Buoyancy
Archimedes Principle
Fluid flow
Viscosity
Buoyancy and Archimedes Principle

One’s body, arms and legs etc, feel lighter under water. Easy to lift someone if they are in a swimming pool.

**Archimedes principle:**
Any object completely or partially immersed in a fluid experiences an upward or buoyant force equal to the weight of the fluid it displaces.

**Archimedes (287 BC – 212 BC)**
Greek Physicist, Mathematician

Buoyant force easily explained

Pressure is greater at greater depth: $P = \rho gh$

therefore the upward force is greater at the bottom of an object than the downward force at the top of the object

$F_2 > F_1$

Net upward force is the **buoyant force** is equal to the weight of the displaced fluid.
Buoyancy and Archimedes Principle

Cube of same fluid

Buoyant force $F_b$

$F_b = P_2 A_2 - P_1 A_1$

Cube remains stationary

$F_b = \text{weight of fluid}
= \text{weight of fluid}
displaced$

$P_1 = \rho g h_1$

$P_2 = \rho g h_2$

$F_b = P_2 A_2 - P_1 A_1 = m_f g$

where $m_f = \text{mass of fluid}$

Cube of different material (only mass of cube changes) buoyant force remains unchanged ($F_b = m_f g$)

$F_b = m_f g = \rho_{\text{fluid}} V_{\text{fluid}} g$

- If an object floats then the buoyant force must equal its weight.
- If an object sinks then its weight must be greater than the buoyant force.
Buoyancy and Archimedes Principle

**Apparent weight** = net downward force

\[ \text{Apparent weight} = \text{net downward force} = w_{\text{object}} - F_b \]

\[ F_b = \text{weight} - \text{apparent weight} \]

\[ F_b = \text{weight of liquid displaced} \]
Buoyancy and Archimedes Principle

Calculate the volume and density of an irregular shaped object.

**Example**

A person has a mass of 75kg in air and an apparent mass of 2kg when submerged in water. Calculate the volume and density of the person.

\[ F_b = \text{weight} - \text{apparent weight} \]

mass of water displaced = Mass – apparent mass

mass of water displaced = 75kg – 2kg = 73kg

Volume of water displaced = \( \frac{m_{\text{water}}}{\rho_{\text{water}}} \)

= \( \frac{73\text{kg}}{1000\text{kgm}^{-3}} \) = 73\( \times 10^{-3} \)m\(^3\)

Therefore volume of person = 73\( \times 10^{-3} \)m\(^3\)

\[ \rho_{\text{person}} = \frac{m_{\text{person}}}{\text{volume}_{\text{person}}} \]

\[ \rho_{\text{person}} = \frac{75\text{kg}}{73\times10^{-3}\text{m}^3} = 1027\text{kgm}^{-3} \]
Viscosity and Fluid flow

One characteristic of fluids is that they flow

Fluid flow in tubes:
examples:
• IV tubes,
• garden hoses,
• circulatory system

Flow rate Q defined as volume flowing per unit time (V/t)

\[ Q = \frac{V}{t} \]

SI units: \( m^3 \) per second

Flow rate depends on
• pressure difference
• other characteristics of the fluid and tube
Viscosity and Fluid flow

Flow rate depends on pressure difference

\[ P_1 = P_2 \text{ no flow} \]

\[ P_1 > P_2 \text{ flow direction} \rightarrow \]

\[ P_1 < P_2 \text{ flow direction} \leftarrow \]

Flow rate \[ Q = \frac{P_1 - P_2}{R} \]

where \( R \) is the resistance to flow.

Resistance is all factors that impair flow; example,
- friction between fluid and tube,
- friction within the fluid*

* known viscosity \( \eta \) (Greek letter eta).

