Lecture 1-2: Properties of the Solar System

- Topics in this lecture:
  - Planetary orbits
  - Mass distribution
  - Angular momentum distribution
Properties of the Solar System

1. Planets orbit roughly in the ecliptic plane.
2. Planetary orbits are slightly elliptical, and very nearly circular.
3. Planets and Sun revolve and orbit in a west-to-east direction. The planets obliquity (tilt of rotation axes to their orbits) are small. Uranus and Venus are exceptions.
4. The planets differ in composition. Their composition varies roughly with distance from the Sun: dense, metal-rich planets are in the inner part and giant, hydrogen-rich planets are in the outer part.
5. Meteorites differ in chemical and geologic properties from the planets and the Moon.
6. The rotation rates of the planets and asteroids are similar (5 to 15 hours).
7. Planet distances from the Sun obey Bode's law.
8. Planet-satellite systems resemble the solar system.
9. The Oort Cloud and Edgeworth-Kuiper Belt of comets.
10. Planets contain ~99% of the solar system's AM but Sun contains >99% of solar system's mass.
Orbits of the planets

- Planets move around the Sun in an orbit affected by the Sun’s mass, and to a less extent, by other bodies in the Solar System.

- Laws governing planetary motion were formulated by Johannes Kepler and based on Tycho Brahe’s observations.

- Kepler’s Laws:
  1. Planets have elliptical orbits with the Sun at one focus.
  2. As a planet orbits, a line connecting the planet to the Sun sweeps out equal areas in equal times.
  3. The square of the orbital period is proportional to the cube of the semimajor axis of the orbit.
Figure 1.1: The orbits of the planets about the Sun, projected onto the plane of the ecliptic. The left part of the figure shows the planets from Jupiter out to Pluto. The inner solar system, too small to be shown adequately with the outer planets, is expanded on the right. The orbits of Mars and Pluto are clearly not centred on the Sun. The position of the main asteroid belt is shown between Mars and Jupiter. The Kuiper belt (a second asteroid belt of icy asteroids), not shown, is outside the orbit of Neptune.

From “Physical Processes in the Solar System” by J. Landstreet
# Planetary Properties

<table>
<thead>
<tr>
<th>Class</th>
<th>Planet</th>
<th>a (A.U.)</th>
<th>M (M_E)</th>
<th>R (R_E)</th>
<th>ρ (g cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terrestrials</td>
<td>Mercury</td>
<td>0.39</td>
<td>0.055</td>
<td>0.382</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>Venus</td>
<td>0.72</td>
<td>0.815</td>
<td>0.949</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>Earth</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>Mars</td>
<td>1.52</td>
<td>0.107</td>
<td>0.532</td>
<td>3.9</td>
</tr>
<tr>
<td>Jovians</td>
<td>Jupiter</td>
<td>5.20</td>
<td>317.8</td>
<td>10.4-11.21</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Saturn</td>
<td>9.54</td>
<td>95.16</td>
<td>9.45</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>Uranus</td>
<td>19.19</td>
<td>14.5</td>
<td>4.01</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Neptune</td>
<td>30.06</td>
<td>17.2</td>
<td>3.88</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Heliosphere (>100 AU from Sun)
Kepler’s 1st Law: Law of Orbits

- Planets have elliptical orbits with the Sun at one focus.

- Equation of ellipse: \( r + r' = 2a \)

- \( a \) is semimajor axis, \( b \) is semiminor axis of ellipse, \( F' \) and \( F \) are focal points.

- Distance of focus from ellipse centre is \( ae \), where \( e \) is the eccentricity:
  - \( e = 0 \Rightarrow \) circle
  - \( 0 < e < 1 \Rightarrow \) ellipse
  - \( e = 1 \Rightarrow \) parabola
  - \( e > 1 \Rightarrow \) hyperbola
Kepler’s 1st Law (cont.)

- Implies that a planet’s distance from the Sun varies during its orbit.
  - Closest point to Sun: *perihelion*.
  - Farthest point from Sun: *aphelion*.
  - Average of perihelion and aphelion is called the *semimajor axis*.
Elliptical orbits

- Consider a point at either end of the semiminor axis where \( r = r' \).

