Lecture 5 - Periodic motion, oscillations

- Todays Topics
  - Circular motion as periodic motion
    - Origin of oscillations
    - Hooke's law
    - Spring constant
    - Periodic motion in 1D - spring
    - Simple harmonic motion
    - Pendulum
    - Dimensional analysis of pendulum
  - Hooke's law
  - Spring constant
  - Periodic motion in 1D - spring
  - Simple harmonic motion
  - Pendulum
  - Dimensional analysis of pendulum
  - Why are oscillations important?
    - Waves
    - Basic wave equations

PY2P30 - Origin of oscillations

Waves are oscillatory motion e.g. simple harmonic motion of spring - consider horizontal (left) or vertical (right) and neglect friction - both produce oscillations

\[ F = -k x \]

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

Note - here \( k \) is the spring constant

Stiffer spring = higher \( k \Rightarrow \) smaller period
PY2P30 - Periodic motion

Can easily visualise this periodic motion

![Graph showing periodic motion](image)

Note - here k is the spring constant

Hooke's law:

\[ F = m \cdot a = -k \cdot A \]

Thus the angular frequency (frequency of oscillation) is dependent upon spring constant k - in turn dependent upon material of spring and the material's elastic modulus E (as we shall see in Lecture 7)

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ a = -\omega^2 A \]

\[ F = m \cdot a = -m \cdot \omega^2 A \]

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Circular motion in 1D

- The periodic circular motion of an object can be considered in 1D - e.g. consider either the x or y axes.

At x in diagram:

\[ v = v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}} \]

At any x:

\[ v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}} \]

As a trigonometric expression

\[ v = \pm v_{\text{max}} \sqrt{1 - (\cos \theta)^2} \]

\[ = \pm v_{\text{max}} \sin \theta \]
**Periodic motion - spring**

- Extension of a spring - 1D periodic motion
- Restoring force on spring - Hookes Law

\[ F = -k x \quad (11-1) \]

For a given amplitude of motion \( A \), at any \( x \):

\[ v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}} \]

The period is given by:

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

Stiffer spring = higher \( k \)?

Will return to simple harmonic motion when considering waves in solids

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**PY2P30 - Foundation Physics - waves**

**Why are oscillations important?** ……..

Recall last years Foundation Physics lectures 2.6 and 2.7

Definitions of

- Plane waves:
- Spherical waves:
- Transverse waves:
- Longitudinal waves:
- Polarised waves:
Recall also the basic wave properties or definitions:

- **Light:**
  \[ c = f \cdot \lambda \]

- **Generally:**
  \[ v = f \cdot \lambda \]

**Equation:**

\[ y(z,t) = A(z,t) \cdot \sin(k \cdot z - \omega \cdot t + \phi) \]

**Wavenumber and angular frequency:**

\[ k = \frac{2\pi}{\lambda} \quad \quad \omega = 2\pi f = \frac{2\pi}{T} \]

**thus also:**

\[ v = \frac{\omega}{k} = f \cdot \lambda \]

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**Periodic motion - pendulum**

- Swing of a pendulum - "1D" periodic motion
- Restoring force on pendulum
  - component of weight in tangential direction

\[ F = -mg \sin(\theta) \]

For small \( \theta \): \( \sin \theta \approx \theta \)

\[ F \approx -\frac{mg}{L} \cdot x \quad \text{where} \quad x = L\theta \]

The period is given by:

\[ T = 2\pi \sqrt{\frac{L}{g}} \]
Dimensional Analysis

- **What is Dimensional Analysis?**
  - Firstly, can check equations e.g. is period $T$ of an oscillating mass $m$ at end of spring with spring constant $k$ equal to:
    
    $T = 2\pi \sqrt{k/m}$ or $T = 2\pi \sqrt{m/k}$?

- Express variables in terms of dimensions [M], [L] and [T] and check for consistency between right and left hand sides of each equation.
  - Use defining equations to obtain dimensions of variables

- Secondly - can obtain the form of an unknown equation
  - Consider a pendulum and assume that an equation for the period has a functional form dependent upon the product of variables raised to differing powers,
    
    $T = C m^\alpha g^\beta l^\gamma \theta^\delta$ where $C$ is a dimensionless constant to be determined by experiment, and $\alpha, \beta, \gamma, \delta$ can be determined by dimensional analysis
  - Can also be applied to water waves and drag forces

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Dimensional Analysis

- **Dimensional Analysis of pendulum**
  - Assume period is some product function $T = C m^\alpha g^\beta l^\gamma \theta^\delta$
  - Determine exponents from dimensional analysis
**Dimensional Analysis**

- Dimensional Analysis of pendulum

**Dimensional Analysis II**

- Work through obtaining formula for period of pendulum in Appendix B
- What about air-resistance or resistance of motion through a fluid?
- What is the form of equation of the drag force in air?
  - Assume drag force in air is in proportion to the following quantities: density of the air, $\rho$; size of object, $r$; speed of object, $v$
  - Use the method of dimensions to obtain a formula.

$$F_D = C r^a \rho^b v^c$$
Dimensional Analysis

• Dimensional Analysis of drag force in air

Kinematics in 1D in fluid - drag forces

Consider an object falling through a fluid:
• A buoyancy force will be exerted on the object
• A drag force will also be exerted on the object
• What is the free-body diagram that would be drawn?
• What is the form of the drag force?
• What is the terminal velocity that would obtain?
Dimensional Analysis III

- What is the form of equation of the drag force in a fluid?
- Assume drag force in a more dense viscous fluid is in proportion to the following quantities:
  - viscosity of the fluid, \( \eta \);
  - size of object \( r \);
  - speed of object \( v \)

\[
F_D = C_D \eta r^p v^n
\]

From the definition of viscosity, \( \eta \) will have dimensions \([M][L]^{-1}[T]^{-1}\).

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PY2P30 - Physics for Earth Sciences

- Next two lectures in mechanics stream
- Rotating bodies
  - Fictitious and real forces
  - Inertial and non-inertial frames
  - Coriolis force
- Tidal motion
  - Fluids under acceleration
  - Tidal forces
  - Centrifugal acceleration
  - Gravitational acceleration due to moon
  - Centrifugal acceleration due to moon
  - Gravitational acceleration due to sun
  - Periodicity
  - Ocean currents
  - Roche limit - asteroids

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CMcG - Lecture 5