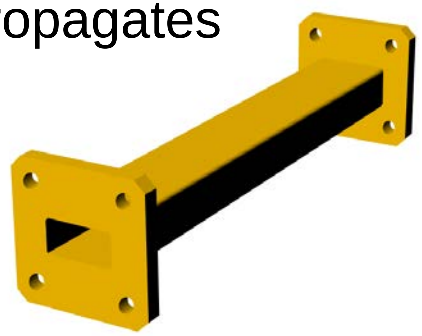
EM Waveguiding

Overview

- ***Waveguide*** may refer to any structure that conveys electromagnetic waves between its endpoints
- Most common meaning is a hollow metal pipe used to carry radio waves
- May be used to transport radiation of a single frequency
- Transverse Electric (TE) modes have $\mathbf{E} \perp \mathbf{k}_g$ (propagation wavevector)
- Transverse Magnetic (TM) modes have $\mathbf{B} \perp \mathbf{k}_g$
- Transverse Electric-Magnetic modes (TEM) have $\mathbf{E}, \mathbf{B} \perp \mathbf{k}_g$
- A cutoff frequency exists, below which no radiation propagates



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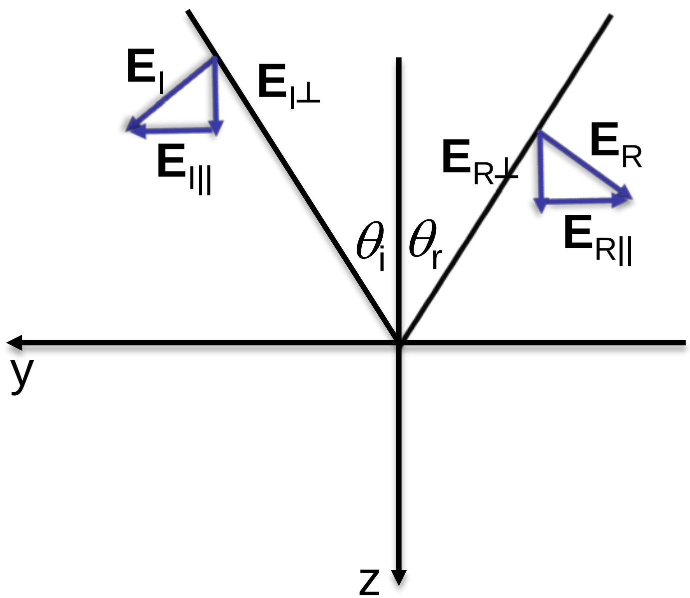
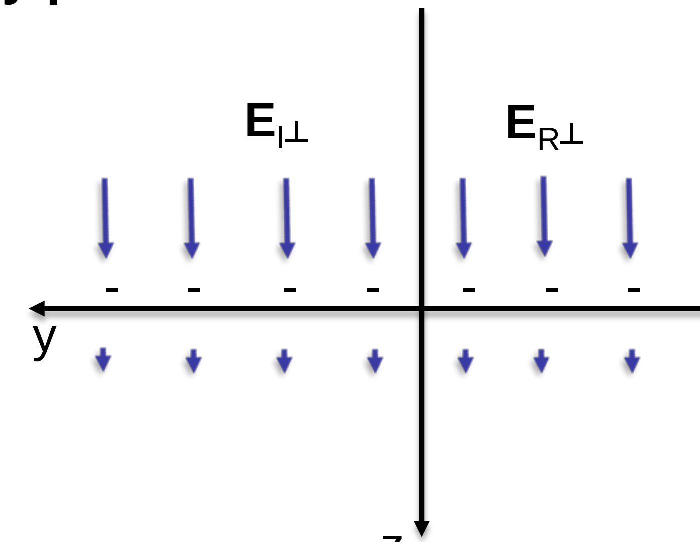
Electromagnetic wave reflection by perfect conductor



$E_\perp$ can be finite just outside conducting surface

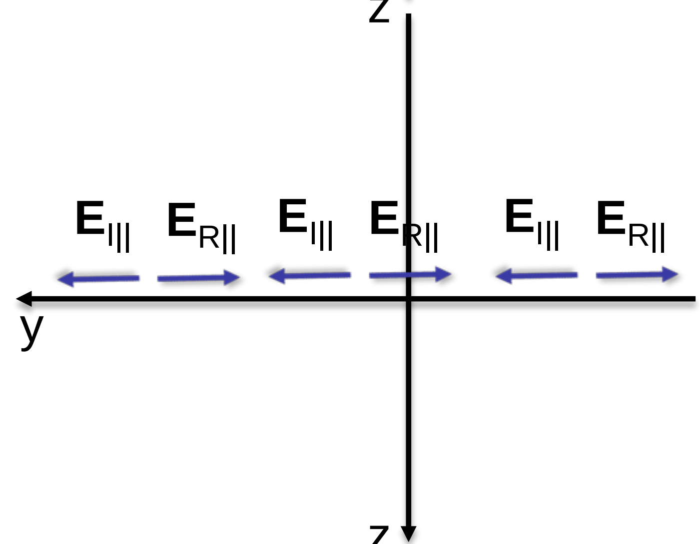
$E_{||}$ vanishes just outside and inside conducting surface



