QUARK MODEL OF HADRONS – I

OVERVIEW OF EVIDENCE FOR QUARKS IN CHRONOLOGICAL ORDER

- Magnetic moment of neutron and proton not conforming to expectation from Dirac equation – thus not a point particle – suggestive of an internal structure. What is that structure?
- Experiments at accelerators produce a great deal of resonances, short lived particles decaying strongly, and long-lived “strange” particles decaying weakly. Was there a rhyme or reason for this “particle zoo”? Patterns in masses/properties of hadron states leading to multiplets – a “quarky” periodic table implying some sub-structure. Higher mass particles reflect internal excitations.
- Quark model proposed ~ early 1960s by both Gell-Mann and Zweig; first verified by prediction of existence and properties of Ω. Group-theoretical methods applied to particle physics. Was this a sleight of hand or were quarks “real”?
- Subsequently electron-proton scattering at high momentum transfer $q^2$ deviates from Rutherford scattering. Elastic $e + p$ scattering (SLAC 1960s Hofstadter) at high $q^2$ proton has sub-structure. Deep inelastic nucleon scattering (SLAC 1967-69 Friedman, Kendall, Taylor) obtained confirmation of three spin-$\frac{1}{2}$ point particles within nucleons (called partons at the time) observed to have differing fractional charges consistent with quark model. Quarks, at least $u$ and $d$, were real!
- Quarks or any fractionally charged particles have never been observed outside the nucleus. Quarks are “confined”? This can be explained in terms of “colour” and the need for antisymmetry. Other puzzles arise however from deep inelastic scattering results such as “asymptotic freedom” and “infra-red slavery”. These are elements of quantum chromodynamics (QCD).
- Puzzling nature of strange quarks and strange particles in general continues – introduction of Cabibbo angle to explain how Weak force gives flavour changing decays across generations of quarks explaining Weak decays. Extension of this model through GIM mechanism (Glashow, Iliopoulos and Maiani) predicts a 4th “charm” quark!
- Direct evidence obtained for 4th quark via observation of $J/\psi$ ($c\bar{c}$) (1974) and the rest of the charmonium resonances and mesons (late 70’s); subsequent discovery of charm containing baryons (late 70’s); discovery of 5th quark via observation of $\Psi$ ($b\bar{b}$) (late 70’s) and the rest of the bottomium resonances and mesons (70’s-80’s), eventually leading to bottom containing baryons (90’s). Charmonium and bottomium spectra give insight into QCD potential.
- The observation of stepwise increase in $R_\mu$ in electron-positron annihilation (70’s-90’s) further bolsters both quark model and colour models. Much more evidence for the colour force, quantum chromodynamics (QCD) – gluons in particular – emerges from “jets”. (Jets observed from 70s’; first direct evidence of gluon ’79)

Of the above the single most convincing experimental evidence of quarks is that obtained from deep inelastic scattering.

Collectively the quark model has been proven.

We examine first the “particle zoo” from collisions…

2017/2018 – Revision 1.5 – Cormac McGuinness
QUARK MODEL OF HADRONS – I - see M&S Chapter 3.1

What is the quark model? In summary it is that:

- Quarks are point-like spin-½ fermions with fractional charges of +⅔ or -⅓.

\[
\begin{align*}
Q &= +\frac{2}{3} \\
Q &= -\frac{1}{3}
\end{align*}
\]

- They interact via the electromagnetic force with all other charged particles, e.g. quarks, charged leptons, charged bosons, via the weak force through coupling to the \( W^\pm \) and \( Z^0 \) bosons and in addition to these by a colour force which acts exclusively between quarks and which is mediated by massless gluons.

- All hadrons are composed of quarks, either baryons of 3 quarks, anti-baryons of 3 antiquarks or mesons of a quark and an antiquark. All hadrons are “colourless” or colour singlets.

In 2014 there was the first confirmation of a third possible type of quark combination: a \( qq\bar{q}q \) while 2015 saw the first confirmation of a fourth possible type of quark combination: a \( qqqq \). These still adhere to either having \( B=0 \) or \( B=1 \) and are still colour singlets.

- In each hadron, that is a composite particle composed of quarks and/or antiquarks, the baryon number of the particle is the sum of the individual baryon numbers of the quarks and antiquarks. Thus from above we only have hadrons whose baryon number is +1 (baryons), -1 (anti-baryons) or 0 (mesons). Baryon number is always observed to be conserved in every interaction.

- This colour force is never observed directly as it is a short range force even though the mediators are massless bosons.

