The Bubble-Verse

For Karen Uhlenbeck, the first woman to win the Abel Prize for math, a backyard phenomenon offers a window onto higher dimensions.

By SIODHAN ROBERTS

PRINCETON, N.J. — On the evening of March 19, the mathematician Karen Uhlenbeck gathered with revelers at the Institute for Advanced Study for a champagne reception. Some hours earlier, she’d been awarded the Abel Prize — the first time a woman had won it — for her discovery of a phenomenon called “bubbling,” among other achievements.

Dr. Uhlenbeck is a professor emerita at the University of Texas at Austin, where she spent the better part of her career (having declined a professorship at Harvard). She retired in 2014 and moved to Princeton. At the institute, she keeps a desk piled with boxes of books. She describes herself as a messy reader, and a messy thinker, and she is stylishly disheveled, with a preference for comfy, colorful clothing with pockets and Birkenstocks with socks.

As a procession of speeches and toasts landed her the prize, Dr. Uhlenbeck stood to the side of the lectern and listened. When it finally came time to make her own remarks (unprepared), she began by simply agreeing: “From the perspective of my late 70s, I find myself as a young mathematician sort of impressive, too.”

She went on to note that, for lack of mathematical candidates, her role model had been the chef Julia Child. “She knew how to pick the turkey up off the floor and serve it,” Dr. Uhlenbeck said.

Jo Nelson, a mathematician at Rice University and a friend of Dr. Uhlenbeck, was thrilled that her visit to the institute coincided with a celebration for one of her mentors. “It’s amazing to hear a woman’s mathematical achievements celebrated and discussed in such detail,” she said.

Even Robert MacPherson, a topologist...
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and faculty member in mathematics at the institute, made a rare social appearance. "This is wonderful in so many ways," he said, holding a mini cosmos of bristly bubbles.

A decade ago, Dr. MacPherson and a collaborator formulated an equation describing how, in three and higher dimensions, infinitely many cellular and quantum objects, such as light sheets and soap films, self-assemble into a structure that resembles a cellular automaton, a simple mathematical model that is used to simulate the growth of tissues and the development of biological systems.

Researchers of all stripes have written "many thousands of papers" on bubbles, Andrea Prosperetti, a mechanical engineer at the University of Houston, has estimated. Bubbles resist for their seeming simplicity, which approaches the existential.

"Bubbles are cornerstones of science," non-liquid, a tiny cloud shedding a mathematical singularity," he wrote. "They differ from chaos, a more

and life itself ending in the union with the

And bubbles are everywhere, on every scale, once you start looking: high-tech drug delivery mechanisms, smoke rings, ad

dressings, soapsuds, black holes and be-

yond. In architecture, the Beijing National Aquatics Center is a box of bubbles. It is an application of the Weaire-Phelan foam, the most efficiently packed foam of equal-volume polyhedral bubbles, discovered in 1994 by the Irish physicist Denis Weaire and his student Robert Phelan (first using a computer simulation, then created in a lab in 2012).

Dr. Uhlenbeck's contribution is less prac-
tical. The Nobel Prize cited "her pioneering achievements in geometric partial differential equations, gauge theory and integrable systems, and for the fundamental impact of her work in quantum field theory and数学物理 systems. The Wilhelmian name for her work—"bubbling"—besides its prickly undertone, "stands for everything else.

"It's much more abstract and theoretical, and about the bubble's role in the

The experiment successfully reproduced "scout configurations in a dry foam sand-

wich" (with a computer simulation to match, embodying only the surface tension forces in a box).

Theorists also contemplate bubbles at the larger end of the scale spectrum. At the University of Cambridge, Adriana Pesci and other applied mathematicians study soap films that form on the frame of a Möbius strip. "The interesting thing about this investigation is that it all started with some bubble films on a table," Dr. Uhlenbeck said.

Astrophysicists have long postulated, on a cosmic scale, that our universe could be a soapbubble structure. Minimal surfaces are important in many areas of physics, and the dynamics driven by a "soap-bubble law.