- Using the Pythagorean Theorem, \( r^2 = b^2 + (ae)^2 \)

- Setting \( r = a \), we may write:
  \[
  a^2 = b^2 + a^2e^2 \\
  \implies b^2 = a^2 (1 - e^2)
  \]

- Relates semiminor axis to eccentricity and the semimajor axis.
Elliptical orbital path

- From figure below, \( r'^2 = r^2\sin^2\theta + (2ae + r\cos\theta)^2 \)  \( \text{Eqn 1} \)

- But as \( r + r' = 2a \) or \( r' = 2a - r \), we may write \( r'^2 = 4a^2 - 4ra + r^2 \)  \( \text{Eqn 2} \)

- Equating the RHS of \( \text{Eqns 1} \) and \( \text{Eqn 2} \):
  \[
  4a^2 - 4ra + r^2 = r^2\sin^2\theta + (2ae + r\cos\theta)^2 \\
  = r^2(\sin^2\theta + \cos^2\theta) + 4a^2e^2 + 4aer\cos\theta 
  \]

- As \( \sin^2\theta + \cos^2\theta = 1 \) => \( 4a^2 - 4ra = 4a^2e^2 + 4aer\cos\theta \)

- Rearranging gives,
  \[
  r(\theta) = \frac{a(1 - e^2)}{1 + e\cos\theta} \quad \text{Eqn 3}
  \]

- For \( 0 < e < 1 \), this is the equation of an ellipse in polar coordinates.
**Perihelion and aphelion distances**

- If $\theta = 0^\circ$, $\cos \theta = 1$ and
  \[ r = \frac{a(1-e^2)}{1+e} = \frac{a(1-e)(1+e)}{(1+e)} \]
  \[ \Rightarrow r = a(1-e) \]
  The planet is at *perihelion*, the closest point to the Sun.

- If $\theta = 180^\circ$, $\cos \theta = -1$ and
  \[ r = \frac{a(1-e^2)}{1-e} = \frac{a(1-e)(1+e)}{(1-e)} \]
  \[ \Rightarrow r = a(1+e) \]
  The planet at the *aphelion*, the most distant point from the Sun.

- Example: The semimajor axis of Mars is 1.5237 AU and the eccentricity is 0.0934. What is the distance of Mars at perihelion?
  \[ r = a(1-e) \]
  \[ = 1.5237(1-0.0934) \]
  \[ = 1.3814 \text{ AU} \]

- What is the distance of Mars at aphelion?
**Kepler’s 2\textsuperscript{nd} Law: Law of areas**

- As a planet orbits, a line connecting the planet to the Sun sweeps out equal areas in equal times.

\[ \frac{dA}{dt} = \text{const} \]

=> Planet moves faster at perihelion.
Kepler’s 2nd Law: Law of areas

- Angular momentum of planet: \( L = r \times p = m (r \times v) \).

- During \( \Delta t \), radius vector sweeps through \( \Delta \theta = v_t \Delta t / r \), where \( v_t \) is the component of \( v \) perpendicular to \( r \).

- During this time, the radius vector has swept out the triangle, of area \( A = rv_t \Delta t / 2 \).

- As \( \Delta t \rightarrow 0 \), \( dA/dt = rv_t / 2 = 1/2 r^2 (d\theta/dt) \).

- Now, the magnitude of \( L \) is given by \( L = m v_t r = m r^2 d\theta/dt \).

\[ \Rightarrow dA/dt = L / 2m = \text{const} \]

i.e. the rate of sweeping out area is a constant.
Kepler’s 3rd Law: Law of Periods

- The square of the orbital period is proportional to the cube of the semimajor axis of the orbit:
  \[ P^2 \sim a^3 \]

- \( P \) is the period measured in years and \( a \) is the semimajor axis in AU.

- Consider \( m_1 \) and \( m_2 \) orbiting at \( r_1 \) and \( r_2 \). Both complete one orbit in period \( P \). Forces due to centripetal accelerations are:
  \[
  F_1 = m_1 \frac{v_1^2}{r_1} = 4 \pi^2 \frac{m_1 r_1}{P^2} \\
  F_2 = m_2 \frac{v_2^2}{r_2} = 4 \pi^2 \frac{m_2 r_2}{P^2}
  \]

  using \( v = \frac{2\pi r}{P} \)
Kepler’s 3rd Law: Law of Periods

- As \( F_1 = F_2 \Rightarrow r_1 / r_2 = m_2 / m_1 \)
  (more massive body orbits closer to centre of mass).

- Separation of the bodies is \( a = r_1 + r_2 \), and
  \[ r_1 = m_2 a / (m_1 + m_2) \]

- Combining with \( F_1 \) and \( F = F_1 = F_2 = Gm_1 m_2 / a^2 \):
  \[ P^2 = 4 \pi^2 a^3 / G(m_1 + m_2) \]

- As \( M_{\text{sun}} (= m_1) >> m_{\text{planet}} (= m_2) \),
  \( \text{const} = 4 \pi^2 / GM_{\text{sun}} \).
Bode’s Law

- Empirical prediction of planet distances from Sun.