SI unit of viscosity \( \text{N.s.m}^{-2} \)
Flow can be characterised as
- laminar
- turbulent

**Turbulent flow**

Fluctuating flow patterns

Caused by constrictions or obstructions

Turbulence causes increased resistance to flow
Laminar flow

Viscosity and Fluid flow

Smooth, streamlined, quiet

Resistance to laminar flow of an incompressible fluid in a tube is a function of
- tube length \((L)\),
- radius \((r)\)
- viscosity \(\eta\)

\[
R = \frac{8\eta L}{\pi r^4}
\]

\[R \propto \frac{1}{r^4}\]

double radius; resistance decreases by a factor of 16
Viscosity and Fluid flow

\[ R = \frac{8\eta L}{\pi r^4} \]

Flow rate \( Q = \frac{P_1 - P_2}{R} \)

\[ Q = \left( \frac{\pi r^4}{8\eta L} \right) (P_1 - P_2) \]

Poiseuille’s Law
French scientist Jean Poiseuille (1797-1869) studied fluid flow in tubes; in particular blood flow
Viscosity and Fluid flow

Fluid flow

Teeth

Sensitivity to cold, heat, air,
Concerned with fluid flow
within the tooth

**Fluid flow**

dentinal tubules (microscopic channels) radiate outward through the dentine from the pulp to the dentine-enamel interface.

If outer enamel develops a crack or cavity when you eat cold food, for example normal fluid flow within the dentine may be disrupted – affecting the pulp (nerves, blood vessels etc), resulting in pain.
Viscosity and Fluid flow

To change flow rate; change radius of tube

Examples

• Blood flow rate in circulatory system changed by **constricting or dilating** blood vessels
• Clamp on IV tubing
• Tap on garden hose

Flow rate \( Q \)

\[
Q = \left( \frac{\pi r^4}{8\eta L} \right) (P_1 - P_2)
\]

\( Q \propto r^4 \)

If effective radius of a vein or artery is reduced by a constriction (deposits)
• blood circulation problems occur.

Result: heart has to work considerably harder to produce a higher blood pressure in order to maintain the required flow rate.
Viscosity

Materials

Resistance to fluid flow

Dentistry

Restorative materials are manipulated in fluid state to achieve desired result.

Viscosity of cement should be low so that it will flow over tooth surface to achieve good retention.

Materials

- Prepared as fluid pastes
- adapted to the required shape
- subsequently solidify

Setting of such materials
Change of viscosity with time
Viscosity

Restorative Dental Material

Initial low viscosity for dispensing and moulding. Followed by large increase in viscosity during setting

Working time – time during which the material can be easily manipulated (low viscosity)

Setting time – time at which viscosity becomes very high

Change of material viscosity with time

- No well defined working time or setting time
- Well defined long working time and sudden setting time
- Long working time reasonable setting time
Example

If effective radius of the artery is halved, by what factor does the blood flow rate change?

\[
Q_0 = \left( \frac{\pi r^4}{8\eta L} \right) (P_1 - P_2)
\]

\[
Q = \left( \frac{\pi \left( \frac{r}{2} \right)^4}{8\eta L} \right) (P_1 - P_2)
\]

\[
\frac{Q}{Q_0} = \frac{1}{2^4} = \frac{1}{16}
\]

\[
Q = \frac{Q_0}{16}
\]

Factor of 16
**Example**

By what percentage would the radius of the arteries have to decrease to reduce the blood flow rate by a factor of 3.

\[
Q_0 = \left( \frac{\pi r^4}{8\eta L} \right) (P_1 - P_2)
\]

\[
\frac{Q_0}{3} = \left( \frac{\pi r_n^4}{8\eta L} \right) (P_1 - P_2)
\]

Dividing the two equations

\[
\frac{Q_0}{3} = \left( \frac{\pi r_n^4}{8\eta L} \right) (P_1 - P_2)
\]

\[
1 = \frac{r_n^4}{3}
\]

\[
\sqrt[4]{\frac{1}{3}} = \frac{r_n^4}{r^4}
\]

\[
\sqrt[4]{3} = \frac{r_n}{r} = 0.76
\]

\[
r_n = 0.76r
\]

Reduction of \((1 - 0.76)\times100\% = 24\%\)
Viscosity and Fluid flow

Example

Patient receiving blood transfusion through a needle of length 2.3cm and radius 0.2mm. The reservoir supplying the blood is 0.7m above the patient’s arm. Determine the flow rate through the needle. Density of blood is $1050\text{kgm}^{-3}$. Viscosity of blood is $2.7\times10^{-3}\text{N.s.m}^{-2}$.

