A list of specific questions from students and their answers that I have received and answered by e-mail

QUESTION 1: asked 30/1/2013

Quoting D:

- I have attached a question I have on lecture 5 slide 10. It's nothing major I just can't figure out one of the lines of maths. In the bottom left of the slide there’s the following equation

\[ \frac{\vec{p}_\mu}{m} = \sqrt{E^2 - m^2 c^2} \]

I presume it’s derived from

\[ E^2 = p^2 c^2 + m^2 c^4 \]

But I can’t workout where the extra \( c^8 \) goes

- A: The equation on the bottom left should have read: 

\[ |\vec{p}_\mu| = \sqrt{E^2 - m^2 c^2} / c \]

My bad – thanks for pointing it out.

QUESTION 2: asked 21/3/2013

Quoting A:

- I feel like this is a very simple question, but I cannot figure it out. I am going back over the Worksheet 2 questions and for the theory part of Q2 "Discuss, in each case, the corresponding scattering amplitudes or matrix elements", I am stuck. I know what the matrix element formula is but I just cannot seem to correctly apply it to the question or more importantly how to discuss it.

- A: I regret that perhaps "matrix element" is the wrong term to use, because in this module we are not discussing the full bra-ket derivation of any of these scattering amplitudes or probabilities, i.e. a proper use of perturbation theory beginning with the Dirac equation. Mostly we discuss things in terms of vertices and the (matrix element/amplitude) probability that a particular vertex contributes to the overall likelihood of the process under discussion.

I have assumed that students are sufficiently conversant with quantum mechanics to know that the probability of any process is dependent upon the matrix element i.e. some bra-ket of the form 

\[ \langle \psi_1 | H' | \psi_2 \rangle \]

where \( H' \) is the perturbing hamiltonian and this quantity is what I refer to as the matrix element. This is also referred to as the quantum mechanical amplitude. The probability is then this matrix element squared or amplitude squared.

In any electron photon 3-pt vertex, the matrix element or amplitude is in proportional to "\( \sqrt{\alpha} \)"; the probability is then for this 3-pt vertex in proportion to "\( (\sqrt{\alpha})^2 = \alpha \)"

The simplest electron-electron scattering will be an exchange of a photon between two electrons and have two such 3-pt vertices. As each contributes "\( \alpha \)" to the probability of the overall diagram, then the overall process has a probability which is proportional to "\( \alpha^2 \)" - sometimes written as of order alpha squared or "\( O(\alpha^2) \)"

The simplest electron-proton scattering is exactly the same if we consider the proton as a composite particle of equal charge to the electron.

My questions may often be complicated, but I am often looking for simple answers...
Quoting A:

- I am presently going back over through Worksheet 3 and I've become unsure on some parts of Q3. I know you did the question in the tutorials recently but I would just like to check my answers, so that I know I have the concepts right in my head.

Q: Apart from the elastic and deep inelastic scattering at very high momentum transfer that could be studied by HERA, describe the likely physics of an electron (or positron) collision with the constituents of a proton

A?: Is it sufficient to say that an electron and quark could solely interact resulting in all the energy of the electron going onto the quark?

- Firstly, not all! Inelastic scattering for sure, with large transfers of momentum, yes, but certainly not all of the energy of the electron being transferred. Conservation of momentum of target and projectile of unequal masses (m and M) will not allow for all the energy of the projectile to be transferred to the target. (simple collision mechanics).

No, it would not be sufficient to simply say the sentence you gave above. A description of the deep inelastic scattering interaction, whether wholly by text or partly by diagram, would indicate the "struck" quark exiting the proton and then an indication of what happens as that quark separates from the two others not involved directly in that momentum exchange. i.e. "struck" quark separate from other two quarks, hadronisation occurs and at least two jets form.

But the question really concerns the manner or method of interaction between projectile and proton constituent, the most likely interaction being electromagnetic with an exchange of a photon between the charged projectile and a charged constituent of the proton (i.e. a quark). Less likely but also possible as first order diagrams are exchanges of W and Z bosons.

- Q: At what ranges of centre of mass energies might one diagram begin to dominate over the other, or to assume greater importance?