- The observable properties of a hadron, including total angular momentum and mass, are due to the interactions of the quarks with each other within the hadron.

- Wavefunctions of hadrons, the Pauli principle, and the colour force will be discussed later.

**Quark properties**

The different flavours of quarks are listed below, together with the additive quantum numbers associated with each quark.

<table>
<thead>
<tr>
<th>Quark</th>
<th>( m ) (MeV/c^2)</th>
<th>Q</th>
<th>B</th>
<th>I(J^P)</th>
<th>I_3</th>
<th>S</th>
<th>C</th>
<th>( \tilde{B} )</th>
<th>T</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>u (up)</td>
<td>1-5</td>
<td>+2/3</td>
<td>1/3</td>
<td>( 1/2(1/2^+) )</td>
<td>( 1/2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
</tr>
<tr>
<td>d (down)</td>
<td>3-7</td>
<td>-1/3</td>
<td>1/3</td>
<td>( 1/2(1/2^+) )</td>
<td>-½</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
</tr>
<tr>
<td>s (strange)</td>
<td>95±25</td>
<td>-1/3</td>
<td>1/3</td>
<td>0(½^+)</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2/3</td>
</tr>
<tr>
<td>c (charm)</td>
<td>1.25 ± 0.09 GeV</td>
<td>+2/3</td>
<td>1/3</td>
<td>0(½^+)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4/3</td>
</tr>
<tr>
<td>b (bottom)</td>
<td>4.20 ± 0.07 GeV</td>
<td>-1/3</td>
<td>1/3</td>
<td>0(½^+)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-2/3</td>
</tr>
<tr>
<td>t (top)</td>
<td>174.2 ± 3.3 GeV</td>
<td>+2/3</td>
<td>1/3</td>
<td>0(½^+)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4/3</td>
</tr>
</tbody>
</table>

- Quantum numbers are as follows: Q =charge in electron units, B =Baryon number, I=Isospin, J=spin, P=parity, \( I_3 = \) Isospin projection, S=Strangeness, C=Charm, \( \tilde{B} = \)Bottom, T=Top, Y=Hypercharge.
**General Properties of Hadrons - see M&S Chapter 3.2**

Although we have never observed isolated quarks more than two hundred bound composite particle states – or hadronic particles – have been observed and all have integer values of electric charge. Consider again the table of properties and quantum numbers that are now associated with each quark:

- All of the above, with the exceptions of angular momentum J or isospin I, are additive quantum numbers whose values are exactly the opposite for the relevant antiquark. i.e. antiquarks have the opposite signs of Q, B, P, I, S, C, B, T, Y
- The charge Q of a composite quark particle or hadron is just the sum of the individual charges of the quarks and antiquarks in a hadron.
- The same is true for all of the additive quantum numbers above in that the total for the composite quark particle or hadron is just the sum of that quantum number from the individual quarks or antiquarks within that hadron.
- For a combination of quarks and antiquarks it is convenient to define $N_u = [N(u) - n(\bar{u})]$, and likewise for the number of down quarks, $N_d = [N(d) - n(\bar{d})]$ etc... but equally this counting of up and down quarks within a hadron can be done by considering the third component of isospin associated with the up and down quarks. $I_3 = \frac{1}{2}(N_u - N_d)$
- The flavour quantum numbers of a hadron or a group of quarks and antiquarks are defined as:
  - Strangeness $S = -N_s =-[N(s) - N(\bar{s})]$
  - Charm $C = N_c = [N(c) - N(\bar{c})]$
  - Bottom $B = -N_b =-[N(b) - N(\bar{b})]$
  - Top $T = N_t = [N(t) - N(\bar{t})]$
  where the signs are in line with the electric charge of the quark
- The baryon number is $B = \frac{1}{3}[N(q) - N(\bar{q})]$ or in terms of the quark numbers
  $$B = \frac{1}{3}[N_u + N_d + N_s + N_c + N_b + N_t]$$
  $$= \frac{1}{3}[N_u + N_d - S + C - B + T]$$
- Isospin and hypercharge were introduced before the quark model came about. Originally introduced by Heisenberg to explain charge–independence of nuclear force i.e. n-n ≈ p-p and because of the near equivalence of the masses of the neutron and proton. Isospin obeys an angular momentum formalism, where the third component of isospin is conserved. Still used in quark model due to u and d quarks.
- Hypercharge is then defined as: $Y = B + S + C + \bar{B} + T$

The third component of isospin as $I_3 = \frac{1}{2}(N_u - N_d)$ $I_3 = Q - \frac{1}{2}Y$

Charge can then be rewritten as: $Q = I_3 + \frac{1}{2}(B+S+C+\bar{B}+T) = I_3 + \frac{1}{2}Y$

Conservation of hypercharge $Y$ and of $I_3$ is equivalent to conservation of $u$ and $d$ quark flavours.
- All flavour quantum numbers, including hypercharge, isospin (I and $I_3$) are conserved in strong interactions. The electromagnetic interaction conserves all except I. The weak interaction conserves charge and baryon number, but not necessarily any quark flavor quantum numbers.