Pressure difference $P_1 - P_2 = \rho gh$

$P_1 - P_2 = (1050\text{kgm}^{-3})(9.8\text{ms}^{-2})(0.7\text{m})$

$= 7.20 \times 10^3\text{Pa}$.

$Q = \left( \frac{\pi r^4}{8\eta L} \right) (P_1 - P_2)$

$Q = \left( \frac{\pi (2\times10^{-4} m)^4}{8(2.7\times10^{-3}\text{Nsm}^{-2}) 2.3\times10^{-2} m} \right) (7.20 \times 10^3 \text{ Pa})$

$Q = 7.28 \times 10^{-8}\text{m}^3\text{s}^{-1}$
Viscosity and Fluid flow

Flow rate and pressure drop

$$Q = \frac{P_1 - P_2}{R}$$

$$P_1 - P_2 = QR$$

Water main

Water supply to homes

If $P_1 = P_2$ no flow

If many users draw water simultaneously

$\rightarrow$ large flow $\rightarrow$ large pressure drop

Hence $P_2$ is smaller than when usage is light.

Solutions: increase $P_1$ and/or decrease $R$ (increase diameter of water main)

Both occur during vigorous exercise in the circulatory system

Blood pressure increases and arteries dilate
Buoyancy and Archimedes Principle

Objects that float

Object’s weight is equal to the buoyant force $F_b$

$$w_{\text{object}} = F_b$$

Applying Archimedes principle

$F_b$ is equal to weight of fluid displaced

therefore

$$w_{\text{object}} = w_{\text{fluid \ (displaced)}}$$

True only if object floats

$$F_b = m_f g = \rho_{\text{fluid}} V_{\text{fluid}} g \quad F_b = m_f g = \rho_{\text{object}} V_{\text{object}} g$$

$$\rho_{\text{fluid}} V_{\text{fluid}} g = \rho_{\text{object}} V_{\text{object}} g$$

$$\frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{V_{\text{fluid}}}{V_{\text{object}}}$$

volume of object submerged = volume of fluid displaced

"The fraction of the object that is submerged is equal to the ratio of the density of the object to the density of the fluid."
Buoyancy and Archimedes Principle

Applications

Why do ships float
Average density is less than that of water
Ship partially submerges until weight of ship equals weight of water displaced.

Hot air (or Helium) balloons
Hot air less dense than cold air resulting in a net upward force on the balloons

Human brain is immersed in cerebrospinal fluid, density $\approx 1007 \text{ kgm}^{-3}$

average density of brain $\approx 1040 \text{ kgm}^{-3}$

Most of brain’s weight is supported by the buoyant force
Buoyancy and Archimedes Principle

Objects that sink (totally submerged)

Applying Archimedes principle
\[ F_b = m_{\text{fluid}}g = (V_{\text{fluid}})(\rho_{\text{fluid}})g \]

But volume of fluid displaced = volume of object

therefore
\[ F_b = (V_{\text{object}})(\rho_{\text{fluid}})g \]

Downward gravitational force on object

\[ m_{\text{object}}g = (\rho_{\text{object}})(V_{\text{object}})g \]

Net force upwards on object = \( F_b - w_{\text{object}} \)

\[ = (V_{\text{object}})(\rho_{\text{fluid}})g - (\rho_{\text{object}})(V_{\text{object}})g \]

Net force upwards = \( (\rho_{\text{fluid}} - \rho_{\text{object}})(V_{\text{object}})g \)

If \( \rho_{\text{fluid}} > \rho_{\text{object}} \) object will float
If \( \rho_{\text{fluid}} < \rho_{\text{object}} \) object will sink
Buoyancy and Archimedes Principle

