Lecture 3 - Minimum mass model of solar nebula

- Topics to be covered:
  - Composition and condensation
  - Surface density profile
  - Minimum mass of solar nebula
Minimum Mass Solar Nebula (MMSN)

- MMSN is *not* a nebula, but a protoplanetary disc.

  Protoplanetary disk

  Nebula

- Gives minimum mass of solid material to build the 8 planets.
Minimum mass of the solar nebula

Can make approximation of minimum amount of solar nebula material that must have been present to form planets. Know:

1. Current masses, composition, location and radii of the planets.

2. Cosmic elemental abundances.

3. Condensation temperatures of material.

Given % of material that condenses, can calculate minimum mass of original nebula from which the planets formed.

Steps 1-8: metals & rock, steps 9-13: ices

Figure from Page 115 of “Physics & Chemistry of the Solar System” by Lewis
Nebula composition

- Assume solar/cosmic abundances:

<table>
<thead>
<tr>
<th>Representative elements</th>
<th>Main nebular Low-T material</th>
<th>Fraction of nebular mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>H, He</td>
<td>Gas H₂, He</td>
<td>98.4 %</td>
</tr>
<tr>
<td>C, N, O</td>
<td>Volatiles (ices) H₂O, CH₄, NH₃</td>
<td>1.2 %</td>
</tr>
<tr>
<td>Si, Mg, Fe</td>
<td>Refractories (metals, silicates)</td>
<td>0.3 %</td>
</tr>
</tbody>
</table>
Minimum mass for terrestrial planets

- **Mercury:** \( \sim 5.43 \, g \, cm^{-3} \) => complete condensation of Fe (\( \sim 0.285\% \, M_{\text{nebula}} \)).
  
  \[
  0.285\% \, M_{\text{nebula}} = 100 \% \, M_{\text{mercury}} \\
  \Rightarrow M_{\text{nebula}} = (100/0.285) \, M_{\text{mercury}} \\
  = 350 \, M_{\text{mercury}}
  \]

- **Venus:** \( \sim 5.24 \, g \, cm^{-3} \) => condensation from Fe and silicates (\( \sim 0.37\% \, M_{\text{nebula}} \)).
  
  \[
  \Rightarrow (100\% / 0.37\% ) \, M_{\text{venus}} = 270 \, M_{\text{venus}}
  \]

- **Earth/Mars:** 0.43\% of material condensed at cooler temperatures.
  
  \[
  \Rightarrow (100\% / 0.43\% ) \, M_{\text{earth}} = 235 \, M_{\text{earth}}
  \]

- **Asteroids:** Cooler temperatures produce more condensation \( \sim 0.5 \% \).
  
  \[
  \Rightarrow (100\% / 0.5\% ) = 200 \, M_{\text{asteroids}}
  \]
Minimum mass for terrestrial planets

- What is the minimum mass required to make the Terrestrial planets?

<table>
<thead>
<tr>
<th>Planet</th>
<th>Factor</th>
<th>Mass ($x10^{26}$ g)</th>
<th>Min Mass ($x10^{26}$ g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>350</td>
<td>3.3</td>
<td>1155</td>
</tr>
<tr>
<td>Venus</td>
<td>270</td>
<td>48.7</td>
<td>13149</td>
</tr>
<tr>
<td>Earth</td>
<td>235</td>
<td>59.8</td>
<td>14053</td>
</tr>
<tr>
<td>Mars</td>
<td>235</td>
<td>6.4</td>
<td>1504</td>
</tr>
<tr>
<td>Asteroids</td>
<td>200</td>
<td>0.1</td>
<td>20</td>
</tr>
</tbody>
</table>

- Total of the 4th column is $29881x10^{26}$ g. This is the minimum mass required to form the Terrestrial planets $=>2.9881x10^{30}$ g $\sim 500 \ M_{\text{earth}}$. 
Minimum mass for jovian planets and pluto

- **Jupiter:** Almost nebula composition due to gas capture ~20%.
  \[ M_{\text{nebula}} = 100 / 20 \times M_{\text{jupiter}} \sim 5 \times M_{\text{jupiter}} \] is minimum mass required.

- **Saturn:** Cooler than Jupiter, with slightly different composition ~12.5%.
  \[ M_{\text{nebula}} = 100 / 12.5 \times M_{\text{saturn}} \sim 8 \times M_{\text{saturn}} \]

- **Uranus:** Less gas capture ~6.7% condensed to form planet.
  \[ M_{\text{nebula}} = 100 / 6.7 \times M_{\text{uranus}} = 15 \times M_{\text{uranus}} \]

- **Neptune:** ~5% of solar nebula material condensed to form planet.
  \[ M_{\text{nebula}} = 100 / 5 \times M_{\text{neptune}} = 20 \times M_{\text{neptune}} \]

- **Pluto:** Main fraction due to ices ~1.4% \[ M_{\text{nebula}} = 100 / 0.14 \times M_{\text{pluto}} = 70 \times M_{\text{pluto}} \]
What is the minimum mass required to make the Jovian planets?

