Lecture 17.

Image formation
  Ray tracing
  Calculation

Lenses
  Convex
  Concave

Mirrors
  Convex
  Concave

Optical instruments
Laws of refraction and reflection can be used to explain how lenses and mirrors operate.

**Convex (converging) lens**

Parallel rays (e.g. from the Sun) passing through a convex lens converge at a focal point F. Sunlight focused by a magnifying glass may burn a hole in paper placed at focal point F.

- **F**: focal point
- **f**: focal length
A farsighted person requires an eyeglass of strength 2.5 diopters. What is the focal length of the eyeglass lens?

\[ D = 2.5 \]

\[ f = \frac{1}{D} = \frac{1}{2.5} = 0.4 \text{ m} = 40\text{ cm}. \]

F is the focal point.
f is the focal length, an important characteristic.

**Power or strength of the lens**

\[
\text{Strength in diopters (D)} = \frac{1}{f} \quad (f \text{ is in metres})
\]

Example

A farsighted person requires an eyeglass of strength 2.5 diopters. What is the focal length of the eyeglass lens?

\[ D = 2.5 \]

\[ f = \frac{1}{D} = \frac{1}{2.5} = 0.4 \text{ m} = 40\text{ cm}. \]

Focal length \( f = 40 \text{ cm} \).
Ray 1 entering lens parallel to optic axis will exit and pass through the focal point.

Ray 2 passing through the focal point will exit the lens and travel parallel to optic axis.

Ray 3 will undergo only a small deviation (not shown) (thin lens).

Real inverted image formed.
Real image (may be projected and displayed on a screen)

Rays reversible.
Image Formation (ray tracing)

Convex lens

Object at a distance less than the focal length from the lens

Simple magnifier

Image

- virtual
- upright
- magnified.
Rays entering lens parallel to axis appear to originate at focal point

Dashed lines indicate the direction from which the rays appear to come
Concave (diverging) lens

Object **outside** the focal length

Virtual image always produced by **concave lens**

Cannot be viewed on screen since rays are diverging on the right of the lens

However can be viewed with the eye since the eye converges the rays onto the retina.
Ray diagrams are useful in sketching the relationship between object and image. Relationship may also be calculated.

Triangles AOB and DOC are similar:
\[
\frac{h}{s} = -\frac{h'}{s'}
\]
\[
\frac{h'}{h} = -\frac{s'}{s}
\]

Triangles EFO and DCF are similar:
\[
\frac{h}{f} = -\frac{h'}{s' - f}
\]
\[
\frac{h'}{h} = -\frac{s' - f}{f}
\]
\[
-\frac{s'}{s} = -\frac{s' - f}{f}
\]
\[
\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}
\]
**Thin lens formula**

\[
\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}
\]

- **Object distance** \(s\)
  - positive if object is in front of lens
  - negative if object is behind lens

- **Image distance** \(s'\)
  - positive if image is formed behind the lens (real)
  - negative if is formed in front of the lens (virtual)

**Focal length** \(f\)
- positive -- convex lens
- negative -- concave lens
Magnification is defined as

\[ M = -\frac{s'}{s} \quad \text{or} \quad M = \frac{h'}{h} \]

M Negative:
- inverted image

M Positive:
- upright image
Object placed inside focal length of converging lens;

image viewed
- virtual,
- magnified
- upright
Example

An object 0.5 cm in height is placed 8 cm from a convex lens of focal length 10 cm. Determine the (a) position, (b) magnification, (c) orientation and (d) height of the image.

\[
\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \quad \frac{1}{s'} = \frac{1}{f} - \frac{1}{s}
\]

\[
\frac{1}{s'} = \frac{1}{10} - \frac{1}{8}
\]

\[
s' = \frac{10 \times 8}{8-10} = -40 \text{cm.}
\]

\[
M = -\frac{s'}{s} = -\frac{-40 \text{cm}}{8 \text{cm}} = 5
\]

\[
h' = M \times h = 5 \times 0.5 \text{cm} = 2.5 \text{cm}
\]

(a) (b) (c) M +ve image upright

(d)
Example
An object is placed 45 cm from a lens of focal length -25 cm. Determine the position, magnification, and orientation of the image.

\[ \frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \]

\[ \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \]

\[ \frac{1}{S'} = \frac{1}{-25 \text{ cm}} - \frac{1}{45 \text{ cm}} \]

\[ S' = -16.1 \text{ cm} \]

\[ M = -\frac{S'}{S} \]

\[ M = -\frac{-16.1 \text{ cm}}{45 \text{ cm}} = 0.36 \]

M +ve image upright
Combining Lenses

Effective focal length \( f_{\text{eff}} \) of combination of a number of thin lenses close together

\[
\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots
\]

Effective strength \( S_{\text{eff}} \) of combination of a number of thin lenses close together

\[
S_{\text{eff}} = S_1 + S_2 + \ldots
\]

Determine the combined strength of a thin convex lens and a thin concave lens placed close together if their respective focal lengths are 10cm and -20cm.

Strength \( S \), in diopters (D) = \( \frac{1}{f} \) (\( f \) is in metres)

\[
\frac{1}{f_{\text{eff}}} = S_{\text{eff}} = \frac{1}{f_1} + \frac{1}{f_2} \quad S_{\text{eff}} = \frac{1}{1\text{m}} - \frac{1}{2\text{m}}
\]

\[
S_{\text{eff}} = 10 - 5 = 5\text{diopters}
\]
Curved mirrors are analogous to lenses. Ray tracing and thin lens equation also valid. Real and virtual images are also formed.