$\mathbf{D}_{\perp 1} = \mathbf{D}_{\perp 2}$

$\mathbf{D}_{\perp 1} = \varepsilon_o \mathbf{E}_{\perp 1}$

$\mathbf{D}_{\perp 2} = \varepsilon_o \varepsilon \mathbf{E}_{\perp 2}$



$E_{||1} = E_{||2}$

$E_{oI} + E_{oR} = 0$

$E_{oT} = 0$

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Electromagnetic wave propagation between conducting plates

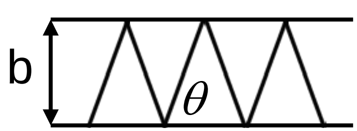
Boundary conditions $B_{\perp 1} = B_{\perp 2}$ $E_{\parallel 1} = E_{\parallel 2}$ (1,2 inside, outside here)

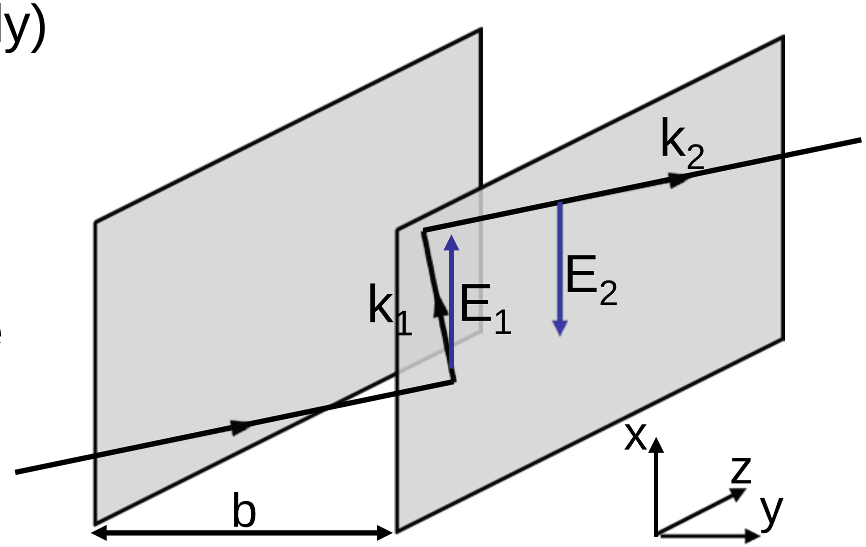
$E_{\parallel}$ must vanish just outside conducting surface since E = 0 inside

$E_{\perp}$ may be finite just outside since induced surface charges allow E = 0 inside (TM modes only)

$B_{\perp} = 0$ at surface since $B_1 = 0$

Two parallel plates, TE mode





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$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$= \mathbf{e}_x E_o e^{i\omega t} (e^{i(-ky \sin\theta + kz \cos\theta)} - e^{i(ky \sin\theta + kz \cos\theta)})$

$= \mathbf{e}_x E_o e^{i\omega t} e^{-ikz \cos\theta} 2i \sin(ky \sin\theta)$

Boundary condition $E_{||1} = E_{||2} = 0$

means that $\mathbf{E} = \mathbf{E}_{||}$ vanishes at y = 0, y = b

$\mathbf{E}_{||}(y=0,b)$ if $ky \sin\theta = n\pi \quad n = 1, 2, 3, ..$

$k = \dfrac{n\pi}{b \sin\theta} \qquad \sin\theta = \dfrac{n\pi}{kb}$

Fields *in vacuum*

$\mathbf{E}_1 = \mathbf{e}_x E_o e^{i(\omega t - \mathbf{k}_1.\mathbf{r})}$

$\mathbf{k}_1 = -\mathbf{e}_y k \sin\theta + \mathbf{e}_z k \cos\theta$

$\mathbf{k}_1.\mathbf{r} = -ky \sin\theta + kz \cos\theta$

$\mathbf{E}_2 = -\mathbf{e}_x E_o e^{i(\omega t - \mathbf{k}_2.\mathbf{r})}$

$\mathbf{k}_2 = +\mathbf{e}_y k \sin\theta + \mathbf{e}_z k \cos\theta$

$\mathbf{k}_2.\mathbf{r} = +ky \sin\theta + kz \cos\theta$

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Allowed field between guides is

$$\mathbf{E} = \mathbf{e}_x E_o e^{i\omega t} e^{-ikz\cos\theta} 2i \sin(ky\sin\theta)$$
$$= \mathbf{e}_x E_o e^{i\omega t} e^{-ikz\cos\theta} 2i \sin(n\pi y/b)$$

