Lecture 12: Atoms in magnetic fields

Topics to be covered:

- Normal Zeeman effect
- Anomalous Zeeman effect
- Plasma physics applications

Zeeman Effect

- First reported by Zeeman in 1896. Interpreted by Lorentz.
- Interaction between atoms and field can be classified into two regimes:
  - Weak fields: Zeeman effect, either normal or anomalous.
  - Strong fields: Paschen-Back effect.
- Normal Zeeman effect agrees with the classical theory of Lorentz. Anomalous effect depends on electron spin, and is purely quantum mechanical.

Photograph taken by Zeeman

Norman Zeeman effect

- Observed in atoms with no spin.
- Total spin of an N-electron atom is
- Filled shells have no net spin, so only consider valence electrons. Since electrons have spin 1/2, not possible to obtain \( S = 0 \) from atoms with odd number of valence electrons.
- Even number of electrons can produce \( S = 0 \) state (e.g., for two valence electrons, \( S = 0 \) or 1).
- All ground states of Group II (divalent atoms) have \( ns^2 \) configurations \( \Rightarrow \) always have \( S = 0 \) as two electrons align with their spins antiparallel.
- Magnetic moment of an atom with no spin will be due entirely to orbital motion.

Norman Zeeman effect

- Interaction energy between magnetic moment and a uniform magnetic field is:
- Assume \( B \) is only in the z-direction:
- The interaction energy of the atom is therefore,
  \[
  E = m_l B \]
  where \( m_l \) is the orbital magnetic quantum number. This equation implies that \( B \) splits the degeneracy of the \( m_l \) states evenly.
But what transitions occur? Must consider selections rules for $m_e$: $\Delta m_e = 0, \pm 1$.

Consider transitions between two Zeeman-split atomic levels. Allowed transition frequencies are therefore,

- Emitted photons also have a polarization, depending on which transition they result from.

Longitudinal Zeeman effect: Observing along magnetic field, photons must propagate in $z$-direction.

- Light waves are transverse, and so only $x$ and $y$ polarizations are possible.
- The $z$-component ($\Delta m_z = 0$) is therefore absent and only observe $\Delta m_z = \pm 1$.
- Termed $\sigma$-components and are circularly polarized.

Transverse Zeeman effect: When observed at right angles to the field, all three lines are present.

- $\Delta m_z = 0$ are linearly polarized $||$ to the field.
- $\Delta m_z = \pm 1$ transitions are linearly polarized at right angles to field.

Last two columns of table below refer to the polarizations observed in the longitudinal and transverse directions.

<table>
<thead>
<tr>
<th>$\Delta m_z$</th>
<th>Energy</th>
<th>Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 1$</td>
<td>$\omega - \mu B$</td>
<td>Longitudinal observation</td>
</tr>
<tr>
<td>$0$</td>
<td>$\omega_0$</td>
<td>Transverse observation</td>
</tr>
<tr>
<td>$-1$</td>
<td>$\omega + \mu B$</td>
<td></td>
</tr>
</tbody>
</table>

Direction of circular polarization in longitudinal case is defined relative to $B$.

Interpretation proposed by Lorentz (1896)

Anomalous Zeeman effect

- Discovered by Thomas Preston in Dublin in 1897.
- Occurs in atoms with non-zero spin => atoms with odd number of electrons.
- In LS-coupling, the spin-orbit interaction couples the spin and orbital angular momenta to give a total angular momentum according to
  \[ \hat{J} = \hat{L} + \hat{S} \]
- In an applied $B$-field, $J$ precesses about $B$ at the Larmor frequency.

$L$ and $S$ precess more rapidly about $J$ to due to spin-orbit interaction. Spin-orbit effect therefore stronger.
Anomalous Zeeman effect

- Interaction energy of atom is equal to sum of interactions of spin and orbital magnetic moments with $B$-field:

\[ E = g_S \mu_B B_S + g_L \mu_B B_L \]

where $g_s = 2$, and the $< \ldots >$ is the expectation value. The normal Zeeman effect is obtained by setting $S = 0$ and $L = mL$.

- In the case of precessing atomic magnetic in figure on last slide, neither $S$ nor $L$ are constant. Only $J = mL$ is well defined.

- Must therefore project $L$ and $S$ onto $J$ and project onto $z$-axis $\Rightarrow$

\[ \hat{L}_z = mL \hbar \]
\[ \hat{S}_z = 0 \]
\[ \hat{J}_z = mJ \hbar \]

The angles $\theta_1$ and $\theta_2$ can be calculated from the dot products of the respective vectors:

\[ \cos \theta_1 = \frac{\hat{S}_z \cdot \hat{J}_z}{\sqrt{\hat{S}_z \cdot \hat{S}_z} \sqrt{\hat{J}_z \cdot \hat{J}_z}} \]
\[ \cos \theta_2 = \frac{\hat{S}_z \cdot \hat{L}_z}{\sqrt{\hat{S}_z \cdot \hat{S}_z} \sqrt{\hat{L}_z \cdot \hat{L}_z}} \]

which implies that

\[ (1) \]

- Now, using $\hat{S} = \hat{J} - \hat{L}$ implies that

\[ \Rightarrow \]

therefore

so that

- Similarly,

Anomalous Zeeman effect spectra

- We can therefore write Eqn. 1 as

\[ E = g_J \mu_B B_J \]

This can be written in the form

where $g_J$ is the Lande $g$-factor given by

This implies that

and hence the interaction energy with the $B$-field is

Classical theory predicts that $g_J = 1$. Departure from this due to spin in quantum picture.

- Spectra can be understood by applying the selection rules for $j$ and $m_j$:

Polarizations of the transitions follow the same patterns as for normal Zeeman effect.

- For example, consider the Na D-lines at right produced by $3p \rightarrow 3s$ transition.
**Sunspot magnetography**

- Measure strength of magnetic field from *spectral line shifts or polarization*.
- Choose line with large Lande g-factor $\rightarrow$ sensitive to $B$.
- Usually use Fe I or Ni I lines.
- Measures field-strengths of $\pm 2000$ G.

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**Tokamak plasma diagnostics**

- Tokamak produces a toroidal magnetic field for confining a plasma.
- Leading candidate for producing stable fusion.
- Can diagnose strength of magnetic field in tokomak using Zeeman effect.
- The figure at bottom shows the subtraction of the two circularly polarised $\sigma$ components for the HeII ion (ie single ionised).
- For further details, see [http://www.plasma.ernet.in/~othdiag/zeeman/pram/pram1.html](http://www.plasma.ernet.in/~othdiag/zeeman/pram/pram1.html)