Lecture 3 - Circular and rotational motion

- Today's Topics
  - Circular Motion
    - Kinetic energy
    - Non-uniform motion - changing accelerations
  - Newton's law of gravitation
    - Inertia
  - Rotational dynamics
    - Torque
    - Rotational inertia
    - Moments of inertia
    - Angular momentum

PY2P30 - Kinetic energy

- Reminder: - Work and Energy (Ch. 6)
  - Work done by a constant force
  - Work done by a varying force
  - Kinetic energy and the work energy principle
  - Potential energy \( W_p = \Delta KE \)
  - Conservative and non-conservative forces
  - Conservation of mechanical energy

- Kinetic energy in circular motion
- Work in circular motion
  - need to define rotational force = torque
  - then can define work done in circular motion
PY2P30 - Centripetal acceleration

- Centripetal acceleration and centripetal force

\[ \sum F_R = ma_R = m \frac{v^2}{r} \]

PY2P30 - Centripetal/ Centrifugal

- Completing the force diagram - centripetal force

Force on ball exerted by string
Force on hand exerted by string

(a) Doesn't happen
(b) Happens

Centrifugal force
Not real!
(in an inertial frame)
PY2P30 - Non uniform circular motion

- Additional force / acceleration gives rise to non-uniform motion

Swinging a mass in a vertical circle

PY2P30 - Uniform circular motion

- Swinging mass in a horizontal circle is uniform
- Example 5-3
**Newton's law of Universal Gravitation**

\[ F = G \frac{m_1 m_2}{r^2} \]

*Q: There is no string holding the moon - why does it not fly away?*

*Q: Does the moon orbit the centre of the earth?*

- **Inertia** - Newton's 2nd law \( F = ma \)
- **Mass** is the measure of **inertia** of a body - and reflects how hard it is to change the state of motion of a body.

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**PY2P30 - Torque**

- To make an object start rotating requires a force acting at a \( \perp \) distance from an axis of rotation.

\[ \text{Torque} = r F \sin \theta \]

Which force \( A, B, C \) exerts the greatest torque?

Note units = N m - but not an energy.
The rotational work done is the product of the applied torque and the rotational displacement:

\[ W = τΔθ \]

- Rotational inertia:
  \[ F = ma \]
  multiply x "r"

  \[ \tau r \]

  \[ mr^2 \]
  represents the "rotational inertia"

- Newtons 2nd law of rotational motion

The moment of inertia (or rotational inertia) of a body is:

Recall:

And now similar to:

\[ \sum F = ma \]

Q: is it easier to accelerate the rotation of a hoop or a disc of the same mass?

Need to calculate I for hoop and disc.
PY2P30 - Calculating moment of inertia

- A hoop – axis of rotation through its centre

- A disk – axis of rotation through its centre
PY2P30 - Calculating moment of inertia

- A sphere

http://hyperphysics.phy-astr.gsu.edu/hbase/isph.html

PY2P30 - Moments of inertia of different shapes

- Note: The relevant moment of inertia depends upon axis of rotation!!

<table>
<thead>
<tr>
<th>Object</th>
<th>Location of axis</th>
<th>Moment of inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin hoop, radius $R$</td>
<td>Through center</td>
<td>$MR^2$</td>
</tr>
<tr>
<td>Thin hoop, width $W$</td>
<td>Through central diameter</td>
<td>$\frac{1}{2}MR^2 + \frac{1}{2}MW^2$</td>
</tr>
<tr>
<td>Solid cylinder, radius $R$</td>
<td>Through center</td>
<td>$MR^2$</td>
</tr>
<tr>
<td>Uniform sphere, radius $R$</td>
<td>Through center</td>
<td>$\frac{2}{5}MR^4$</td>
</tr>
<tr>
<td>Long uniform rod, length $L$</td>
<td>Through center</td>
<td>$\frac{1}{3}ML^2$</td>
</tr>
<tr>
<td>Long uniform rod, length $L$</td>
<td>Through end</td>
<td>$\frac{1}{3}ML^2$</td>
</tr>
</tbody>
</table>

Moments of inertia also dependent upon distribution of mass in object.

Formulae here assume constant density.
Angular momentum defined by:

\[ L = I \omega \]

(analagous to linear momentum)

Total torque = rate of change of angular momentum

\[ \sum \tau = \frac{d}{dt} L \]

If the net torque is zero, then the angular momentum is constant.

Vector direction of angular velocity and momentum:

\[ \vec{L} \propto \vec{\omega} \]

\[ \vec{L} \propto \vec{m} \times \vec{v} \]

Conceptual problems (ii)

A rider in a "barrel of fun" finds herself stuck with her back to the wall.

Which free-body diagram correctly shows the forces acting on her?

Choose from 1-6
Conceptual problems (iii)

Two wheels with fixed hubs, each having a mass of 1 kg, start from rest, and forces are applied as shown. Assume the hubs and spokes are massless, so that the rotational inertia is $I = mR^2$

In order to impart identical angular accelerations, how large must $F$ be?

![Diagram of two wheels with forces applied](image)

1) 0.25 N
2) 0.5 N
3) 1 N
4) 2 N
5) 4 N