1. Consider the 2-body decay of the $\pi^0$ – the two massless photons must have equal and opposite momenta and equal and opposite energies when considering the $\pi^0$ in its rest frame, the center-of-mass frame of the two photon system. Nothing complicated here.

Consider the 2-body decay of a $\Lambda^0$ (uds) by the weak interaction into a p and a $\pi^-$ where the masses are 1115.7, 938.3 and 139.6 MeV/c$^2$ respectively. Calculate the momenta of the decay products in the centre of mass (CM) frame. This is a two body decay and thus the momenta in the CM frame are equal and opposite.

[Hint: this will involve some complicated algebra and cancellations to obtain the momenta of the decay products. If necessary can then determine the energies in the CM frame of each of the decay products.]

[Further hint: It is useful to know the identity: $(a+b+c+d)^2 = (a+b+c)^2 + 2d(a+b+c) + d^2$]

[Detailed hints for obtaining momenta: note this could be shorter but the unnecessary steps illustrate a point....]

a. start in the CM frame, or rest frame of decaying particle.
   b. then $\mathbf{p}_p = -\mathbf{p}_\pi$, or taking the magnitudes $p_p = p_\pi = p$. Simpler to refer to $p$.
   c. expand $m_\pi^2 c^4 = (E_p + E_\pi)^2 - (\mathbf{p}_p + \mathbf{p}_\pi)^2 c^2$
   d. use $E_p^2 - p^2 c^2 = m_p^2 c^4$ and similarly $E_\pi^2 - p^2 c^2 = m_\pi^2 c^4$
   e. obtain an expression for the energy cross term $2E_p E_\pi$, i.e. put this on left hand side
   f. square both sides
   g. replace $E_p^2$ with $m_p^2 c^4 + p^2 c^2$ and similarly for $E_\pi^2$ (from identities in d above)
   h. apply the identity $(a+b+c+d)^2 = (a+b+c)^2 + 2d(a+b+c) + d^2$
   i. look for cancellations
   j. obtain a formula for $p^2$ which is only in terms of the masses $m_\Lambda, m_p, m_\pi$ and is uniquely determined by the kinematics of the decay.
   k. check that the CM momenta $p$ obtained matches that listed in the excerpt from the particle data listing
   l. repeat for the second listed decay channel.]

2. Consider the 2-body decay of a $\Delta^{++}$ (uuu) by the strong interaction into a p and a $\pi^+$ where the masses are 1232, 938.3 and 139.6 MeV/c$^2$ respectively. Calculate the energy and momenta of the decay products in the centre of mass frame.

[Hint:]

a. determine $p^2$ in CM frame as above for this two body decay
b. from this determine $E_p^2$ where $E_p^2 = m_p^2 c^4 + p^2 c^2$ is the energy of the proton in the CM frame, similarly for $E_\pi^2$
c. thus determine $E_p$ and $E_\pi$ ]
Consider the laboratory frame when the $\Delta^{++}$ is observed to have an energy before decay of 1500 MeV. If the decay occurs such that the decay products in the centre of mass frame move away perpendicular to the direction of motion in the laboratory frame, then calculate the observed spatial opening angle between the paths of the two decay products as seen in the laboratory frame.

[Detailed hints for second part:

d. calculate $\gamma$, remembering that for a single particle $E_{\text{laboratory}} = \gamma m_{\text{rest mass}} c^2$.
e. from $\gamma$ calculate $\beta$
f. can then apply the Lorentz transformation to obtain laboratory energy and momenta of decay products in particular directions
g. in general $\mathbf{p}' \neq -\mathbf{p}$ unless the dashed frame is the rest frame. regardless of direction of decay products: $m_{\Delta}^2 c^4 = (E_{p} + E_{\pi})^2 - (p_{p} + p_{\pi})^2 c^2$ holds in any frame – whether the center of mass frame, or the laboratory frame.
h. it is useful to recall that $\mathbf{p}' \cdot \mathbf{p}' = p_{p}' p_{\pi}' \cos \alpha$ where $\alpha$ is the angle between the two momenta]

Will the pion ever be observed to be emitted back along the path of the $\Delta^{++}$ in the laboratory. If not explain why?

[Detailed hints:
i. can reapply the Lorentz transformation to obtain laboratory energy and momenta of decay products in particular directions, but be careful of signs]

3. [Hint: to conserve baryon number – fixed target proton-proton collision implies that initial baryon number sums to two – must have at least two baryons as products of collision. From notes, mass of $J/\psi$ is $\sim 3.097$ GeV/c$^2$. You may need to make assumptions about motions of particle to obtain a lower bound. Energy and momenta are conserved in both centre of mass frame and in laboratory frame]

4. [Hint: need to calculate $\gamma$ to obtain the relativistic “boost” that the $B_d^0$ meson has. Need to obtain an estimate for the mass of the $B_d^0(d\bar{d})$ meson. Look this up from previous handouts, in textbook, or from particle data group, or from observation of the $2B_d^0$ threshold in the $\Upsilon$ spectrum of $b\bar{b}$ bottomium mesons seen in question 5. Then $E_{\text{laboratory}} = \gamma m_{\text{rest mass}} c^2$. Calculate the observed decay time in the laboratory frame of reference, and the observed decay length in the laboratory frame.]

5. [Hint: First calculate the CM energy of the electron-positron collision.

Is the CM energy coincident with a state in the $\Upsilon$ spectrum of states in the diagram shown? These are $b\bar{b}$ states. Consider how a $B_d^0(b\bar{d})$ could be formed as a result of the collision. What does the BB threshold imply for the energy of the $B_d^0$ (or $\bar{B}_d^0$) arising as a result of the decay?

The remainder is similar to Q4 above, but must now determine the average boost after making
assumptions about the decay process above. Note that you can calculate the boost of the $B^0_d$ in the rest frame of the $\Upsilon$, but you should also consider the boost of the centre of mass frame of the decaying $\Upsilon$ particle. Consider what is the boost of the $\Upsilon$ particle that is created.]

6. [Hint: the first part is the inverse problem of Q2.]

The second part is similar to Q3.]

7. [Hint: $M_x c^4 = \left( \sum E_i \right)^2 - \left( \sum p_i \right)^2$ where $\left( \sum p_i \right)^2 = \Sigma p_x^2 + \Sigma p_y^2 + \Sigma p_z^2$]

[Trivial to calculate in this instance.]

8. Hint: Always consider baryon number conservation first when looking at a jet of particles. Thus you can immediately determine which particles are mesons and which are baryons.

Then when looking at both jets in a two jet event (not exactly the case here as we are only looking at one of the two jets) – then the baryon number must also be conserved across the entirety of the two jets. If the two jets are observed in an $e^+e^-$ collision, e.g. as in LEP, then the sum of baryon numbers over all particles in the two jets must sum to zero, because there were zero baryons or zero net baryon number to begin with. The inverse is also true if one looks at all the final states and considers their baryon numbers.

9. It may be useful to indicate that it is possible that where an energetic particle is not completely absorbed in a detector, such as a calorimeter, then the energy of the particle may be underestimated. What is reported may only be the measured amount of energy observed to be deposited.