The Bubble of Inflation

Over the 1980-1990 academic year, re-

searchers at the Institute for Advanced Study for one of its annual thematic deep drives—a "Spe-
cial Year in Geometric Methods in Geometric Meth-
ods" devoted to minimal surfaces and relat-
ed tributaries. It's the world's biggest bub-
ble around here lately," said Helmut Ho-

theorists. Dr. Uhlenbeck's contributions were cited often in the seminars and workshops.

Dr. Uhlenbeck's review of the book in Nature, the Scottish physicist David MacKay Maxwell lamented, "Can the poetry of bubbles survive this?"

A soap bubble is the physical world's solution for a mathematical challenge: to minim-

ize, a surface area—in this case, one that is curtailed by a prescribed volume of air. Na-

ture is always seeking to optimize, to maxi-

mize against the cost of doing so. So "minimal surfaces" problems are all around, even in higher dimensions, and all describe the governing rules.

He said a founder of a field called geometric analysis, Dr. Uhlenbeck approaches min-

imal surfaces essentially, under the banner of "variational methods in geometry."

"I'll give you a problem," he said. "Take a

fixed length of string, lie it down on the

plane—as a table—and enclose the

largest area you can inside. I wobble the

string a little bit, it's only a little, to see

whether the area increases or decreases.

The optimal answer is no square but a cir-

cle. You might try a square, she said: "But

by shrinking and smoothing the cor-

ners, you can enclose more area. That's a

variations process."

In a classic paper from 1976, Cyril Iken-

berg declared the soap film "an analog computer." He used wire frames of poly-

dra, such as a tetrahedron or cube, and
dipped them in a soap solution in order to

compute—faster than a mathematician
certainly—the solution to those particular three-dimensional minimization problems.

This method was popularized in the 1990s
by Richard Courant, the founding director of
the Courant Institute of Mathematical Sciences at New York University. Recently, Courant researchers in the applied mathe-
matics lab conducted a study that con-

cluded that "there is more than one way
to blow a bubble."

Their experiment involved blowing large

bubbles of oil/gas (Bottella) in a water

tank, creating what investigators called "a

mathematical crisis." This led to a "neces-

sary formula" describing the critical flow speed needed to blow and pinch off a bubble.

But they also discovered another way to

blow a bubble: impose a gentle flow, below the critical speed, on a film that is already

somehow inflated.

"We found this second result surprising."

Previously, the best result dated to 1981
and showed that in any three- or higher-di-

mensional space, there is always at least one minimal surface. "We were sure there were more," Dr. Marques said.

There are, according to the two new

proofs by Antoine Song, a doctoral stu-

dent of Dr. Marques, rounded out a series of results simply blown away by the two

expected revelations: "I discovered the

fascinating fact that the problem of scale

survives in bubble and in mathematics and in
drawing."

In physics, she noted, quantum theory

deals with the very small, whereas relativity

deals with the very large, and physi-
cians don't see how the two

Dr. Uhlenbeck's bubble-drawing displayed a

similar challenge: She observed intricate,

funny-looking bubbles and wondered how she

could use those tools to investigate regions of interest at a larger, more accessible scale.

"It's working as an artist," she said, as if under a magnifying glass, and then you can see the details.

With this approach she also enabled other
thorists to tackle some messy, tur-

ey-on-the-floor situations.

"I've had several runs with soap-bubbling," said Dr. Uhlenbeck, a former director of the

Institute's school of natural sciences, bouncing around with the kids before her speech at the Abel Prize ceremony.

He tried to explain how the "bubbling of

inflations has various important applica-
tions and implications in both mathematics

and physics. An inflationary universe is an

inflationary universe that is not a

inflationary universe. It was the key mystery to under-

standing the inflationary universe."

After blowing, Dr. Uhlenbeck moved on
to other mathematical mysteries for a few

minutes, including the so-called "monotonic return to minimal surfaces. Every Friday, her collaborator Penny Smith would

Lehigh University to talk math. After the

Abel, she, the second Dr. Uhlenbeck, ex-
hustated, took two Fridays off. But when

their sessions resume, they'll dive deep into

some even more high-dimensional and

bubbling.

Here, Dr. Uhlenbeck said, "It gets me more.

inventor of the theory.

inventor of the theory.

inventor of the theory.

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