Since $\sin\theta = \frac{n\pi}{kb}$ $\cos\theta = \left(1 - \frac{n^2\pi^2}{k^2b^2}\right)^{1/2}$

The wavenumber for the guided field is

$$k_g = k\cos\theta = \left(k^2 - \frac{n^2\pi^2}{b^2}\right)^{1/2} \quad n = 1, 2, 3, ..$$

Fields

$$\mathbf{E}_1 = \mathbf{e}_x E_o e^{i(\omega t - \mathbf{k}_1.\mathbf{r})}$$

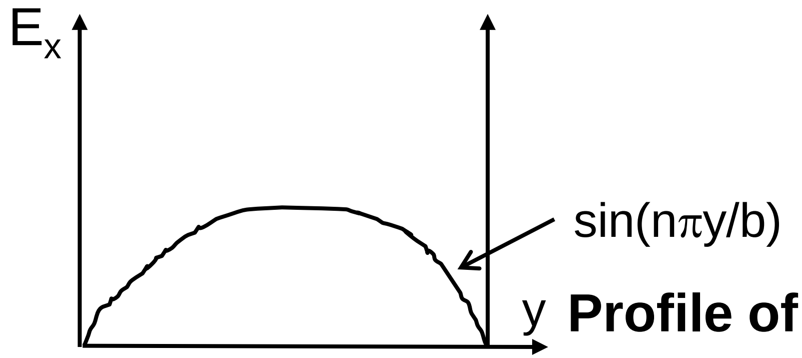
$$\mathbf{k}_1 = -\mathbf{e}_y k\sin\theta + \mathbf{e}_z k\cos\theta$$

$$\mathbf{k}_1.\mathbf{r} = -ky\sin\theta + kz\cos\theta$$

$$\mathbf{E}_2 = -\mathbf{e}_x E_o e^{i(\omega t - \mathbf{k}_2.\mathbf{r})}$$

$$\mathbf{k}_2 = +\mathbf{e}_y k\sin\theta + \mathbf{e}_z k\cos\theta$$

$$\mathbf{k}_2.\mathbf{r} = +ky\sin\theta + kz\cos\theta$$



Profile of the first transverse electric mode (TE_1)

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Magnetic component of the guided field from Faraday's Law

$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t = -i\omega\, \mathbf{B}$ for time-harmonic fields

$\mathbf{B} = i\nabla \times \mathbf{E} / \omega = 2\, E_o / \omega\, (0,\, ik_g \sin(n\pi y/b),\, \sqrt{(k^2 - k_g^2)} \cos(n\pi y/b)\,)\, e^{i(\omega t - k_g z)}$

The BC $B_{\perp 1} = B_{\perp 2} = 0$ is satisfied since $B_y = 0$ on the conducting plates. The **E** and **B** components of the field are perpendicular since $B_x = 0$.

The **phase velocity** for the guided wave is $v_p = \omega / k_g = c\, k / k_g$

$k_g = \left(k^2 - \dfrac{n^2\pi^2}{b^2}\right)^{1/2}$ Hence $v_p = c\left(1 - \dfrac{n^2\pi^2}{k^2 b^2}\right)^{-1/2}$

The **group velocity** for the guided wave is $v_g = \partial\omega / \partial k_g = c\, \partial k / \partial k_g = c\, k_g / k$