**Isospin multiplets**

- Isospin originally introduced to describe p and n as a doublet of multiplicity $2I+1$ where $I=\frac{1}{2}$ i.e. two different “projections” where proton has $I_3 = +\frac{1}{2}$ and neutron has $I_3 = -\frac{1}{2}$. This was principally introduced because their mass was similar to each other.
- Isospin multiplets are then composite particles with the same total number of up and down quarks or antiquarks. Particles within these multiplets have essentially the same masses.
Arriving at the Quark model

The first discovered baryon other than \( p \) or \( n \), was a \( \Lambda^0 \)... **Baryons (B=1):**

\[
\begin{align*}
\mathbf{p}(udd) \quad Q &= +\frac{1}{2}; \quad J = \frac{1}{2}; \quad I = \frac{1}{2}; \quad I_3 = \frac{1}{2}; \quad S = 0; \quad B = 1 \\
\mathbf{n}(ud\bar{d}) \quad Q &= 0; \quad J = \frac{1}{2}; \quad I = \frac{1}{2}; \quad I_3 = -\frac{1}{2}; \quad S = 0; \quad B = 1 \\
\mathbf{\Lambda^0}(uds) \quad Q &= 0; \quad J = \frac{1}{2}; \quad I = 0; \quad I_3 = 0; \quad S = -1; \quad B = 1 \\
\end{align*}
\]

and many more followed ...

The first discovered mesons were as follows: **Mesons (B=0):**

\[
\begin{align*}
\pi^- (\bar{u}\bar{d}) \quad Q &= -1; \quad J = 0; \quad I = 1; \quad I_3 = -1; \quad S = 0; \quad B = 0 \\
\pi^+ (u\bar{d}) \quad Q &= +1; \quad J = 0; \quad I = 1; \quad I_3 = 1; \quad S = 0; \quad B = 0 \\
\pi^0 (u\bar{u} / d\bar{d}) \quad Q &= 0; \quad J = 0; \quad I = 1; \quad I_3 = 0; \quad S = 0; \quad B = 0 \\
K^- (\bar{u}s) \quad Q &= -1; \quad J = 0; \quad I = \frac{1}{2}; \quad I_3 = -\frac{1}{2}; \quad S = -1; \quad B = 0 \\
K^+ (us) \quad Q &= +1; \quad J = 0; \quad I = \frac{1}{2}; \quad I_3 = \frac{1}{2}; \quad S = +1; \quad B = 0 \\
\end{align*}
\]

and many more followed....

Some flavoured baryons and mesons:

Listed are both some baryons and mesons. The antiparticles of all the particles listed have the opposite quark composition and the opposite flavour quantum numbers, but same mass, spin \( J \).