- A: The interaction involving the photon occurs at lower energies, while the interactions with the Z and W boson occur at higher energies. A diagram will dominate when the centre of mass energy rest mass energy of the exchange boson. Photons have no rest mass while the rest masses of the Z and W bosons are 91 and 80GeV/c^2, respectively. Therefore if 80>CMS>0 GeV/c^2, photon exchange will dominate, while with 91>CMS>80 GeV/c^2 W boson exchange will dominate and Z boson exchange will dominate when CMS>91GeV/c^2.

(Note all energies should be quoted in GeV, and masses in GeV/c^2. You are mixing the two here but I follow your meaning. Further note added: CMS in this discussion means Centre of Mass System energy.)

It would be unfair to say that massive boson exchange will automatically dominate at higher CMS energies, as $\alpha_{EM}$ will still be greater than $\alpha_{W}$ or $\alpha_{Z}$ at all energies. However these massive boson exchange diagrams will certainly assume greater importance as corrections as these diagrams become kinematically more and more favourable. However, for CMS >91 GeV, the W boson exchange will continue to be more important than Z boson exchange.

Why? $\alpha_{Z} = \alpha_{W} \tan^2(\theta_{W})$ i.e. $\alpha_{Z} < \alpha_{W}$ dependent of course on the value of the weak mixing angle. The weak mixing angle is not very large, in fact $\theta_{W} \sim 28^\circ$, so $\tan^2(\theta_{W}) \sim 0.16$.
In regards to the very last question part of Q3:

Q: How does this contrast to electron-positron or proton-antiproton collisions?

A few of us, have become confused about what happens during p-pbar annihilation. We know that jets are created and I presume it is done in the same way as the e+e- annihilation, that is:

During e-e+ annihilation a qqbar pair is created and they travel in the opposite direction. As they separate the energy stored within the colour field begins to increase. This increase is linear with the distance of separation. If E>2 m_q a new qqbar pair is formed and the process further continues. Then as the energy decreases you are left with hadrons (hadronisation), resulting in jets.

I then go on to say that the main contrast is that the jets occur when the centre of mass energy is low and in the case of e-e+ --> hadrons the CMS range is 15-40GeV.

In e’e’ annihilation, in and around CMS ~ 91.2 GeV - a Z^0 diagram dominates via Breit Wigner resonance production of a real Z^0 which then decays, with the Z^0 replacing the photon at the vertex where the e’e’ annihilate. Once you go above CMS > 2* 80 GeV, then W^-W^+ production occurs via any one of three diagrams involving photon (\gamma), Z^0 or neutrino (the latter a differently shaped diagram –see Lecture 17 slide 5). At all CMS energies direct q-q'bar production is possible.

In p-pbar collisions, at low CMS energies, fragmentation, scattering and jets result from the p-pbar interactions, mostly via strong interactions involving gluons, but also through electromagnetic annihilation of quarks with their respective antiquarks (of same flavour). No W's are formed from q-q'bar interactions until the constituent quarks and antiqarks themselves have sufficient CMS energies for some such q-q'bar collisions to occur at the correct mass energy corresponding to the production of either a W^- or W^+ boson. (Here q denotes one flavour, and q' denotes a different flavour.)

At similar slightly higher CMS interaction energies between a quark and an antiquark a q-q'bar interaction can produce a Z^0 boson.

Note that the distribution of parton momenta within a proton means that the CMS energies of individual q-qbar or q-q'bar interactions are not well defined at a single value but are instead spread over a wide range of energies, as each q can take a fraction of momentum f(x) almost anywhere between 0 and 1.

As the nominal energies of p-pbar interactions increases the "strong" background dominates more and more as the average CMS energy of q-qbar interactions moves to higher energies and away from the resonances where Z's and W's can be directly produced.

(Note at higher interaction energies the fraction f(x) shifts slightly lower, but this is not especially relevant for this discussion here.)

The above could all be expressed in one or two diagrams and a sentence or two much more quickly than I have done so here.

In short, the differing collisions change in different ways as the CMS energy is increased.
QUESTION 4: asked 12/4/2013

Quoting R:

- Q: I am just wondering is there a definitive list of particles for which you'd advise us to know their quark composition? I imagine the list on page 3 of handout 3 is a good place to start, but I'm not sure as to whether that includes all of them, or not.

- The list you mention may be where you should start, but not where you should end!