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass ($x10^{26}$ g)</th>
<th>Factor</th>
<th>Min Mass ($x10^{26}$ g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jupiter</td>
<td>19040</td>
<td>5</td>
<td>95200</td>
</tr>
<tr>
<td>Saturn</td>
<td>5695</td>
<td>8</td>
<td>55560</td>
</tr>
<tr>
<td>Uranus</td>
<td>890</td>
<td>15</td>
<td>13050</td>
</tr>
<tr>
<td>Neptune</td>
<td>1032</td>
<td>20</td>
<td>20640</td>
</tr>
</tbody>
</table>

Total mass is therefore $= 184450 \times 10^{26}$ g $= 3085 \, M_{\text{earth}}$.

This is minimum solar nebula mass required to make the Jovian planets.
**Minimum nebula mass**

- The minimum mass required to condense the nine planets is therefore:

<table>
<thead>
<tr>
<th>Planet</th>
<th>( M (x M_{earth}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terrestrial</td>
<td>500</td>
</tr>
<tr>
<td>Jovian</td>
<td>3085</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.119</td>
</tr>
</tbody>
</table>

  **3585 \( M_{earth} \)**

- This is the minimum mass required to produce the planets.

- As \( M_{sun} \sim 2 \times 10^{33} \ g \), the mass required to make the planets is therefore \( \sim 0.01 M_{sun} \).

- Disk contained 1/100 of the solar mass.
Nebular surface density profile

- To make a more precise estimate, distribute min mass requirements over series of annuli, centred on each planet.

- Choose boundaries of annuli to be halfway between the orbits of each planet. i.e., Mercury @ 0.38 AU and Venus @ 0.72 AU => (0.72-0.38)/2 = 0.17 AU.

- We therefore estimate that Mercury was formed from material within an annulus of 0.38±0.17 AU => 0.33 - 0.83 x 10^{13} cm.

- The surface density of an annulus, $\frac{\text{mass}}{\text{area}}$, where $\text{area} = \pi r_{\text{outer}}^2 - \pi r_{\text{inner}}^2$
  
  $\quad = \pi [(0.83 \times 10^{13})^2 - (0.33 \times 10^{13})^2]$
  
  $\quad = 1.82 \times 10^{26} \text{ cm}^2$

- Surface density of disk near Mercury is therefore:

  $1160 \times 10^{26} / 1.82 \times 10^{26} = 637 \text{ g cm}^{-2}$
Nebular surface density profile

- For Venus at 0.72 AU, Mercury is at 0.38 AU and Earth is at 1 AU => Venus’ annulus extends from
  \[(0.72 - 0.38)/2 = 0.17\] to \[(1 - 0.72)/2 = 0.14\]

- The material that formed Venus was located between
  0.72 - 0.17 AU and 0.72 + 0.14 or 0.55-0.86 AU. This is 0.83-1.29 x 10^{13} cm.

- Area is then \[\pi r_{outer}^2 - \pi r_{inner}^2 = 3.06 \times 10^{26} \text{ cm}^2\].
  \[\Rightarrow \pi = 13150 \times 10^{26} / 3.06 \times 10^{26} = 4300 \text{ g cm}^{-2}\].

- This is the approximate surface density of the disk where Venus formed.

- For Jupiter at 5.2 AU, the Asteroids are at 3 AU and Saturn is at 9.6 AU. The annulus therefore ranges from 4 - 7.2 or 6 - 11 x 10^{13} cm.

- As the area = 267 x 10^{13} cm^2 => \[\pi = 95200 \times 10^{26} / 267 \times 10^{26} = 356 \text{ g cm}^{-2}\]
## Minimum mass and density