<table>
<thead>
<tr>
<th>Particle</th>
<th>( m ) (MeV/c^2)</th>
<th>( q )</th>
<th>( B )</th>
<th>( I(J^{PC}) )</th>
<th>( I_3 )</th>
<th>( S )</th>
<th>( C )</th>
<th>( B )</th>
<th>( T )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>938</td>
<td>uud</td>
<td>1</td>
<td>( \frac{1}{2} (\frac{1}{2}^+ )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( n )</td>
<td>940</td>
<td>udd</td>
<td>1</td>
<td>( \frac{1}{2} (\frac{1}{2}^+ )</td>
<td>-( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \Lambda^0 )</td>
<td>1116</td>
<td>uds</td>
<td>1</td>
<td>( \frac{1}{2} (\frac{1}{2}^+ )</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \Lambda_c^+ )</td>
<td>2285</td>
<td>udc</td>
<td>1</td>
<td>( \frac{1}{2} (\frac{1}{2}^+ )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( \Lambda_b^0 )</td>
<td>5624</td>
<td>udb</td>
<td>1</td>
<td>( \frac{1}{2} (\frac{1}{2}^+ )</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \pi^0 )</td>
<td>134.97</td>
<td>( (u\bar{u} - d\bar{d})/\sqrt{2} )</td>
<td>0</td>
<td>1 (0^+)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \pi^+ / \pi^- )</td>
<td>139.57</td>
<td>u\bar{d} / \bar{u}d</td>
<td>0</td>
<td>1 (0^+)</td>
<td>+1/-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( K^- )</td>
<td>493.68</td>
<td>( s\bar{u} )</td>
<td>0</td>
<td>( \frac{1}{2} (0^-) )</td>
<td>-( \frac{1}{2} )</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( D^- )</td>
<td>1869.3</td>
<td>( d\bar{c} )</td>
<td>0</td>
<td>( \frac{1}{2} (0^-) )</td>
<td>-( \frac{1}{2} )</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( B_d^0 )</td>
<td>5279</td>
<td>( b\bar{d} )</td>
<td>0</td>
<td>( \frac{1}{2} (0^-) )</td>
<td>+( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( B_s )</td>
<td>5279</td>
<td>( b\bar{u} )</td>
<td>0</td>
<td>( \frac{1}{2} (0^-) )</td>
<td>-( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

- In addition the charge conjugation parity C is shown for the neutral pion.
- Parity of the hadrons is defined by the parity of the neutron and proton: \( P_p = P_n \equiv 1 \) and that of the higher mesons by \( P_{K} = P_{D} = P_{B} \equiv -1 \)

Some flavourless mesons:

- i.e. all flavour quantum numbers are zero. These mesons are their own antiparticles.
Leptons, Pions and Nucleons - see M&S Chapter 3.3

The first generation of experiments: natural sources

- **Radioactive sources for experiments:** discovery of proton (1919), neutron (1932). Secondary beam experiment in the case of the discovery of the neutron.

- **Cosmic ray sources – cosmic rays explained:** mostly incident protons (~86%) – thus fixed target proton-proton collisions in upper atmosphere create a cosmic ray shower. Contents of individual shower may be dominated by pions, producing as they decay muons, neutrinos, gamma rays, positrons and electrons.

  e.g. diagram to right arising from an example

  \[ p + p \rightarrow p + n + \pi^+ + \pi^+ + \pi^- \]

  Pi-meson or pion decays were quickly characterized as:

  \[
  \begin{align*}
  \pi^+ & \rightarrow \mu^+ \nu_\mu \\
  \mu^+ & \rightarrow e^+ \bar{\nu}_\mu \\
  \pi^- & \rightarrow \mu^- \bar{\nu}_\mu \\
  \mu^- & \rightarrow e^- \nu_\mu \bar{\nu}_e \\
  \pi^0 & \rightarrow \gamma \gamma
  \end{align*}
  \]

- **Cosmic ray sources – first discoveries:** discovery of positron (1932), of muon (1937). Both used Wilson cloud chambers, with triggering mechanisms for photographic detection

- Discovery of pions (pi-mesons) (1946) – balloon experiments with stacks of photographic plates. The process observed was: \[ \pi^+ \rightarrow \mu^+ + \nu_\mu \] followed by \[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]
Pions were then produced at synchrotron in Berkeley by processes such as:
\[ p + p \rightarrow p + n + \pi^+ \]

The second generation of experiments: artificial sources

- Artificial sources such as Linacs, cyclotrons and synchrotrons could then produce intense \( p \) beams, which could then result, through fixed target interactions, in large numbers of pions through interactions such as:
  \[ p + p \rightarrow p + n + \pi^+ \]
  \[ \rightarrow p + p + \pi^0 \]
  \[ \rightarrow p + p + \pi^+ + \pi^- \]

The subsequent interaction of pions with protons (or neutrons) could be studied in hydrogen bubble chambers where the protons (or deuterons) in liquid H\(_2\) (or D\(_2\)) are the fixed targets. Pions, produced in fixed target collisions with metal targets, would be selected from the forward debris by charge-to-mass energy filters using magnets and then accelerated or directed towards the H\(_2\) (or D\(_2\)) bubble chambers.

- The observed rate of pion-nucleon collisions or rate of “events” could be measured and plotted against the CM energy of the pion-nucleon collision thus obtaining a plot such as:

![Graph showing invariant mass vs. cross-section for pion-nucleon collisions]

Each peak or “resonance” corresponds to the resonant production of a short lived stable state according to: \( \pi^+ + p \rightarrow X \rightarrow \) stable hadrons where the width of the resonance is inversely proportional to its lifetime.