$$v_p\, v_g = c^2$$

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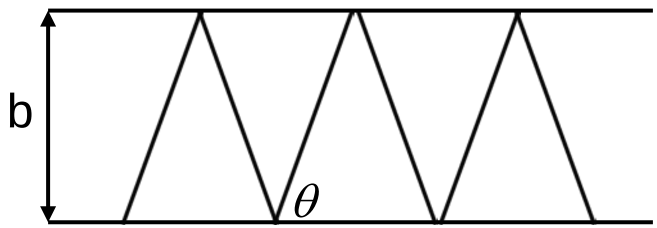
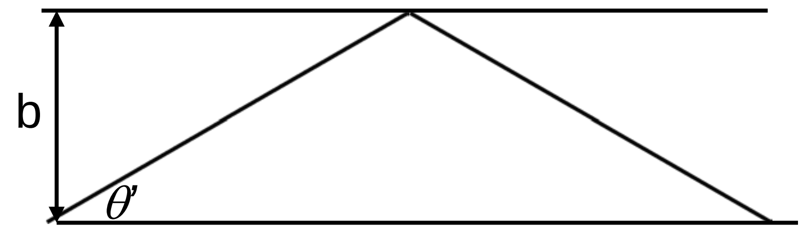
Frequency Dispersion and Cutoff



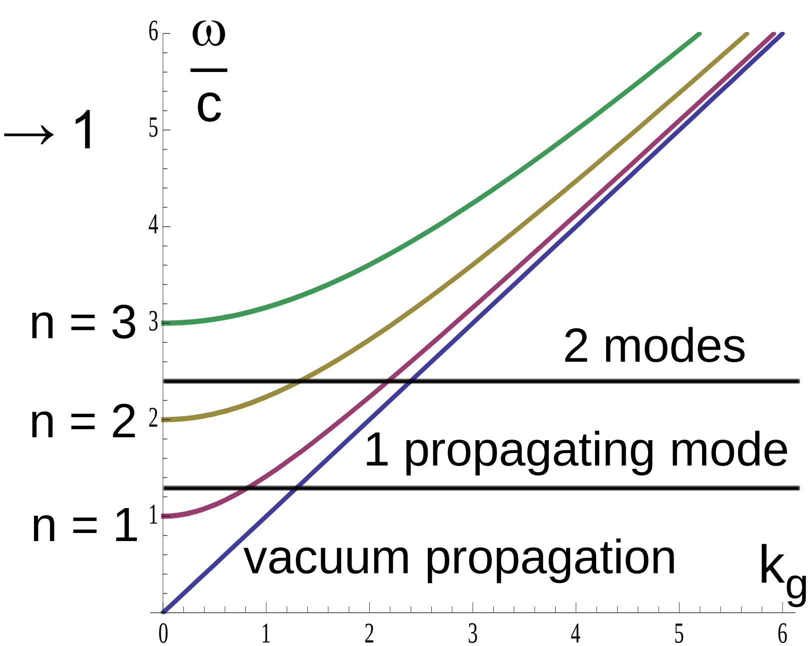


$$k = \frac{n\pi}{b\sin\theta} \qquad \sin\theta = \frac{n\pi}{kb} \qquad \text{cutoff when } \sin\theta \rightarrow 1$$

$$\omega = ck = 2\pi\nu \quad \nu = \frac{ck}{2\pi} = \frac{cn}{2b\sin\theta}$$

$$\nu_{cutoff} = \frac{c}{2b} \quad \text{or} \quad \frac{\omega_{cutoff}}{c} = \frac{\pi}{b} \qquad (n = 1)$$

$$k_g = \left(k^2 - \frac{n^2\pi^2}{b^2}\right)^{1/2} = \left(\frac{\omega^2}{c^2} - \frac{n^2\pi^2}{b^2}\right)^{1/2}$$



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Summary of TE_n modes

$\mathbf{E} = 2\,E_o\,(i\,\sin(n\pi y/b),\,0\,,0)\,e^{i(\omega t - k_g z)}$

$$k_g = \left(k^2 - \frac{n^2\pi^2}{b^2}\right)^{1/2}$$

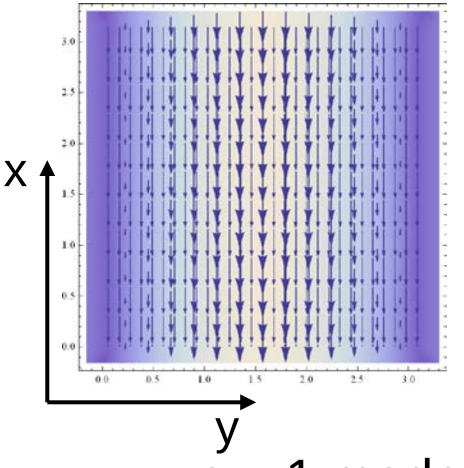
$\mathbf{B} = 2\,E_o\,/\,\omega\,(0,\,i k_g \sin(n\pi y/b),\,\sqrt{(k^2 - k_g^2)}\,\cos(n\pi y/b)\,)\,e^{i(\omega t - k_g z)}$