- Begin with:
  
  $0, 3, 6, 12, 24, 48, 96, 192, 384$

- Now add 4:
  
  $4, 7, 10, 16, 28, 52, 100, 196, 388$

- Then divide by 10:
  
  $0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0, 19.6, 38.8$

- Sequence is close to mean distances of planets from the Sun.

- Bode’s Law or Titus-Bode’s Law:
  
  $r_n = 0.4 + 0.3 \times 2^n$

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance (AU)</th>
<th>Bode’s Law (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Venus</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Earth</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Mars</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Ceres</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.2</td>
<td>5.2</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.6</td>
<td>10.0</td>
</tr>
<tr>
<td>Uranus</td>
<td>19.2</td>
<td>19.6</td>
</tr>
</tbody>
</table>
**Bode’s Law (cont.)**

\[ r_n = 0.4 + 0.3 \times 2^n \]
Bode’s Law (cont.)

- Law lead Bode to predict existence of another planet between Mars and Jupiter – asteroids belt later found.
- Uranus fitted law when discovered.
- Neptune was discovered in 1846 at the position predicted by Adams, to explain the deviation of Uranus from its predicted orbit.
- Pluto’s orbit when discovered in 1930 did not fit the relation. Not a “planet” anymore!

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance (AU)</th>
<th>$r_n$ (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranus</td>
<td>19.2</td>
<td>19.6</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.07</td>
<td>38.8</td>
</tr>
<tr>
<td>Pluto</td>
<td>39.5</td>
<td>77.2</td>
</tr>
</tbody>
</table>
The Solar System to scale
Mass distribution

- The density of a planet is measured in $g \text{ cm}^{-3}$ (cgs units).
  
  $$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

- Convenient because the density of water is $1 \text{ g cm}^{-3}$.

- To determine volume, need:
  
  1. Distance from Earth.
  2. Angular extent of the planet.

- To determine the mass (from Kepler’s 3rd Law) we need:
  
  1. Mean Sun-planet separation.
  2. Orbital period.
Mass Distribution (cont.)

- The volume is determined from:
  \[ V = \frac{4}{3} \pi R^3 \]
  where \( 2R = \frac{2\pi d\theta}{360} \).

- Mass determined from Newton’s form of Kepler’s 3rd Law:
  \[ P^2 = 4 \pi^2 a^3 / G (m + M) \]
  \[ \Rightarrow m = \left( 4 \pi^2 a^3 / G P^2 \right) - M \]

- \( \rho = m / V \quad g \text{ cm}^{-3} \)

- Compare to:
  - Cork: 0.2
  - Wood: 0.5
  - Water: 1.0
  - Basalt: 3.3
  - Lead: 11.0
  - Gold: 19.0
Properties of the planets

- From consideration of size and density, can divide the planets into two categories:

1. Terrestrial Planets
   - Small size, high density and in the inner solar system.
   - Mercury, Venus, Earth, Mars.

2. Jovian Planets
   - Large size, low density and in outer solar system:
     - Jupiter, Saturn, Uranus, Neptune.

- Pluto
  - Pluto is in category of own. It has small size and low density.
Compression vs. composition: The inner planets

- From their densities, inner planets likely to be composed of rock and some metal in cores.

- Might expect that planets less massive than the Earth would have lesser densities, because they are less compressed at the center by gravity.

- Amongst the terrestrial planets, this is true for both Mars and the Moon, which are both smaller and less dense than the Earth.

- Venus is roughly the same size and density as Earth.

- But, Mercury is both less massive and more dense than the Earth.
  => Has Mercury a different composition than the Earth?
What about the densities of the outer planets?

Might expect the outer planets, which are very massive, to be much more compressed than the inner planets, and so more dense.

In fact, these heavier bodies are less dense than the inner, terrestrial planets.

The only composition which we can use to construct such massive bodies with such low densities is a mixture of hydrogen and helium, the two lightest elements.

The composition of the outer planets is hence more similar to the Sun and stars than to the inner planets.
Angular momentum distribution

- The Sun has a relatively slow rotational period of ~26-days.
  => Like most G-, K- and M-class stars.

- The orbital AM is: \( h_{\text{orb}} = mvr = 2\pi mr^2/P \quad (r = \text{distance}) \)

- The spin AM of a inhomogeneous nonspherical rotating body is more difficult to evaluate.