The decay of the resonant state \( X \) also observed in bubble chambers with the decay products also giving the invariant mass associated with the fixed-target pion-nucleon collision.
• **Breit-Wigner resonance formula:**

\[
\sigma_\gamma = \frac{\pi}{q_i^2} \frac{2j+1}{(2s_1+1)(2s_2+1)} \frac{\Gamma_i \Gamma_f}{E - E_0 \mp \frac{\Gamma^2}{4}}
\]

Note that different resonances are accessible for $\pi^- + p$ compared to $\pi^+ + p$

Examine labeling of resonances: Name (CM energy) $J^P$
where $J$ is the angular momentum and $P$ the parity.

• What of the first of these resonances? Named a “$\Delta$”

Width (FWHM) of resonance: $\Gamma \sim$ because ………
Position of resonance: $E_0 \ldots \ldots \ldots$ — interpret as rest mass energy of unstable particle

The observed angular momentum $J$ is: \ldots \ldots
The parity $P$ of the particle is: \ldots \ldots +

• In the pion-nucleon cross section plot above the typical FWHM $\Gamma \approx 120\text{MeV} \rightarrow \Delta \approx 10^{-23}\text{s}$
- a timescale typical of a strong interaction. For the “$\Delta$”, the decay rate is $\Gamma \approx 195\text{MeV}$

• As we will learn later the $\Delta$ particles have an “isospin” of $\frac{3}{2}$ while those resonances labeled “$N$” have “isospin” of $\frac{1}{2}$.

• Many pion-nucleon processes are very straightforward to picture as they are excited states of nuclear matter composed of $u$ and $d$ quarks. e.g. $\pi^+ + p \rightarrow \Delta^+(1236) \rightarrow \pi^+ + p$

Draw the Feynman diagram showing the quark lines that would describe the production and decay of this short lived resonant state:

• What about: $\pi^- + p \rightarrow \Delta^0(1236) \rightarrow \pi^- + p$

or also: $\pi^- + p \rightarrow \Delta^0(1236) \rightarrow \pi^0 + n$

• When the fixed target was changed to use deuterium ($D_2$), each of whose nuclei contains both a proton and a neutron, then could observe both:

$\pi^- + n \rightarrow \Delta^-(1236) \rightarrow \text{stable hadrons}$
and
\[ \pi^- + n \rightarrow \Delta^+ (1236) \rightarrow \text{stable hadrons} \]

- Thus from such experiments there emerged four \( \Delta \) particles of the same rest mass energy, same angular momenta (or spin), but differing charges, these forming a family or “multiplet” of \( \Delta, \Delta^+, \Delta^0, \) and \( \Delta^{++} \).

How to explain these? Isospin was introduced and used to explain the reaction pathways and the existence of these “isospin multiplets”.

- These isospin multiplets, by definition had the same “isospin” \( I \) but with \( 2I+1 \) possible manifestations or differing particle states which all had the same mass, but had differing charges, and had differing values of \( I_3 \) the third component of isospin (\( I_3 \) is the projection of the isospin \( I \) onto an arbitrary axis, much like total angular momentum \( J \) is projected onto an arbitrary \( z \) axis to obtain \( J_z \) where both \( J \) and \( J_z \) quantum numbers are conserved.)

- Isospin \( I \) – to an extent – just counts the number of the quarks which have either up or down flavours, but allows for groupings of these where each additional quark can produce two distinct multiplet groupings. \( I_3 \) in effect then just indicates the “net” up/down flavour.

“Particle zoo” and complexity:

How also to explain all of the other higher energy resonances or later those created with differing projectiles or targets, or those other particles created in “associated production” events?

Rules suggested were too complex and failed; needed a simpler unifying mechanism.

Quark model allowed such a satisfactory description e.g.:

- \( \Delta \) multiplet \( I=3/2 \Rightarrow 2I+1=4 \) states – note also \( J=3/2 \)
  \[
  \begin{align*}
  \Delta^- & \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++} \\
  \text{quarks} & \quad (\quad) \quad (\quad) \quad (\quad) \quad (\quad) \quad \text{MeV/c}^2 \\
  I_3 & \quad \_ \quad \_ \quad \_ \quad \_
  \end{align*}
  \]

- \( N \) multiplet \( I=1/2 \Rightarrow 2I+1=2 \) states – note also \( J=1/2 \)
  \[
  \begin{align*}
  p & \quad n \\
  \text{quarks} & \quad (\quad) \quad (\quad) \quad \text{MeV/c}^2 \\
  I_3 & \quad \_ \quad \_
  \end{align*}
  \]
Remarks on rates of processes and lifetimes of particles:

- Rates of reactions for different processes dependent upon mechanism of interaction, similarly lifetime of particles dependent upon mechanism of decay.
- Strong interactions and decays: For the nucleon-nucleon exchange the Yukawa model gives a range \( R \approx 1.4 \times 10^{-15} \text{ m} \), thus for a pion traveling at \( c \), the expected rate of this “strong” process would be \( \sim 10^{-13} \text{s}^{-1} (\sim 120 \text{MeV}) \) and the lifetime of a particle decaying by this “strong” process should be \( \sim 10^{-23} \text{s} \)
- Electromagnetic interactions: Typical lifetimes would be \( \sim 10^{-20} \text{s} \)
- Weak interactions: much longer lifetimes \( \sim 10^{-10} \text{s} \) but ranging up to \( \sim 10^3 \text{s} \) (n)
- **Example**: charged and neutral pion decays

Diagrams shown as Feynman diagrams with the quark composition of the pions.

\[ W \]

(Label the plots above and give the lifetimes)

The first generation of experiments: (continued)

- Discovery of strange particles occurred in cosmic ray experiments – kaons (\( K^\pm, K^0, \bar{K}^0 \)) and hyperons (\( \Lambda^0 \)).

The kaons are strange mesons (\( K^+(u\bar{s}), K^- (\bar{u}s), K^0 (d\bar{s}), \bar{K}^0 (\bar{d}s) \)) while the hyperon is a strange baryon (\( \Lambda^0 (uds) \)), where their quark composition is given in brackets.
Strange particles:

- This “strange” matter was the first evidence of matter containing the third quark. Other low mass strange baryons are the $\Sigma^+ (uus)$, $\Sigma^0 (uds)$ and $\Sigma^- (dds)$.

- Unexpectedly long lifetimes: $\tau_{K^0} = 1.2 \times 10^{-8} \text{ s}$ and thus $c\tau = 3.7 \text{ m}$ with a wide variety of observed decays:

\[
\begin{align*}
K^+ & \rightarrow \mu^+ + \nu_{\mu} & \quad 63\% \\
& \rightarrow \pi^0 + e^+ + \nu_e & \quad 5\% \\
& \rightarrow \pi^0 + \mu^+ + \nu_{\mu} & \quad 3\% \\
& \rightarrow \pi^+ \pi^0 & \quad 21\% \\
& \rightarrow \pi^+ \pi^- & \quad 6\% \\
& \rightarrow \pi^+ \pi^0 \pi^0 & \quad 2\% \\
\end{align*}
\]

In particular the neutral kaon ($\tau_{K^0} \approx 5 \times 10^{-8} \text{ s}$) had an odd range of decay channels. Gell-Mann and Pais surmised that two neutral kaons existed with differing properties and decay channels and very different lifetimes.

\[
\begin{align*}
\tau_{K^0} &= 5 \times 10^{-8} \text{ s} \quad \text{or} \quad \tau_{K^0} = 0.8 \times 10^{-10} \text{ s}, \\
K^0_S & \rightarrow \pi^0 \pi^0 & \quad 31\% \\
& \rightarrow \pi^- \pi^+ & \quad 69\% \\
K^0_L & \rightarrow \pi^+ e^- + \nu_e & \quad 41\% \\
& \rightarrow \pi^+ \mu^- + \nu_\mu & \quad 27\% \\
& \rightarrow \pi^0 \pi^0 \pi^0 & \quad 20\% \\
& \rightarrow \pi^+ \pi^- \pi^0 & \quad 12\% \\
\end{align*}
\]

- Along with the odd lifetimes, these strange particles were always produced two at a time in what became known as “associated production”. To account for this “strangeness” a new additive quantum number $S$ was introduced called “strangeness”.

- Shown here is a hydrogen bubble chamber photograph from the 10” chamber at LBL.

\[
\pi^- + p \rightarrow K^0 + \Lambda^0
\]
All strong interaction were observed to “conserve” this strangeness – this concept being an empirical mechanism introduced to explain the associated production. No strong or electromagnetic process was ever observed to break this rule.

We now explain this in terms of simultaneous creation of equal numbers of strange quarks and strange antiquarks though this model did not exist at that time.