Phase velocity $v_p = \omega / k_g = c\,k / k_g$

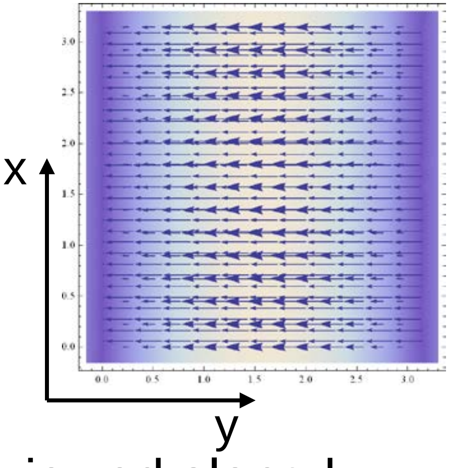
Group velocity $v_g = \partial\omega / \partial k_g = c\,k_g / k$

$$\nu_{cutoff,n} = \frac{c k_{cutoff,n}}{2\pi} = \frac{c}{2b}\,n$$

E



B



n = 1 mode viewed along k_g

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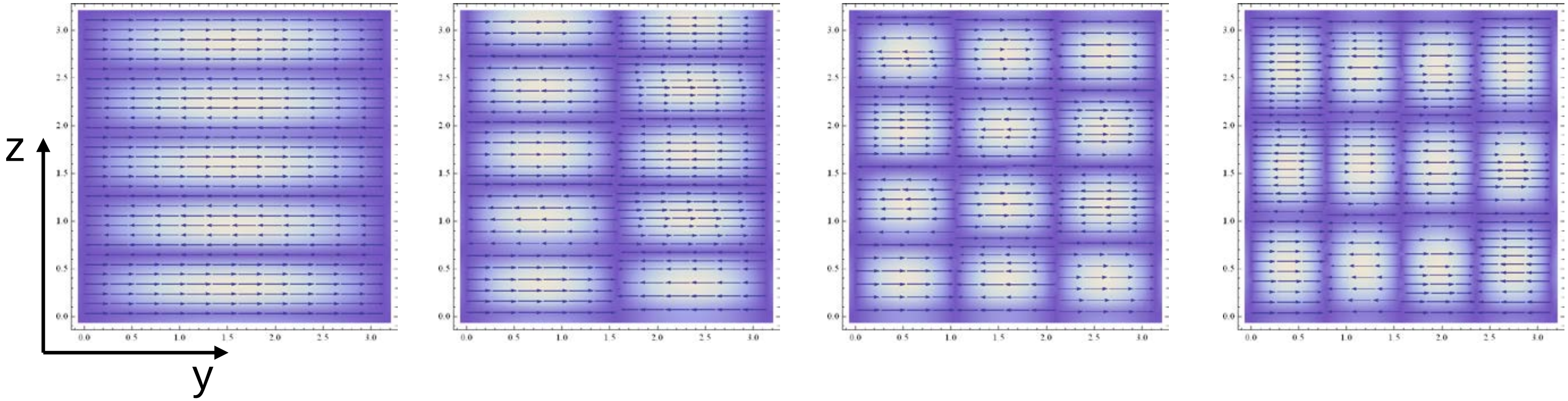
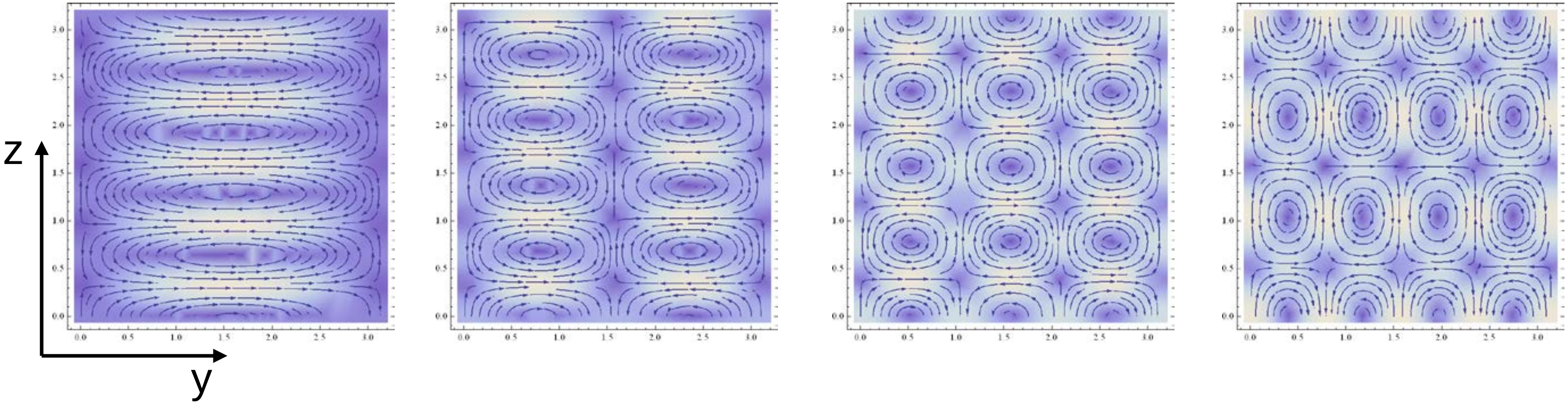
Electric components of TE_n guided fields viewed along x (plan view)