  Pages 9, 10 and 11 of the same handout describing the low mass baryon octet and baryon decuplet and the two meson nonets are very necessary.

  (By "low mass" is meant that all of the internal orbital angular momentum numbers of the quarks within these hadrons are all zero i.e. L=0.)

  I don't expect that you know the names and symbols of each and every one of these hadrons (nor their masses), but I do expect that you know the quark compositions as you need to know the organising principles behind these diagrams and why there are nine, eight and ten in these different multiplet diagrams. i.e. how quarks can combine to form hadrons.

  Not knowing these organising principles and the reasons behind this quark-like periodic table would be a severe handicap.

QUESTION 5: asked 17/4/2013

Quoting C:

- Q: Would it be possible to get PDFs of the slides from all of the lectures. While I really appreciate the stuff online, (it is really helpful), PDFs would be handy if you don’t want to have your computer to hand and just wanted to check a quick quark diagram or something.

- Dear C

  As I explained at the beginning of the course, the handouts (of which I only ever handed out in hardcopy format the first at the first lecture), are prepared to be duplicates of the material presented in the lectures - they contain all the relevant materials on the slides from each lecture, but without all the fluff and the repetition that may be found in some of the lecture slides. As such they are more compact, and less wasteful of paper than the lecture material itself if those slides were printed out on paper. Further, the handouts also, purposely have gaps for students to fill in the details of equations or of quark line diagrams as they arise during the lecture or as they re-encounter the material when revising or reviewing the lecture. Further annotations can be made on these notes by each student as and when necessary. These ARE the printed material which I am happy to provide.

  I purposely avoid giving PDFs of the lecture slides for this reason because I want students to consult these handouts, and PREFERABLY also consult the source material for the course which is the textbook. I do NOT want to make available TWO sets of notes, which students may then feel compelled to study both, when as far as I know the handouts ARE the set of notes - the contents of the lectures clearly, particularly the spoken part allows me to give added nuance or explanations or visual material where appropriate that adds to the information in the handouts or adds to the students understanding especially when I am asked questions.
(If you find an area or section where the handouts are lacking when compared to the lecture slides then please let me know.)

I apologise for saying no, but perhaps you were unaware of the existence of the prepared HANDOUTS?

If the later handouts have gotten lost on the busy course webpage then see perhaps a less confusing webpage at https://www.tcd.ie/Physics/people/Cormac.McGuinness/Teaching/PY4P02bare.html

QUESTION 6: asked 18/4/2013

Quoting C:
- Q: I was doing your 2005 Q5 question and in it you ask why the long and short lived neutral kaons have no corresponding antiparticle.

Upon checking your notes and a quick glance through M&S I can’t find an answer. Has this been taken off the course and if not what is the reason for it.

- The answer is that particle and antiparticle must have opposite additive quantum numbers (e.g. charge, etc...), the exact same rest mass as each other, the exact same lifetime as each other, the exact same decay routes (charge conjugate decay routes) as each other, and the exact same branching ratios into these decay routes as each other.

Now ask yourself, is it the case that the long and short kaons satisfy all (or indeed any) of these?

Perhaps there is no explicit statement of these reasons why we can state that these have no antiparticles in M&S, but I certainly implicitly describe this as being the case in the discussion in the lecture on CP violation. In fact I do explicitly state this on slide 10.

As it happens I have made it clear in that particular lecture that I would not be asking about CP violation this year, thus sparing students from at least 5% of the course material.

QUESTION 7: asked 20/4/2013

Quoting A:
- I have a question about 2011 Q4b the second reaction to be precise: muon neutrino + proton -> muon + proton + pi-plus.
  \( \nu_\mu + p \rightarrow \mu^- + p + \pi^+ \)

I see that you have answered a similar question before but I am still unsure of the d dbar part. In my understanding gluons are not involved in Weak processes, however they are clearly essential here. X and I have been discussing it and X came up with a similar diagram as (to the right). We are wondering if this is correct? If this is correct, I then presume that the diagram for the first reaction is pretty much the same except with a Z^0 boson. (\( \nu_\mu + p \rightarrow \nu_\mu + p + \pi^0 \))
It is a Weak process in that for it to happen it entirely depends upon the muon exchanging a W with a quark in the proton. This process is always unlikely to happen, i.e. it has a low probability and there is a low cross-section for observing these events as a result of the requirement of the exchange of a massive boson (range of Yukawa exchange). These processes are observable because one can have beams of $10^6$-$10^{10}$ neutrinos/sec crossing a detector for a very long time, and can check for these unlikely events. The majority of neutrinos pass by without interactions but then the interactions when they are observed can (potentially) stick out like a sore thumb (provided there is no other background processes which could be confused with the process that is desired to be observed). See lecture 15 and last few slides for a brief discussion of such experiments and also intro to Ch 8 in M & S.

