Lectures 5-6: Magnetic dipole moments

- Orbital dipole moments.
- Orbital precession.
- Spin-orbit interaction.
- Stern-Gerlach experiment.
- Total angular momentum.
- Fine structure, hyperfine structure of H and Na.
- The Lamb shift.

Orbital magnetic dipole moments

Consider electron moving with velocity \( v \) in a circular Bohr orbit of radius \( r \). Produces a current

\[ I = \frac{e v r}{2} \]

where \( T \) is the orbital period of the electron.

- Current loop produces a magnetic field, with a moment

\[ \mu = I L = \frac{e v r}{2} \times L \]

- Specifies strength of magnetic dipole.

- Magnitude of orbital angular momentum is \( L = m v r = m v r^2 \).

Combining with Eqn. 1 =>

\[ (1) \]

- An electron in the first Bohr orbit with \( L = \hbar \) has a magnetic moment defined as

\[ = 9.27 \times 10^{-24} \text{ Am}^2 \]

- Bohr magneton

Sodium D-line doublet

- Grotrian diagram for doublet states of neutral sodium showing permitted transitions, including Na D-line transition at 589 nm.

- D-line is split into a doublet:

\[ D_1 = 589.59 \text{ nm}, \ D_2 = 588.96 \text{ nm}. \]

- Many lines of alkali atoms are doublets. Occur because terms (bar \( s \)-term) are split in two.

- This fine structure can only be understood via magnetic moments of electron.

Orbital magnetic dipole moments

- Magnetic moment can also be written in terms of the Bohr magneton:

\[ \mu = g_l \hbar \]

where \( g_l \) is the orbital g-factor. Gives ratio of magnetic moment to angular momentum (in units of \( \hbar \)).

- In vector form, Eqn 2 can be written

- As

- The components of the angular momentum in the z-direction are

\[ L_z = m_l \hbar \quad \text{where} \quad m_l = -l, -l+1, ..., 0, ..., +l-1, +l. \]

- The magnetic moment associated with the z-component is correspondingly
**Orbital precession**

- When magnetic moments is placed in an external magnetic field, it experiences a torque:

\[ \text{(3)} \]

which tends to align dipole with the field. The potential energy associated with this force is

- Minimum potential energy occurs when \( \mu \parallel B \).

- If \( \Delta E = \text{const.}, \mu \) cannot align with \( B \) \( \Rightarrow \) \( \mu \) precesses about \( B \).

- But from Eqn. 3,

- Setting this equal to Eqn. 4 \( \Rightarrow \)

- Called Larmor precession. Occurs in direction of \( B \).

**Electron spin**

- Electron also has an intrinsic angular momentum, called spin. The spin and its z-component obey identical relations to orbital AM:

\[ \frac{s}{\hbar} = \pm \frac{1}{2} \]

- Therefore two possible orientations:

\( \Rightarrow \) spin magnetic quantum number is \( \pm 1/2 \).

- Follows that electron has intrinsic magnetic moments:

\[ \mu_s = g_s m_s \]

where \( g_s \approx 2 \) is the spin g-factor.

**The Stern-Gerlach experiment**

- This experiment confirmed the quantisation of electron spin into two orientations.

- Potential energy of electron spin magnetic moment in magnetic field in z-direction is

\[ \frac{1}{2} m_s \mathbf{B}^2 \]

- The resultant force is

\[ \mathbf{F} = m_s \frac{d\mathbf{S}}{dt} = \hbar m_s \frac{d\mathbf{S}}{dt} = \frac{\hbar}{2} \frac{d\mathbf{S}}{dt} \]

- The deflection distance is then,

\[ \Delta x = \frac{\hbar B L}{2 m_s} \]

- As \( g_s m_s = \pm 1 \),

- The deflection distance is then,

**The Stern-Gerlach experiment**

- Conclusion of Stern-Gerlach experiment:

  - With field on, classically expect random distribution at target. In fact find two bands as beam is split in two.

  - There is directional quantisation, parallel or antiparallel to \( B \).

  - Atomic magnetic moment has \( \mu_z = \pm \mu_F \).

  - Find same deflection for all atoms which have an \( s \) electron in the outermost orbital \( \Rightarrow \) all angular momenta and magnetic moments of all inner electrons cancel. Therefore only measure properties of outer \( s \) electron.

  - The \( s \) electron has orbital angular momentum \( l = 0 \) \( \Rightarrow \) only observe spin.
The Stern-Gerlach experiment

- Experiment was confirmed using:

<table>
<thead>
<tr>
<th>Element</th>
<th>Electronic Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1s^1</td>
</tr>
<tr>
<td>Na</td>
<td>{1s^22s^2}2p^23s^1</td>
</tr>
<tr>
<td>K</td>
<td>{1s^22s^22p^6}3s^23p^6^4s^1</td>
</tr>
<tr>
<td>Cu</td>
<td>{1s^22s^22p^6}3s^33p^6^4s^1</td>
</tr>
<tr>
<td>Ag</td>
<td>{1s^22s^22p^6}3s^33p^6^4s^1 5s^1</td>
</tr>
<tr>
<td>Cs</td>
<td>[{Ag}5s^25p^6]6s^1</td>
</tr>
<tr>
<td>Au</td>
<td>[{Cs}5d^104f^2]6s^1</td>
</tr>
</tbody>
</table>

- In all cases, l = 0 and s = 1/2.
- Note, shell penetration is not shown above.