n = 1 n = 2 n = 3 n = 4



Magnetic components of TE_n guided fields viewed along x (plan view)



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Rectangular waveguides

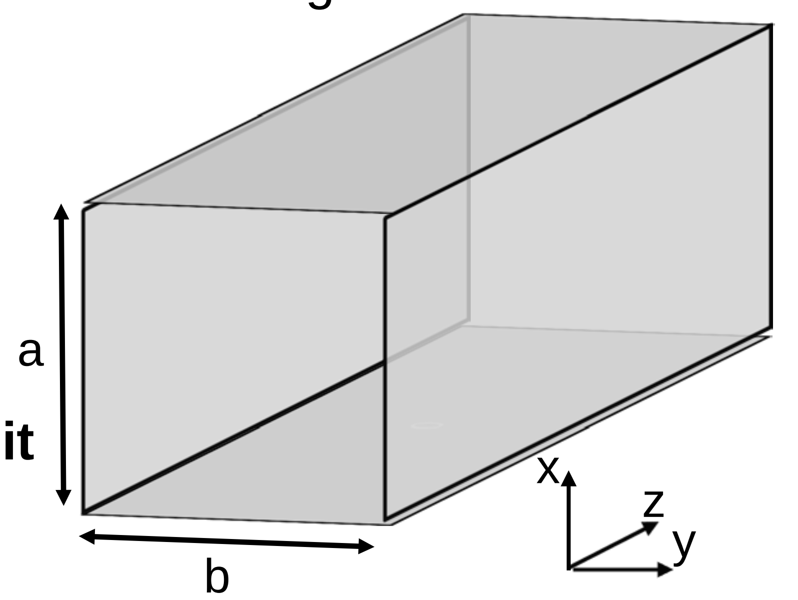
Boundary conditions $B_{\perp 1} = B_{\perp 2}$ $E_{\parallel 1} = E_{\parallel 2}$

$E_{\parallel}$ must vanish just outside conducting surface since E = 0 inside

$E_{\perp}$ may be finite just outside since induced surface charges allow E = 0 inside

$B_{\perp}$ = 0 at surface

Infinite, rectangular conduit



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TE_{mn} modes in rectangular waveguides

TE_n modes for two infinite plates are also solutions for the rectangular guide

E field vanishes on xz plane plates as before, but not on the yz plane plates

Charges are induced on the yz plates such that **E** = 0 inside the conductors

Let $E_x = C\, f(x) \sin(n\pi y/b)\, e^{i(\omega t - k_g z)}$

In free space $\nabla.\mathbf{E} = 0$ and $E_z = 0$ for a TE_{mn} mode and $\partial E_z/\partial z = 0$

Hence $\partial E_x/\partial x = -\partial E_y/\partial y$

$f(x) = -n\pi / b \cos(m\pi x/a)$
satisfies this condition

By integration

$E_x = -C\, n\pi / b \cos(m\pi x/a) \sin(n\pi y/b)\, e^{i(\omega t - k_g z)}$

$E_y = C\, m\pi / a \sin(m\pi x/a) \cos(n\pi y/b)\, e^{i(\omega t - k_g z)}$

$E_z = 0$

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Dispersion Relation

Substitute into wave equation $(\nabla^2 - 1/c^2\, \partial^2/\partial t^2)\mathbf{E} = 0$

$$\nabla^2 E_{x,y} = \left[-\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 - k_g^2 \right] E_{x,y}$$

$$\partial^2/\partial t^2\, E_{x,y} = -\,\omega^2\, E_{x,y}$$

$$-\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 - k_g^2 - \omega^2 / c^2 = 0$$

$$k_g^2 = k^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}$$

Magnetic components of the guided field from Faraday's Law

$$B_x = -C\, m\pi / a\; k_g / \omega\; \sin(m\pi x/a)\cos(n\pi y/b)\, e^{i(\omega t - k_g z)}$$

$$B_y = -C\, n\pi / b\; k_g / \omega\; \cos(m\pi x/a)\sin(n\pi y/b)\, e^{i(\omega t - k_g z)}$$

$$B_z = i\, C\, (k^2 - k_g^2) / \omega\; \cos(m\pi x/a)\cos(n\pi y/b)\, e^{i(\omega t - k_g z)}$$

