7. Motivation for Einstein's Theories of Relativity

The theory of Special Relativity - 1905
The theory of General Relativity - 1915

7.1 Unification of Electricity and Magnetism

Brief History of findings:
- Greek philosophers: electric shocks, magnetic minerals
  - Thought of as unrelated phenomena until 19th century
- Cavendish (1771-73), Coulomb (1785-91): force between two charges (similar to Newton's gravitational law)
- Oersted (1819): electric current produces magnetic field
- Ampère: forces between two parallel currents, magnetic in nature
- Faraday: magnetism produces electricity (induction); concept of fields

7.2 Maxwell's equations

Maxwell (1873) summarises these results in four equations (here stated for vacuum)

\[
\begin{align*}
\nabla \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \quad \text{(Gauss's law for } \vec{E}) \\
\nabla \cdot \vec{B} &= 0 \quad \text{(Gauss's law for } \vec{B}) \\
\n\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law)} \\
\n\nabla \times \vec{B} &= \mu_0 \left( \vec{j} + \frac{\partial \vec{E}}{\partial t} \right) \quad \text{(Ampere's law)}
\end{align*}
\]

VECTOR NOTATION

\[
\nabla \cdot \vec{E} &= \rho / \varepsilon_0 \\
\nabla \cdot \vec{B} &= 0 \\
\n\nabla \times \vec{E} &= -\varepsilon_0 \frac{\partial \vec{B}}{\partial t} \\
\n\nabla \times \vec{B} &= \mu_0 \left( \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)
\]

7.2 (ctd) - Maxwell's equations

- Maxwell showed that these can be reduced to wave equations for E and B:

\[
\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0, \quad \nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0; \quad (32.15)
\]

- The speed \( c \) of the wave is given by:
  - and is identified as the speed of light
  \[
  \frac{1}{c^2} = \varepsilon_0 \mu_0
  \]

- Solutions of wave equations

\[
\begin{align*}
\vec{E} &= \hat{E} e^{ik(x-ct)}, \\
\vec{B} &= \hat{B} e^{ik(x-ct)}
\end{align*}
\]

(Read YF Sections 32.1 & 32.2)

Recall wave equation in 1D:

\[
\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2} \quad (15.12)
\]

(Read YF Sections 15.3)

7.3 The speed of light

Measurements of the speed of light

- First attempts by Galilei (lanterns at mountain tops!)
- Ole Roemer (1676): orbital period of Jupiter moon Io fluctuates with distance between Jupiter and earth (22 minutes)
  - speed of light \( c \sim 2.14 \times 10^8 \text{ m/s} \)
- Fizeau (1849), using spinning cogwheel: \( c \sim 3.133 \times 10^8 \text{ m/s} \)
- Heinrich Hertz (1887) measure speed of light by sending and receiving radio waves
- Today: \( c \sim 2.99792458 \times 10^8 \text{ m/s} = 299792.458 \text{ km/s} \)
7.3 The question of the ether

- Sound waves and water waves require a medium, what about light?
- Aristotle: "nature abhors a vacuum"
- Descartes: light as an instantaneous disturbance in the ether (plenum)
- Maxwell: "We can scarcely avoid the conclusion that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."

- Following the notions of Aristotle and Descartes ⇒ some medium is required to propagate a disturbance - the "ether"

But ether could not be found experimentally and was not required theoretically.

- Michelsen and Morley (1887): attempt to detect the passage of earth through the ether by measuring speed of light waves in different directions; no effect!
- Fitzgerald (1889), Lorentz (1892) explained this null result by suggesting that an object moving through ether would be contracted!

7.3 Michelson-Morley experiment

- Interferometer - observe interference fringes dependent upon optical path length difference between the two arms of the interferometer.

If ether existed, then if the earth was travelling through the ether it was expected that there would be a difference in the effective speed of light in one arm of the interferometer ⇒ change in interference fringes

No such change seen ⇒ a null result.

(Fitzgerald (1889), Lorentz (1892) explained this null result by suggesting that an object moving through ether would be contracted!)

7.3 Einstein's thought experiment

- Galilean transformation (Young and Freedman, Figure 37.02)
- Galilei transformation in principle valid for all velocities V of moving frame of reference, but what happens to Maxwell's equations at V=c?
  - Time dependence drops out!
  \[ E = E e^{ik(x-(c-V)t)} \]
  - But if E doesn't change, also B doesn't, Maxwell's electromagnetic theory would collapse!
  → Einstein decides to reinvestigate Galilean transformation

7.4 Einstein's postulates

1905: Two Postulates
1. all physical laws valid in one frame of reference are equally valid in any other frame of reference moving uniformly relative to the first (inertial frame)
2. speed of light in vacuum is the same in all inertial frames, regardless of the motion of the light source.