Strangeness was not conserved in decays of these particles by the Weak interaction, but rules were observed related to exactly how the strangeness was not conserved. For example if a hadron was observed to change strangeness with \( \Delta S = \pm 1 \) through a decay involving a lepton, then the charge state of the hadron was also observed to change with \( \Delta Q = \pm 1 \), with the rule being \( \Delta S = \Delta Q = \pm 1 \) and no observation of \( \Delta S = -\Delta Q = \pm 1 \), where the latter processes are said to be “forbidden”.

E.g: \( \Sigma^- \rightarrow n + e^- + \bar{\nu}_e \) is observed but \( \Sigma^+ \rightarrow n + e^+ + \nu_e \) is not

(Explain by quark model and Weak decay with diagrams given that \( S=1 \) for the \( \Sigma^- \) and \( \Sigma^+ \))

- Many, many resonances discovered, each having different CM energy and other properties such as angular momentum, spin, isospin, strangeness quantum numbers and parity and charge conjugation eigenstates as well as decay modes and lifetimes giving a particle zoo.

Experimental validation of the quark model:

- Quark model introduced by Gell-Mann explaining the “zoo” in terms of three quarks: up, down and strange. Prediction (1962) of the \( \Omega^- \) baryon composed only of strange quarks along with its properties – verified in its discovery (1964).

- Formation and decay of \( \Omega^- \) (sss) (BNL 1964):
Reactions for the
Formation: ______________________________

Decay chain: ______________________________
____________________________
____________________________
____________________________
____________________________

When revising draw quark line diagrams of the $\Omega^-$ formation and of each decay:

Formation diagram:

$\Omega^-$ decay diagram:

$\Xi^0$ decay diagram:

$\Lambda^0$ decay diagram:

$K^0$ decay diagram:
Quark structure of low mass baryons composed of u, d, s quarks only

LIGHT BARYONS (L=0)

- The quark models success lay in predicting the Ω and was obtained by realizing the “eightfold way”. This arrangement of all of the then known baryons into an octet and a decuplet as shown firstly according to spin.
- These multiplet diagrams arrange the hadrons according to their strangeness on the y-axis and their third component of isospin on the x-axis.
- The arrangements become more persuasive when it is clear that the rows correspond to particles of much the same mass – these are termed isospin multiplets.
- The lightest baryons are those where the internal orbital angular momentum of the quarks within the baryon is zero, i.e. the quarks within the baryon are not orbiting each other and are relatively stationary within the baryon.
- In general resonances at higher observed rest mass often simply correspond to “excited” baryons where the internal motions of the quarks or antiquarks relative to one another gives non-zero internal angular momentum, and this internal excited state increases the observed rest mass energy of the composite particle that is the baryon.

Table of properties of all 18 light baryons

<table>
<thead>
<tr>
<th>BARYONS</th>
<th>B</th>
<th>I₃</th>
<th>I</th>
<th>Y</th>
<th>S</th>
<th>Q</th>
<th>J⁺ = 1/2</th>
<th>J⁺ = 3/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>uuu</td>
<td>1</td>
<td>+1/2</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>+2</td>
<td>Δ⁺(1236)</td>
<td>Δ⁺(1236)</td>
</tr>
<tr>
<td>uud</td>
<td>1</td>
<td>+1/2</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>+1</td>
<td>p(938)</td>
<td>Δ⁻(1236)</td>
</tr>
<tr>
<td>udd</td>
<td>1</td>
<td>-1/2</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>n(940)</td>
<td>Δ⁰(1236)</td>
</tr>
<tr>
<td>ddd</td>
<td>1</td>
<td>-1/2</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>Δ⁻(1236)</td>
<td>Δ⁻(1236)</td>
</tr>
<tr>
<td>uus</td>
<td>1</td>
<td>+1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
<td>Σ⁺(1189)</td>
<td>Σ⁺(1385)</td>
</tr>
<tr>
<td>uds</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>Σ⁰(1192), Λ⁰(1116)</td>
<td>Σ⁰(1385)</td>
</tr>
<tr>
<td>dds</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>Σ⁻(1197)</td>
<td>Σ⁻(1385)</td>
</tr>
<tr>
<td>uss</td>
<td>1</td>
<td>+1/2</td>
<td>1/2</td>
<td>-1</td>
<td>-2</td>
<td>0</td>
<td>Ξ⁰(1314)</td>
<td>Ξ⁺(1530)</td>
</tr>
<tr>
<td>dss</td>
<td>1</td>
<td>-1/2</td>
<td>1/2</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>Ξ⁻(1321)</td>
<td>Ξ⁻(1530)</td>
</tr>
<tr>
<td>sss</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>-3</td>
<td>-1</td>
<td>Ω⁻(1672)</td>
<td>Ω⁻(1672)</td>
</tr>
</tbody>
</table>