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Cutoff Frequency

$$k_g^2 = k^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}$$

$$\nu_{cutoff} = \frac{ck_{cutoff}}{2\pi} = \frac{c}{2\pi}\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)^{1/2} = c\left(\frac{m^2}{4a^2} + \frac{n^2}{4b^2}\right)^{1/2}$$

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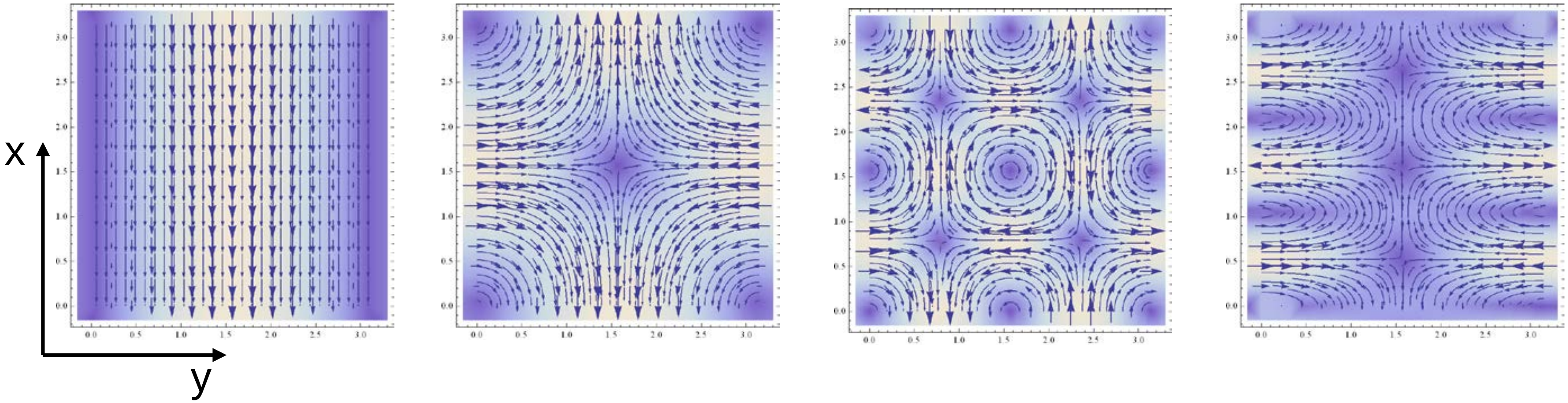
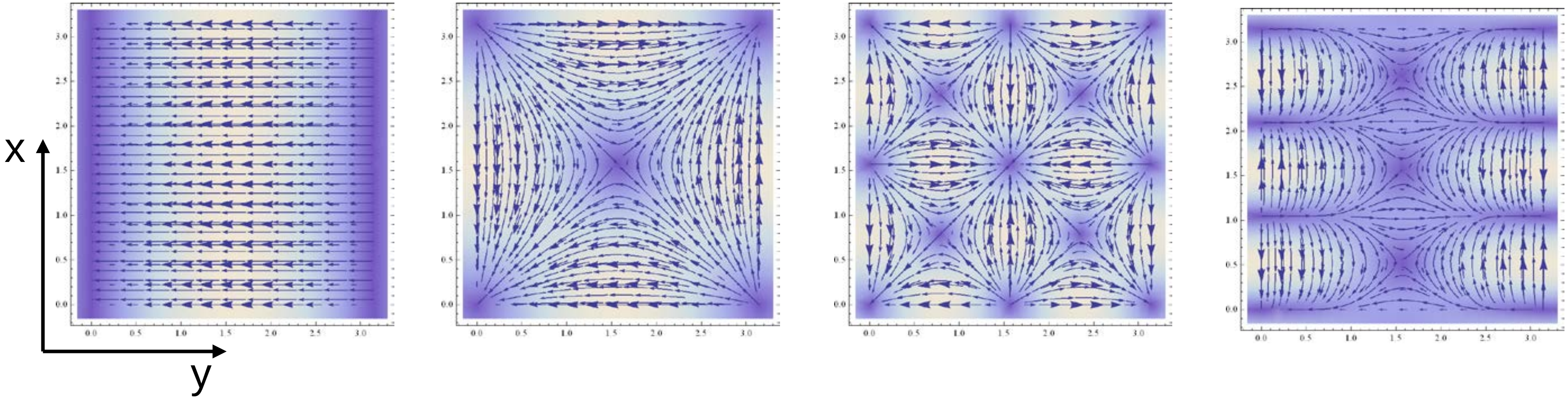
Electric components of TE_{mn} guided fields viewed along k_g