Thus there is no frame at which c=0, no frozen electromagnetic wave.

Starting consequences:
- Space and time are relative - change in space and time must make up for motion of light source in frame where source is moving
- Relativity of simultaneity
- Time dilation
- Length contraction
- \( E=mc^2 \) as consequence of the new Lorentz transformation

(PY1P20 Origins of Modern Physics Lecture 7)

2009/2010 - Cormac McGuinness
7.4. Motivation for Einstein’s Theory of Relativity

All these predictions of special relativity have been confirmed by experiments.

In particular, the equivalence of mass and energy: i.e.
\[ E = mc^2 \]
as consequence of the new Lorentz transformation

This will be relied upon in our next two lectures on nuclear physics.

7.5. Motivation for Einstein’s General Theory of Relativity

- Newton’s Law of Gravity

\[ F = G \frac{m_1 m_2}{r^2} \]

- Instantaneous force: action at a distance
  - Surely the signal indicating that one mass has moved must propagate through space? (to Einstein instantaneous action at a distance should not be possible)

- But how is this force transmitted?

- Also: strictly speaking two definitions of mass: inertial mass (Newton’s second law, \( F = m_{\text{inertial}} a \)) and then also gravitational mass, but experiments show they are the same.

7.5. Principle of equivalence

Recall postulate of special relativity (1905):
1. all physical laws valid in one frame of reference are equally valid in any other frame of reference moving uniformly relative to the first (inertial frame)

The more general case relates two frames of reference which are not moving uniformly with respect to one another, i.e. one frame of reference may be accelerating with respect to the other.

Thus: general relativity (1907-1915)
1. all physical laws valid in one frame of reference are equally valid in any other inertial frame of reference

This is the EQUIVALENCE PRINCIPLE

Impossible for astronaut to distinguish whether they are in a gravitational field or in an accelerating rocket.

7.5. General Theory of Relativity

Recall postulate of special relativity (1905):
1. all physical laws valid in one frame of reference are equally valid in any other frame of reference moving uniformly relative to the first (inertial frame)

The more general case relates two frames of reference which are not moving uniformly with respect to one another, i.e. one frame of reference may be accelerating with respect to the other.

Thus: general relativity (1907-1915)
1. all physical laws valid in one frame of reference are equally valid in any other inertial frame of reference

This is the EQUIVALENCE PRINCIPLE

Einstein needed to use more complicated tensor calculus to express this mathematically as well as a new flexible coordinate frame or geometry.
7.5. General Theory of Relativity

- Einsteins field equation for General Relativity is often written as:

\[ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

Space time curvature tensor expressed as $G$ - related to the stress-energy tensor $T$ relating to the distribution of mass in the spacetime.

Described by Wheeler as:
“spacetime tells matter how to move: matter tells spacetime how to curve”

Space is distorted by gravitational fields (stars, black holes),
Or: space is no longer absolute, but defined by matter

Young and Freedman, Figure 37.25)

Gravitational waves

An implication of Einstens' general theory is the existence of gravitational waves. Pulsars had previously been discovered by Anthony Hewish in 1967 (Nobel 1974) and Jocelyn Bell-Burnell. Pulsars periodically emit bursts of radio waves.

In 1974 Russell Hulse and Joseph Taylor Jr. (Nobel 1993) discovered a binary pulsar.
The pulsar bursts observed showed a periodic Doppler effect indicating the source was orbiting another body, whose mass could be calculated.

By 1978, their observations of the period was decreasing; thus the mutual orbit of these two massive bodies was decaying. The energy-loss of the gravitational system is not explainable by Newton but only via emission of energy as gravitational waves.

The observed rate of energy loss exactly matches the predictions of Einstein's general theory of relativity for these two massive objects in mutual orbit.

We have still to detect gravitational waves, but we know they must be present.


Gravity Probe B

Visit http://einstein.stanford.edu for the story of what was planned to be the most sensitive test of two other predictions of General Relativity - geodetic effect and inertial frame dragging.

Gravity Probe B was a relativity gyroscope satellite experiment by NASA launched in 2004. It is said to contain four of the most perfect spheres ever made.

These form gyroscopes that should maintain their axis of spin throughout the lifetime of the satellite - thus defining a constant vector in an orbiting body.

The precession of these axes of spin in the four gyroscopes would be measurable and compared to predictions of General Relativity.

However - an experimental problem in manufacture left the spheres being charged which introduced an unwanted electromagnetic effect.

After several years this source of error has been fully measured and these two additional predicted effects of General relativity have been measured to better than 0.5% and 10% accuracy respectively and found to agree with predictions.