MATHEMATICS STACKS ORANGES

Written by David Larousserie in Le Monde (19th May 2012)
Translated by Arthur Marguerite and Paco Maurer (25th May 2012)

In science, one sometimes needs to return to his or her own happy moments of childhood to progress. Thus, two independent research groups played a little game of stacking oranges, Christmas balls or ball bearings to study the properties of those systems.

Any fruit or vegetable seller could have explained to them the best way to stack oranges is to make little pyramids by putting 3 spheres in contact with each other to form a triangular stack and then putting a fourth in the middle, followed by a repetition of this procedure.

Okay. But why, in many experiments, this structure is favoured over some others which are as compact as this one ? And what happens if we try to put as many spheres as possible into a cylinder of a particular diameter? Here are the two questions that have been answered respectively by a French team from the *Laboratoire de Physique des Solides* of Orsay in the journal *Physical Review letters* on the 6th of April and by an Irish team from Trinity College Dublin in the journal *Physical Review E* on the 11th of May.

Various applications

Of course these researchers are looking for something more than the creation of beautiful fruit piles in a fruit store; for instance, they are looking for the realisation of textile fibre in foams for a better thermal insulation, or the creation of porous materials whose spherical cavities would have the proper repartition to eliminate acoustic waves.

"Everybody noticed that, in the experiments, the spherical objects under study, often soap bubbles in foams, tend to arrange themselves according to the classical structure of the orange seller’s piles. But nobody could understand why", recalled Wiebke Drenckhan of CNRS, who was in charge of the first study.

Indeed, there are two main possible ways that enable us to put the same number of oranges into a finite volume. The market gardener's one is called face centred cubic (or FCC). The other one is called hexagonal close packed (or HCP); Its base is composed of seven spheres. Six of them form an hexagon and the seventh one is put in the centre. Upper balls are put in the 3 holes formed by the hexagon, and so on.

In the seventeenth century, Johannes Kepler calculated that these structures are the densest possible. They contain only 26% of vaccum.

But it only happened at the end of the 90's that the mathematician Thomas Hales came to prove this conjecture. However he needed a computer to achieve his proof. It turned out that this problem of packing oranges isn't that trivial.

Wiebke Drenckhan remembers: "We have explored a lot of different hypotheses, for example an attribution to the flow of fluid between bubbles in foams. Without success". Her student Sascha Heitkam from Orsay University and Dresde Technic University (Germany) triggered the whole new idea. He started piling up ball bearings on his desk and then patting them. To their surprise, the FCC pyramids are steadier than the HCP pyramids. The problem wasn't about thermodynamics but about mechanics. The supervisor said: "We were almost ashamed to publish it!". Because of the repartition of different spheres on the edges of the pyramid, a force pushing from the top of the pyramid doesn't have the same consequences for the HCP than for the FCC. In the latter case, the force is only transmitted to the ground, whereas in the HCP case a part of the transmitted force has a tendency to push the sphere out of the edifice. Bang, collapses!

This had no chance to happen for their Irish colleagues because, here, the spheres are held inside a cylinder. The important issue was more about how to fill the cylinder's space as much as possible with various objects. Ho-Kei Chan, at the end of 2011, found the solution, and then improved it with his directors, Denis Weaire and Stefan Hutzler, in their recently published article. Not only does his theory explain how to put a sphere on the others, taking into account the positions of the previously filled ones, but it is also very efficient." It only takes 15 minutes, instead of one week that was necessary with one of the previous methods", recalls Ho-Kei Chan. Moreover, all the previous methods employed were not sequential, which means while they can tell one structure being denser than another, they cannot tell how to build those structures. Last but not least, the method opens a new field. "We call it columnar crystallography", added his director, Denis Weaire, author of *The Pursuit of Perfect Packing* (Taylor & Francis), a book for the general public on the secrets of packing, at the boundaries between art and science.

Besides known structures like the "pearl necklace" (spheres up on each other), the spiral or the double-helix, these physicists were discovering structures that people had never seen before. They have begun the exploration of cases where the spheres don't touch the cylindrical boundaries. Secretly, they hope it will help to understand why all molecular "natural helices" seem to curl up in only one orientation and also hope to control this orientation to influence the chemical and optical properties of such-molecules.

So, childish men or great architects ?"