Definition: \[ Y \equiv B+S \text{ (or strictly: } Y \equiv (B+S+C+\tilde{B}+T)) \]
\[ I_3 = \frac{1}{2}(N_u - N_d) \quad I_3 = Q - \frac{1}{2}Y \quad \Rightarrow \quad Q = I_3 + \frac{1}{2}(B+S+C+\tilde{B}+T) \]
BARYONS: $J=1/2$  
(low mass baryon octet)

BARYONS: $J=3/2$  
(low mass baryon decuplet)
MESONS: $J=0$ (low mass (uds) meson nonet)

MESONS: $J=1$ (low mass (uds) meson nonet)
What is the organizing principle of these multiplet diagrams…?

- Horizontal axis is $I_3$
- Vertical axis is either strangeness $S$, or almost equivalently hypercharge $Y=B+S$
- Composite particle values of $I_3$ and $S$ purely dependent upon the number of up, down or strange quarks or antiquarks within the composite particle, and determines the position of the particle on the plot.
- The masses of the isospin multiplets of the composite particles or hadronic states depend in a non-trivial manner on:
  - Firstly, upon the sum of the masses of the individual quarks
  - Secondly upon the sum of pairwise terms that depends upon the expectation values of the spins of the quarks coupled with the quark masses via a colour magnetic interaction…
    - This is more complicated if there is additional orbital angular momentum corresponding to motion of the quarks relative to one another internally within the composite particle.
    - These terms are thus strongly dependent upon the total angular momentum of the particle.
    - Compare e.g. mass difference between $p$ and $\Delta^+$. 

Some discussion points on identification of particles:

- Observing new particles is dependent on centre of mass energy of collision.
- Identifying new particles dependent upon centre of mass energy of collision.
- Expect an energy threshold for production, and an invariant mass for the new particle.
- Reaction products: their identification and the measurement of their vector momenta and energy – all of these are needed for reconstruction of decay.
- Decay pathways, identification of momenta and energy – can reconstruct rest mass of decaying particle.
- Rules for production, parity ? J ? isospin ?
- Empirical observations of new conservation laws e.g. conservation of strangeness – now all explained by the quark “flavours” and their interactions.

Note that not every decay is a convenient 2-body decay!
When considering the kinematics of a 3-body decay there is a whole lot of complexity in the relativistic kinematics whereby you must calculated the center of mass energies of pairs of the three decay particles. Not convenient for an undergraduate course, and no 3-body decay will be examined to this level of detail in this course, but the basics of the procedure are given here for your interest.

Not for examination but download the online pdf of this handout to see the extra page on 3-body decays and Dalitz plots. Hence we will stick to 2-body decays for any exercises.

Consider again the 3-body decay of the muon from the above analysis (earlier exercise). It can be seen clearly why the momentum of the electron produced in the decay is almost uniquely determined in the light of two (~) zero mass neutrinos as the allowed area shrinks to nearly a point
Extreme example of energy-momentum analysis: Analysis of 3-body decay via Dalitz plot

In rest frame of particle $M$, the momenta of three decay particles lie in a plane. Defining $p_1 = p_i + p_j$ and $m_{ij}^2 = p_i^2$ and $m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2$ and $m_{12}^2 = (P - p_1)^2 = M^2 + m_1^2 - 2M_E$ where $E_3$ is in the rest frame of $M$.

Plotting the ranges of the possible combined momenta of each pair of particles is a Dalitz plot.

![Dalitz plot example](image)

**Figure 38.3:** Dalitz plot for a three-body final state. In this example, the state is $\pi^+ \pi^0 \eta$ at 3 GeV. Four-momentum conservation restricts events to the shaded region.

There is uniform scatter in the shaded region if there is direct three-body decay.

Otherwise non-uniform scatter reveals an indirect process through competing intermediate states and thus fingerprints details of the matrix elements involved and the constructive or destructive interference in the amplitude due to the intermediate resonance states.

Hence we will stick to 2-body decays for any exercises.

Consider again the 3-body decay of the muon from the above analysis (earlier exercise). It can be seen clearly why the momentum of the electron produced in the decay is almost uniquely determined in the light of two ($\sim$) zero mass neutrinos as the allowed area shrinks to nearly a point.