**Table IV.8** Minimum Mass of the Primitive Solar Nebula

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass ((10^{26} \text{g}))</th>
<th>(F^a)</th>
<th>(M_{\text{solar}} \ (10^{26} \text{g}))</th>
<th>(r_{\text{ann}} \ (10^{13} \text{cm}))</th>
<th>(A_{\text{ann}} \ (\text{cm}^2) \ (x10^{-26}))</th>
<th>(\sigma = M/A) (\text{(g cm}^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>3.3</td>
<td>350</td>
<td>1,160</td>
<td>0.33–0.83</td>
<td>1.82</td>
<td>637</td>
</tr>
<tr>
<td>Venus</td>
<td>48.7</td>
<td>270</td>
<td>13,150</td>
<td>0.83–1.29</td>
<td>3.06</td>
<td>4300</td>
</tr>
<tr>
<td>Earth</td>
<td>59.8</td>
<td>235</td>
<td>14,950</td>
<td>1.29–1.89</td>
<td>6.00</td>
<td>2500</td>
</tr>
<tr>
<td>Mars</td>
<td>6.4</td>
<td>235</td>
<td>1,504</td>
<td>1.89–3.20</td>
<td>20.95</td>
<td>72</td>
</tr>
<tr>
<td>Asteroids</td>
<td>0.1</td>
<td>200</td>
<td>20</td>
<td>3.2–6.0</td>
<td>80.9</td>
<td>0.25</td>
</tr>
<tr>
<td>Jupiter</td>
<td>19,040</td>
<td>5</td>
<td>95,200</td>
<td>6.0–11.0</td>
<td>267</td>
<td>355</td>
</tr>
<tr>
<td>Saturn</td>
<td>5,695</td>
<td>8</td>
<td>55,560</td>
<td>11.0–21.5</td>
<td>1072</td>
<td>42.4</td>
</tr>
<tr>
<td>Uranus</td>
<td>870</td>
<td>15</td>
<td>13,050</td>
<td>21.5–36.8</td>
<td>2802</td>
<td>4.7</td>
</tr>
<tr>
<td>Neptune</td>
<td>1,032</td>
<td>20</td>
<td>20,640</td>
<td>36.8–52.0</td>
<td>4240</td>
<td>4.9</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.1</td>
<td>70</td>
<td>7</td>
<td>52–70</td>
<td>6900</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\(F^a\) is the factor by which the planetary mass must be multiplied to adjust the observed material to solar composition.
Surface density of solar nebula

- Surface density of the drops off as:
  \[ \sigma(r) = \sigma_0 r^{-\beta} \]

- \( \beta \approx 1.5 \) and \( \sigma_0 \approx 3,300 \text{ g cm}^{-3} \).

- Local deficit of mass in asteroid belt. Mars is also somewhat deficient in mass.

- Inside Mercury’s orbit, nebula material probably cleared out by falling in on Sun or blown out.

- Outer edge may be due to a finite scale size of the original nebular condensation.
Surface density of solar nebula

- Hayashi (1981) widely used:
  \[ \sigma(r) = 1700 \left(\frac{r}{1\text{AU}}\right)^{-3/2} \text{ g cm}^{-2} \]

- Weidenschilling (1977) produced figure at right which shows similar trend.

- Mars and asteroids appears to be under-dense.
Minimum Mass Extrasolar Nebula


- Surface density near exoplanet is:
  \[ \sigma_{\text{exoplanet}} = \frac{M_{\text{exoplanet}}}{2\pi a_{\text{exoplanet}}^2} \]

  where
  \[ M_{\text{exoplanet}} = \left( \frac{R_{\text{exoplanet}}}{R_{\text{Earth}}} \right)^{2.06} M_{\text{Earth}} \]

  is best fit power-law relationship for the Solar System.

- Semimajor axis from Kepler’s laws is:
  \[ a_{\text{exoplanet}} = \left( \frac{P}{\text{yr}} \right)^{2/3} \text{ AU} \]
Minimum Mass Extrasolar Nebula

- MMEN: $\sigma = 620 \left(\frac{a}{0.2 \text{AU}}\right)^{-1.6}$
Minimum mass estimate

- Can also estimate minimum mass from:

\[ M = \int \sigma(r) dA = \int_0^{2\pi} \int_{R_S}^{R_F} \sigma(r) r dr d\theta \]

where \( R_S \) is the radius of the Sun and \( R_F \) is the max distance of Pluto.

- Assume that \( \sigma(r) = 3300 \left( \frac{r}{R_E} \right)^{-2} \), where \( R_E = 1 \text{ AU} \). Therefore,

\[ M = \int_0^{2\pi} \int_{R_S}^{R_F} (3300\left( \frac{r}{R_E} \right)^{-2}) r dr d\theta \]

\[ = 3300R_E^2 \int_0^{2\pi} d\theta \int_{R_S}^{R_F} r^{-1} dr d\theta \]

\[ = 6600\pi R_E^2 \ln\left( \frac{R_F}{R_S} \right) \]

- Setting \( R_E = 1.49 \times 10^{13} \text{ cm} \), and \( R_S = 6.96 \times 10^{10} \text{ cm} \), and \( R_F = 39 \text{ AU} \) =>$ M \approx 0.02 \, M_{\text{Sun}} $

- Ie approximately a factor of two of previous estimate.
"I think you should be more explicit here in step two."