- The mass of the body is \( 4\pi \rho_{\text{ave}} R^3/3 \), where \( R \) is the radius and \( \rho_{\text{ave}} \) is the mean density.
Angular momentum of the Sun

- An average density adopted for Sun and it is assumed that mass is mostly within 0.6R. This is 0.72 for a perfect sphere, but the Sun is oblate.

- Mean density = 1.41 g cm\(^{-3}\)
- Mass = 2 \times 10^{33} g
- Period at equator = 26.5 days = 2289600 s
- Radius = 6.96 \times 10^{10} cm

- The spin AM is therefore:

\[ h_{\text{Sun}} = m v R = \frac{4 \pi \rho R^3}{3} \frac{2 \pi r}{P} R \]

- Setting \( R = 0.6R \Rightarrow \)

\[ h_{\text{Sun}} = m v R = \frac{4 \pi \rho}{3} \frac{2 \pi}{P} (0.6R)^5 \]

\[ = 1.9 \rho R^5 / P = 2 \times 10^{48} \text{ g cm}^2 \text{ s}^{-1} \]

- Detailed modelling gives \( \sim 1.7 \times 10^{47} \text{ g cm}^2 \text{ s}^{-1} \).
Angular momentum of the planets

- The orbital AM is: \( h_{\text{orbit}} = \frac{2\pi m r^2}{P} \)

- **Earth**
  - \( m = 5.97 \times 10^{27} \) g
  - \( r = 1 \text{ AU} = 1.5 \times 10^{13} \) cm
  - \( P = 1 \) year
  \[
  h_{\text{orbit}} = \frac{2\pi (5.97 \times 10^{27})(1.5 \times 10^{13})^2}{365 \times 24 \times 60 \times 60} = 2.6 \times 10^{47} \text{ g cm}^2 \text{ s}^{-1}
  \]

- **Jupiter**
  - \( m = 1898 \times 10^{27} \) g
  - \( r = 5.2 \text{ AU} = 5.2 \times 1.5 \times 10^{13} \) cm
  - \( P = 11.86 \) years
  \[
  h_{\text{orbit}} = 1.9 \times 10^{50} \text{ g cm}^2 \text{ s}^{-1}
  \]

- **Saturn**
  - \( m = 586 \times 10^{27} \) g
  - \( r = 9.61 \text{ AU} \)
  - \( P = 29.5 \) years
  \[
  h_{\text{orbit}} = 7.8 \times 10^{49} \text{ g cm}^2 \text{ s}^{-1}
  \]

- Jupiter therefore carries ~50% of the total AM of the Solar System, while the Jovian planets together make up ~99.27% of the total!
**Angular momentum distribution**

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass ($x10^{27}$ kg)</th>
<th>Period (years)</th>
<th>AM (gcm$^2$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.33</td>
<td>0.24</td>
<td>8.6x10$^{45}$</td>
</tr>
<tr>
<td>Venus</td>
<td>4.87</td>
<td>0.61</td>
<td>1.9x10$^{47}$</td>
</tr>
<tr>
<td>Earth</td>
<td>5.97</td>
<td>1</td>
<td>2.6x10$^{47}$</td>
</tr>
<tr>
<td>Mars</td>
<td>0.64</td>
<td>1.88</td>
<td>3.4x10$^{47}$</td>
</tr>
<tr>
<td>Jupiter</td>
<td>1898.8</td>
<td>11.86</td>
<td>1.9x10$^{50}$</td>
</tr>
<tr>
<td>Saturn</td>
<td>568.41</td>
<td>9.5</td>
<td>7.8x10$^{49}$</td>
</tr>
<tr>
<td>Uranus</td>
<td>86.97</td>
<td>19.31</td>
<td>1.7x10$^{49}$</td>
</tr>
<tr>
<td>Neptune</td>
<td>102.85</td>
<td>30</td>
<td>2x10$^{49}$</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.0129</td>
<td>39.91</td>
<td>3.7x10$^{45}$</td>
</tr>
</tbody>
</table>

- Sun only has 0.4% of the total AM in solar system.
Orbital angular momenta of the planets

- Note the overwhelming importance of the Jovian planets.

- The symbol associated with each planet:
Mass and AM distributions

- Although Sun contains 99.9% of the mass of Solar System, the outer planets have 98% of system’s angular momentum.

- This is a serious problem: material accreting onto the Sun cannot have retained all its original AM.

- There are two parts to the problem:
  1. How does material lose AM and fall into the star in the first place?
  2. How does the star lose AM and slow down? Solar-type stars all rotate at about the same speed at the Sun.