Spin-orbit interaction

- Fine-structure in atomic spectra cannot be explained by Coulomb interaction between nucleus and electron.
- Instead, must consider magnetic interaction between orbital magnetic moment and the intrinsic spin magnetic moment.
- Called spin-orbit interaction.
- Weak in one-electron atoms, but strong in multi-electron atoms where total orbital magnetic moment is large.
- Coupling of spin and orbital AM yields a total angular momentum, \( \hat{J} \).

Spin-orbit interaction

- Consider reference frame of electron: nucleus moves about electron. Electron therefore in current loop which produces magnetic field. Charged nucleus moving with \( v \) produces a current:

- According to Ampere’s Law, this produces a magnetic field, which at electron is

- Using Coulomb’s Law:

\[
\Rightarrow \quad (5)
\]

where \( \epsilon = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \)

- This is the magnetic field experienced by electron through \( E \) exerted on it by nucleus.

Spin-orbit interaction

- We know that the orientation potential energy of magnetic dipole moment is \( \Delta E = -\hat{\mu}_s \cdot \hat{B} \) but as

- Transforming back to reference frame with nucleus, must include the factor of 2 due to Thomas precession (Appendix O of Eisberg & Resnick):

\[
(6)
\]

- This is the spin-orbit interaction energy.

- More convenient to express in terms of \( S \) and \( L \). As force on electron is

\[
\text{can write Eqns. 5 as}
\]
Spin-orbit interaction

- As
- Substituting the last expression for $B$ into Eqn. 6 gives:
- Evaluating $g_s$ and $\mu_B$, we obtain:
- For hydrogenic atoms,
- Substituting into equation for $\Delta E$:
- Expression for spin-orbit interaction in terms of $L$ and $S$. Note, $\alpha = e^2/4\pi\varepsilon_0\hbar c$ is the fine structure constant.

Sodium fine structure

- Transition which gives rise to the Na D-line doublet is 3P$\rightarrow$3S.
- 3P level is split into states with total angular momentum $j=3/2$ and $j=1/2$, where $j = l \pm s$.
- In the presence of additional externally magnetic field, these levels are further split (Zeeman effect).
- Magnitude of the spin-orbit interaction can be calculated using Eqn. 7. In the case of the Na doublet, difference in energy between the 3P$3/2$ and 3P$1/2$ sublevels is:
  $$\Delta E = 0.0021 \text{ eV} \text{ (or 0.597 nm)}$$

Hydrogen fine structure

- Spectral lines of H found to be composed of closely spaced doublets. Splitting is due to interactions between electron spin $S$ and the orbital angular momentum $L \Rightarrow$ spin-orbit coupling.
- Hα line is single line according to the Bohr or Schrödinger theory. Occurs at 656.47 nm for H and 656.29 nm for D (isotope shift, $\Delta \lambda = 0.2$ nm).
- Spin-orbit coupling produces fine-structure splitting of ~0.016 nm. Corresponds to an internal magnetic field on the electron of about 0.4 Tesla.

Total angular momentum

- Orbital and spin angular momenta couple together via the spin-orbit interaction.
- Internal magnetic field produces torque which results in precession of $\hat{L}$ and $\hat{S}$ about their sum, the total angular momentum:
- Called L-S coupling or Russell-Saunders coupling. Maintains fixed magnitude and $z$-components, specified by two quantum numbers $j$ and $m_j$.
  $$m_j = j, j + 1, \ldots, -j + 1, -j.$$  
- But what are the values of $j$? Must use vector inequality.
Total angular momentum

- From the previous page, we can therefore write

- Since, \( s = 1/2 \), there are generally two members of series that satisfy this inequality:
  \[ j = l + 1/2, \ l - 1/2 \]

- For \( l = 0 \Rightarrow j = 1/2 \)

- Some examples vector addition rules
  
  \( J = L + S \), \( L = 3, S = 1 \)
  
  \( L + S = 4, |L - S| = 2 \), therefore \( J = 4, 3, 2 \).

  \( L = l_1 + l_2, l_1 = 2, l_2 = 0 \)
  
  \( l_1 + l_2 = 2, |l_1 - l_2| = 2 \), therefore \( L = 2 \)

  \( J = j_1 + j_2, j_1 = 5/2, j_2 = 3/2 \)
  
  \( j_1 + j_2 = 4, |j_1 - j_2| = 1 \), therefore \( J = 4, 3, 2, 1 \)

Total angular momentum in a magnetic field

- Total angular momentum can be visualised as precessing about any externally applied magnetic field.

- Magnetic energy contribution is proportional \( J_z \).

- \( J_z \) is quantized in values one unit apart, so for the upper level of the sodium doublet with \( j = 3/2 \), the vector model gives the splitting in bottom figure.

- This treatment of the angular momentum is appropriate for weak external magnetic fields where the coupling between the spin and orbital angular momenta can be presumed to be stronger than the coupling to the external field.

Total angular momentum

- For multi-electron atoms where the spin-orbit coupling is weak, it can be presumed that the orbital angular momenta of the individual electrons add to form a resultant orbital angular momentum \( L \).

- This kind of coupling is called \( L-S \) coupling or Russell-Saunders coupling.

- Found to give good agreement with observed spectral details for many light atoms.

- For heavier atoms, another coupling scheme called \( j-j \) coupling provides better agreement with experiment.