The diagram you have drawn for $\nu_\mu + p \rightarrow \mu^- + p + \pi^+$ is fine and is correct in all essentials.

To explain: Firstly lepton family number must be conserved by the muon neutrino, and hence it can only change into a muon by emitting a $W^+$ and thus also conserving charge at the $\nu_\mu W$ (numuW) vertex. The $W^+$ boson then interacts with one of the quarks in the proton. The "struck" quark, that is the $d$ quark which absorbs the $W^+$, is catapulted out of the proton because of the momentum exchange between the incoming muon and the quark. The momentum transferred is mediated by the exchange boson. If the momentum transfer to the quark is high enough, then this momentum transfer may fragment the proton smashing it apart. As we have learned if you input energy to forcibly separate quarks, the potential energy of the quark-antiquark combinations increases as they separate. This potential energy and the colour confinement principle leads to hadronisation or formation of new quark-antiquark pairs (of the same flavour) through this acquired potential energy where the just created anti-quark accompanies the separating quark and the just-created quark must hang out with the other two quarks. Only in this way can colour confinement hold with a meson and a baryon emerging from the collision each of which have a net colour quantum numbers of zero.

Clearly this "part" of the diagram is a QCD or strong process which requires gluons. However, one should recognise that the gluons in this diagram are "after the fact" of the original Weak interaction, that is the exchange of the $W$ boson.

The diagram for the first part of that question: $\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0$ is indeed an exchange of a $Z^0$ with one of the quarks in the proton, again knocking out the struck quark.

Finally, the exam question asks: "Should the rate or cross section of these processes be similar to each other? Explain."

The simple answer is YES, the probabilities or cross sections of each process should be similar at least to an order of magnitude for the same incoming muon neutrino energy. This was proven experimentally by muon beam experiments in bubble chambers which measured both W exchange processes and Z exchange processes where these could be seen to be different observable events as noted above.

The probability for the W exchange diagram will be proportional to:

$$(\alpha_W)^2 \cdot |V_{ud}|^2 \cdot (\alpha_S)^2 \cdot \text{(number of d quarks in proton)} \quad \text{[as u quarks cannot absorb a } W^+ \text{!]}$$

The probability for the Z exchange diagram will be proportional to:

$$(\alpha_Z)^2 \cdot (\alpha_S)^2 \cdot \text{(number of u and d quarks in proton)}$$
One should be able to state that $\alpha_Z$ is of the same order of magnitude as $\alpha_W$, but is a little less as determined by the Weak mixing angle. One could be more exact but this is sufficient for the answer in the exam.

(For exactness one could state that $\alpha_Z = \alpha_W \cdot \tan(\theta_W)^2$; As $\theta_W \sim 30^\circ \Rightarrow \tan(\theta_W)^2 \sim 0.333$ - thus once one takes into account that any of the three quarks in the proton can absorb the $Z^0$ (multiplying that probability by three) then one might expect the rates of the two processes to be equal excepting for the factor of $|V_{ud}|^2$. However, there is in fact two subtly different ways of drawing the W exchange diagram involving the ordering of gluon interactions which multiplies this by a factor of two. This then gives the ratio quoted in Lecture 15 Slide 23 which demonstrated that $Z^0$ exchanges happened about as frequently as $W^\pm$ exchanges.)

