the plasma frequency.