Calculate the fraction of an iceberg’s volume that is submerged when it floats in water. Density of ice $917 \text{kgm}^{-3}$

$$ V_{\text{water}} = \frac{\rho_{\text{iceberg}}}{\rho_{\text{water}}} \frac{m_{\text{water}}}{m_{\text{iceberg}}} = \frac{\rho_{\text{iceberg}}}{\rho_{\text{water}}} $$

$$ \frac{\rho_{\text{iceberg}}}{\rho_{\text{water}}} = \frac{V_{\text{water}}}{V_{\text{iceberg}}} = \frac{917 \text{kgm}^{-3}}{1000 \text{kgm}^{-3}} = 0.917 $$

$$ F_b = \rho_{\text{fluid}} V_{\text{fluid}} g $$

Weight of object (w) = $\rho_{\text{object}} V_{\text{object}} g$

$$ w = F_b $$

$$ w_{\text{iceberg}} = w_{\text{water}} $$

$$ m_{\text{iceberg}} = m_{\text{water}} $$

$V_{\text{water}} = \text{volume of water displaced}$
Buoyancy and Archimedes Principle

Example
Determine the density of a liquid if an object of known volume (50cm$^3$) and mass (0.150kg) has an apparent mass of 0.105kg in the liquid.

Applying Archimedes principle
Buoyant force is equal to weight of liquid displaced

Weight – apparent weight = weight of liquid displaced

Mass – apparent mass = mass of liquid displaced

\[ 0.150kg - 0.105kg = 0.045kg \text{ is displaced by the object} \]

Volume of object = 50cm$^3$ = 50x10$^{-6}$m$^3$ = Volume of liquid displaced

Therefore density of liquid =

\[ \frac{0.045kg}{50x10^{-6}m^3} \]

= 900kgm$^{-3}$
A 70 kg statue lies at the bottom of the sea (Density 1.025x10^3 kg m\(^{-3}\)). Its volume is 3.0x10^4 cm\(^3\). How much force do you need to lift it?

**Weight - Apparent weight**

= weight of fluid displaced

\[ mg - W_{app} = \text{weight of fluid displaced} \]

\[ W_{app} = mg - m_{\text{fluid}} g \]

\[ W_{app} = mg - \rho_{\text{fluid}} Vg \]

\[ W_{app} = g (m - \rho_{\text{fluid}} V) \]

\[ W_{app} = 9.8 \text{ms}^{-2} (70\text{kg} - 1.025 \times 10^3 \text{ kg m}^{-3} \ 3.0 \times 10^{-2} \text{m}^3) \]

\[ W_{app} = 9.8 \text{ms}^{-2} (70\text{kg} - 30.75\text{kg}) \]

384.65N
Buoyancy and Archimedes Principle

You immerse an object in water and measure the apparent weight. You then repeat the process in salt solution (density slightly higher than water) would you expect the new apparent weight to be

a) Higher
b) Same
c) Lower

Applying Archimedes principle

\( F_b \) is equal to weight of fluid displaced

\( F_b \) is equal to \( mg \) of fluid displaced

\( F_b \) is equal to \( \rho Vg \) of fluid displaced
A small car ferry measures 4.00 metres wide by 6.00 metres long. When several cars of total weight $9.41 \times 10^3 \text{N}$ drive on to it, it sinks an additional depth $(d)$ into the water. Calculate the additional depth. Density of water $= 1000 \text{kgm}^{-3}$

It floats, therefore:

object’s weight is equal to the buoyant force $F_b$

$$W_{\text{object}} = F_b$$

Applying Archimedes principle

$F_b$ is equal to weight of fluid displaced

$$W_{\text{object}} = W_{\text{fluid}}$$

Additional volume $(V)$ of ferry below water line

$$V = 4 \text{m} \times 6 \text{m} \times d$$

Additional mass of ferry $= \text{density} \times \text{Volume}$

Additional weight of ferry $= \text{density} \times \text{Volume} \times g$

$$4 \text{m} \times 6 \text{m} \times d \times (1000 \text{kgm}^{-3}) \times (9.8 \text{ms}^{-2}) = 9.41 \text{kN}$$

$$d = 4 \text{cm}$$