QUESTION 8: asked 22/4/2013

Quoting A’:

- I have two questions from the 2010 examination paper. Q3 (b) I said the lifetime is approx $10^{-23}$ seconds and that this is the typical timescale for a strong interaction. Is this enough information?
- No. That is a statement but not an explanation. To explain you must give a reason for that number. A sufficient explanation might be that the limit of the range of the strong interaction is of the order of the size of the nucleus, i.e. about $10^{-15}$ m as otherwise nuclei would be bigger. The maximum speed that any particle could possibly have while travelling over that distance would be the speed of light, so the typical time for a strong interaction might be given by the distance of $10^{-15}$ m divided by the speed of light giving $\sim 10^{-23}$ s

- Q3(c) I can’t seem to find the answer for this and I was wondering if you could point me in the right direction.
- Negative kaons have strange quarks and up anti-quarks; positive kaons have strange anti-quarks and up quarks.
If one of these kaons were to undergo an absorption event, as opposed to simply a scattering event, when interacting with a nucleon (i.e. either a proton or neutron), then it would require the anti-quark in that meson being able to annihilate with a quark of the same flavour in the nucleon. This can happen for one charged kaon, but not for another. You should be able to fill in the rest...?

In my notes it is mentioned in the section on strange interactions with matter in handout 5 page 19, and again in the context of neutral kaons systems and the unexpected regeneration of the short lived neutral kaon when a long lived neutral kaon beam interacts with matter in the lecture on CP violation in neutral kaon systems.

QUESTION 9: asked 22/4/2013

Quoting D:

- I am in your SS High Energy course and just wanted to ask you a question on something in handout 7.
When talking about Pion decays you say that pions decay into a muon and neutrino is much more likely then pion decay into positron and neutrino because the chirality of the two decay products of the pion must be opposite and because muons are non-relativistic both chirality...
states are allowed. But if the pion decays into a positron and neutrino, the positron will be right handed and the neutrino will be left handed and so I don't see why this is a less likely decay for the pion? I hope I've described my question in enough detail.

- Firstly I should emphasise that pions decay into muons of same charge and two neutrinos.

Secondly, the answer is due to the spin-dependence of the Weak interaction which does not get discussed in any detail in our course other than the helicity of the neutrino as ALL fermions (not just neutrinos) prefer to be lefthanded in a relativistic limit when produced by a Weak interaction. There are two pages of the M & S textbook that specifically deal with pion and muon decay and discuss this in general. This is section 10.1.3. If you wish to learn more about the "V-A" or vector/axial nature of the Weak interaction then consult Perkins.

**QUESTION 10: asked 22/4/2013**

Quoting C:

- In 2008 when you ask to prove that the photon cannot initiate pair production in a vacuum, is a mathematical derivation needed, or will a simple statement that it's needs a third party particle in to interact with and this isn't available in vacuo suffice?
- I think consideration of the definition of the word "prove" is sufficient to answer your question?

If you can mathematically show that you cannot conserve momentum and energy simultaneously in an e-e-gamma 3pt vertex then you will have proven that a photon cannot initiate pair production in vacuum - provided of course one accepts the principles of the conservation of energy and momentum. So far these principles have never been disproved!

This exercise is completed in the second lecture, on pages 12 and 13 of Martin and Shaw, and on pages 10 and 11 of my first handout.

**QUESTION 11: asked 22/4/2013**

Quoting C:

- Just when it comes to the 2011 Q1c, when you asked for a very rough estimate of the threshold energy to produce the W boson, would \( E^2 = (mc^2)^2 + p^2c^2 \) be sufficient, or is it necessary to use the following formula? [http://i.imgur.com/QfWnsCs.png](http://i.imgur.com/QfWnsCs.png)

- Burn that formula! Where did you get it? That formula is wrong on so many fronts and can certainly not apply to this. Why is there a pion involved anyway?

The answer does not require a formula but just some reasoning.

Imagine in a thought experiment that isolated free quarks were possible. Now imagine a linear collider where an up quark is accelerated to collide with a down anti-quark say (or an anti-up to collide with a down). If an up were to combine with an anti-down they could form a \( W^+ \) boson (alternatively if a down were to combine with an anti-up then it could form a \( W^- \) boson).

Now, in the case of an electron -positron collider the production of \( Z^0 \) particles only occurs when the combined center of mass energy of the colliding particles is tuned at or near to the rest mass energy of the \( Z^0 \) particle and this occurs via an electron-positron-\( Z^0 \) 3pt-vertex.
Just as in the case of an electron positron collider, in our imaginary up-antidown collider the two colliding quarks could only produce a $W^+$ boson via an up-antidown-$W^+$ 3pt vertex if the combined center of mass energy of the colliding quarks was essentially tuned to be equal to (or near to) the rest mass energy of the $W^+$ boson (or certainly if the center of mass energy would be within the Breit-Wigner resonance lineshape, then there would be a finite non-zero probability of the $W^+$ being produced.)