*Flat Mirror*

Object and image distance equal
Object and image same size
Image, upright, virtual
Spherical Mirrors

Hollow sphere

Spherical mirror is a section of hollow sphere

Radius of curvature $R = 2f$
Curved Mirrors (Spherical)

**Concave mirror** (converging)
- Positive focal length

**Convex mirror** (diverging)
- Negative focal length

Thin lens formula may be used to determine object and image distances and focal lengths etc.

**Lenses and mirrors**

**Real image**: inverted (h’ negative), positive image distance s’

**Virtual image**: upright (h’ positive), negative image distance s’
Mirrors

Concave shaving/makeup mirrors

C is the centre of curvature

Object placed at distance < f from mirror

Image is virtual, upright and enlarged.

Application: searchlight
Example
An object is positioned 5 cm in front of a concave mirror of focal length 10 cm. Determine the location of its image and its characteristics.

\[ \frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \]

\[ \frac{1}{s'} = \frac{1}{10cm} - \frac{1}{5cm} \]

\[ \frac{1}{s'} = \frac{1}{10cm} - \frac{2}{10cm} = -\frac{1}{10cm} \]

\[ s' = -10cm \]

Characteristics.
• Image virtual
• Located behind mirror

\[ M = -\frac{s'}{s} \]

\[ M = -\frac{-10cm}{5cm} \]

\[ M = 2 \]
Optical instruments

Microscopes, telescopes, cameras etc

System may have many optical elements (example, lenses and mirrors)

**Thin lens formula or ray tracing** may be used to analyse behaviour of such systems

**Simple compound microscope**
two convex lenses

![Diagram of a simple compound microscope with labeled parts: Objective lens, Object (height h), F₀, Fₑ, h', eyepiece, Fₑ, Final image, h'', image formed by objective lens is inside focal length of eyepiece lens.]
Optical instruments

Dental loupes

Multiple lenses

Important Characteristics

• Resolution*
• Field width *
• Field depth
• Magnification

Resolution
• Ability to see fine detail

Field width
Size of operating site when viewed through loupe
Function of lens system diameter and magnification

Field depth
Depth or range of focus
Depends on, available light, optical design, magnification, and accommodation

Magnification,
important but not at the expense of resolution. Large fuzzy image of little use.
Dental Loupes

Galilean Design

Typically Magnification

\[ m \approx 2.5 \rightarrow 4.5 \]

Typical working distance

28-38 cm

Optical design allows observer focus at infinity thereby relieving eyestrain
Simple refracting telescope

Objective lens forms image (real, inverted) at focal point $F_0$ which is also the focal point $F_e$ of the eyepiece; virtual image is then formed at infinity.

Virtual image at infinity, Magnified and inverted

\[ M = - \frac{f_0}{f_e} \]
Example

Effective focal length of the objective in the Hubble telescope is 57.8 m. What focal length eyepiece is required to give a magnification of \(-8.0 \times 10^3\).

\[
M = -\frac{f_0}{f_e} \quad f_e = -Mf_0 = -(-8.0 \times 10^3) \times 57.8 \text{ m} \\
= 7.23 \times 10^{-3} \text{ m}
\]
Question 2.

An object of height 3 cm is positioned 40 cm from a concave lens with a focal length of -20 cm. Determine the position of the image, its magnification, height, and orientation.

\[
\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \quad \frac{1}{s'} = \frac{1}{f} - \frac{1}{s}
\]

S = 40 cm (object in front of lens)

f = -20 cm (concave lens)

\[
\frac{1}{s'} = \frac{1}{-20} - \frac{1}{40} = -\frac{3}{40} \quad S' = -13.33 \text{ cm}
\]

\[
M = -\frac{s'}{s} \quad M = -\frac{-13.33}{40} = \frac{1}{3} \quad \text{(Image upright)}
\]

\[
M = \frac{h'}{h}
\]

\[
h' = Mh = \frac{1}{3} \times 3.0 \text{ cm} = 1.0 \text{ cm}
\]
Question 2.

An object of height 3 cm is positioned 40 cm from a concave lens with a focal length of -20 cm. Determine the position of the image, its magnification, height, and orientation.

Virtual image always produced by a concave lens
Lens translated to change image distance $S'$ to adjust for different object distances $S$. **Focal length of lens is fixed.**

Real, inverted image formed on CCD array

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$$
Endoscope

Endoscope for medical investigations—inserted through small incision or orifice to inspect and facilitate operation on interior parts of the body.

Flexible shaft includes:
- Light source to illuminate area,
- Image channel to view area under investigation,
- Air or water conduit to clear debris,
- Instrument conduit

Typical endoscope

- Eyepiece
- Instrument entry
- Flexible shaft
- Transmits light, air, water
Optical instruments

Optical fibre: Total Internal Reflection

Applications

Image transport, Coherent fibre bundle

Optical communications:

Optical fibres used to transmit modulated laser beams; carrying information

Telephone and internet communications
Rate at which information can be transported proportional to frequency of light

Single fibre: many millions of phone conversations simultaneously.
Cable has many fibres