What is the rest mass energy of the $W^+ / W^-$ boson? Its mass is about 80 GeV/c$^2$. Thus the center of mass energy of the system of two colliding quarks must be 80 GeV.

Some algebra: For two particles colliding head on the value of the center of mass energy of the system is given by $s = M^2 c^4 = m_1^2 c^4 + m_2^2 c^4 + 2E_1 E_2 + 2p_1 p_2 c^2$ (some simple cancellations have already occurred after expanding $s = M^2 c^4 = (E_1 + E_2)^2 - (\vec{p}_1 c + \vec{p}_2 c)^2$ and taking the beams to be colliding and hence the angle between their momenta as being $\pi$).

If $m_1 \sim m_2$ and $E_1 >> m_1$ and $E_2 >> m_2$ then we can discount the rest mass terms, and also equate $E_1 \sim p_1 c$; and $E_1 \sim p_1 c$. Thus $s = M^2 c^4 = 2E_1 E_2 + 2E_1 E_2$

Finally if $E_1 \sim E_2 = E$ then $s = M^2 c^4 = 4E^2$ or $Mc^2 = 2E$. Thus if $Mc^2$ has to be 80 GeV, then $E$ must be 40 GeV and clearly the momentum of the quark (and antiquark) must be 40 GeV/c

But quarks or antiquarks cannot be isolated as free quarks. Instead we have them bound by threes into say protons, or anti-protons respectively. If on average, we were to say that the momentum of a quark inside a proton was typically a third of the momentum of the proton (recall the parton distribution functions $f(x)$), then to get the average momentum/energy of the quarks (antiquarks) within a proton (antiproton) to 40 GeV, clearly we have to give an average momentum/energy to the proton (antiproton) of at least 120 GeV. Then the center of mass energy of the colliding proton and anti-proton would be 240 GeV, but as the component quarks (antiquarks) are free to take almost any value of the momentum of the parent quark, there would quark-antiquark collisions with a range of possible energies from 0 GeV (unlikely) to 240 (GeV) also unlikely, but peaking at ~ one third of that at about 80 GeV.

All of the above is as clear as an explanation as I can give. A written answer to the question could be MUCH MUCH more succinct in outlining the reasoning without diverting into a "thought experiment" as I have.

Finally, the experimental signatures of the discovery of the W boson are as detailed in my handout 5 page 5 or in Lecture 10 or in the relevant section in M & S.

**QUESTION 12: asked 22/4/2013**

**Quoting D:**

- I have another question, hope you don't mind.
  - In the 4th worksheet Q1 (g) I don't fully understand why this decay cannot occur. Can the strange quark not emit a $W^-$ and become an up quark and then the $W^-$ decays into an anti up and a down and then the up and anti up annihilate to produce a photon?
You said: “In the solution it says it does not conserve quark flavour but I thought for weak interactions you don't have to conserve quark flavour?”

To be precise charged weak current interactions, those involving W bosons, CANNOT conserve quark flavours. By contrast neutral weak currents, those involving Z bosons, CANNOT change quark flavours (even from s->d).

The solution actually says "(g) Does not conserve quark flavours, => Weak interaction; how?" i.e. it makes the statement that it does not conserve quark flavours which IMPLIES that IT IS a Weak interaction. (My ADDED EMPHASIS) and clearly it cannot be solely an electromagnetic decay (as electromagnetism by itself would conserve quark flavours).

I do not state in the solution sheet that it cannot occur - did I misspeak in the tutorial?

I have to concede that is an inventive diagram you describe and could indeed explain this decay. A simpler way of describing an equivalent diagram is this: the s emitting a W- becoming an up, and then that up quark reabsorbing the W- and becoming a down quark. The photon is emitted by any of the charged particles along the way, including the W-.

Consider how likely this process is to occur? Firstly is it kinematically possible? If the mass of the Sigma was greater than the mass of the proton, which it is, then there is no barrier on that front.

Clearly it is a Weak process in the diagram you describe which implies the transition rate for this process will be very small leading to a long partial lifetime. Are there other decay routes? - in fact there are namely whereby a Weak decay of the strange quark occurs producing a W- and an up quark. Then depending upon how one rearranges which quark accompanies the antiquark one can get a decay into: (a) proton + neutral pion or (b) neutron + positive pion.

These two routes account for 99.88% of the branching ratios. The other 0.12% of the time we see this decay. Why is it so unlikely? Probably because of phase space arguments as it is less likely that the W be reabsorbed into the emitting quark than go wandering off and decay somewhere else.

QUESTION 13: asked 29/4/2013

Quoting R:

- I was hoping you could help me with the second part of 2009 Q3c, it asks what can be inferred from the 20:1 ratio of the two decay rates.
  I have a few ideas here but I'm not sure which/if any of them are correct.
  1. The CKM matrix gives a much greater value for V_{ud} than V_{us}
  2. The kaon decays to other higher mass products preferentially
  3. The pion is less massive than the kaon and so we see more of them in the product of the reaction.

- The Feynman diagrams describing the two decays differ only in the quark-antiquark flavours combining to give a W boson as either an udW or an usW 3-pt vertex in the pion or kaon decay respectively. For the pion decay the term V_{ud} enters into the transition matrix element describing the quantum mechanical amplitude for the decay. The probability of the decay is in proportion to the square of the transition matrix element. Thus the decay rate of
the pion to muon and muon neutrino is in proportion to $|V_{ud}|^2$, similarly the decay rate of the kaon to muon and muon-neutrino is in proportion to $|V_{us}|^2$.

In a four flavour mixing model, as originally suggested by Cabibbo, the absolute values of $V_{ud}$ and $V_{us}$ are just equal to $\cos(\theta_c)$ and $\sin(\theta_c)$ respectively where $\theta_c$ is the Cabibbo angle which specifies the degree of mixing between quark eigenstates in the Weak interaction.

Thus your idea 1 is on the right track - but you are far from understanding the solution if that is all you can state. To bolster your understanding you should consult page 7 of handout #7 - https://www.tcd.ie/Physics/people/Cormac.McGuinness/Teaching/PY4P02_Handout_Number7.pdf and Lecture 16 slides 7-10 or M & S 8.1.2.

Your point 2 is interesting but you clearly need to re-read the question. The question effectively asks you or directs you to only consider THAT SPECIFIC MUONIC decay of the negative kaon, it does not mention any other decay pathways. You are then told that the rates of the two SPECIFICALLY identified processes are in this ratio of 20:1. As it happens the negative pion can decay in no other way, and yes while there are other possible ways in which a negative kaon can also decay, as with any decaying particle the total decay rate of the kaon would be the sum of the rates of all of the possible decay paths. However you are only asked to consider this particular decay pathway. Thus your point 2 is irrelevant to the question that is asked.

Your point 3 is valid as the decay rate in a Weak decay is given by a dimensional argument introduced early in the lecture course (See handout Number 1 page 20 - https://www.tcd.ie/Physics/people/Cormac.McGuinness/Teaching/PY4P02_Handout_Number1.pdf, Lecture 4, or M & S Section 1.5). There the decay rate, on dimensional grounds, should be in proportion to the fifth power of the mass of the decaying particle, as well as in proportion to the square of the Fermi coupling constant. That is $\Gamma = K \times (G_F)^2 \times m^5$ where $\Gamma$ is the decay rate of the process in question and $m$ the mass of the decaying particle and $G_F$ the Fermi coupling constant.

Clearly this should be taken into account, but 2009 Q3c was only intending to refer to the factors arising due to the quark-antiquark-W couplings at the 3-pt vertex in the respective diagrams and the relative probabilities or decay rates arising from these factors.

As a result of 2009 Q3c not being specific enough, you will find last years 2012 Q2b having the latter part of the question rephrased as: "What can one infer if the ratio of these decay rates, after taking into account the masses of the involved quarks, were 20:1?"

The specific answer one should give in the examination is clearly one that relates the observed ratio 20:1 to $|V_{ud}|^2:|V_{us}|^2$ and hence to $|\cos(\theta_c)|^2:|\sin(\theta_c)|^2$ from which the Cabibbo angle $\theta_c$ or mixing angle between the first and second generation $\theta_{12}$ can be approximated.

I hope that detailed answer is complete enough for you.