m = 0 n = 1 | m = 1 n = 1 | m = 2 n = 2 | m = 3 n = 1



Magnetic components of TE_{mn} guided fields viewed along k_g



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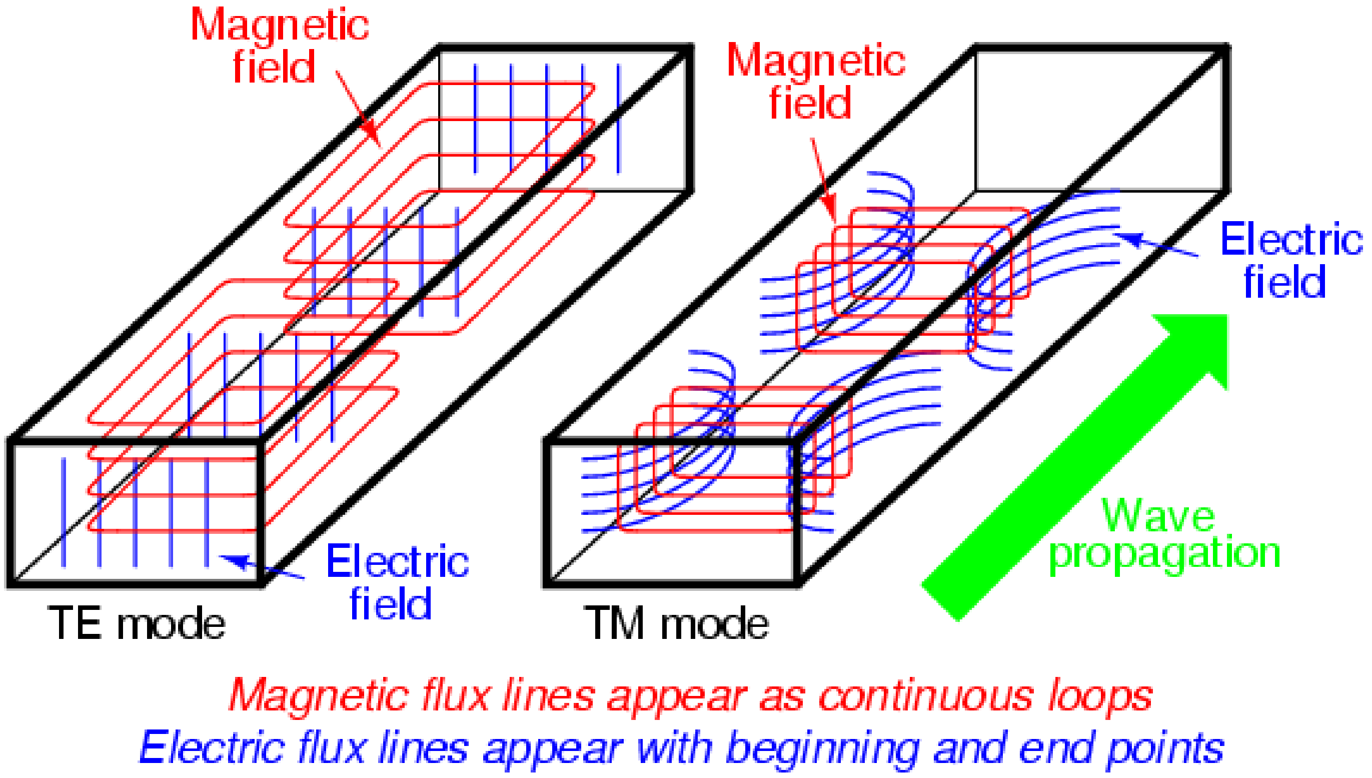
Comparison of fields in TE and TM modes



www.opamp-electronics.com/tutorials/waveguides_2_14_08.htm

EM Waveguiding

The TE_{01} mode

Most commonly used since a single frequency $\nu_{cutoff,02} > \nu > \nu_{cutoff,01}$ can be selected so that only one mode propagates.

Example 3 cm radar waves in a 1cm x 2 cm guide

$$\nu_{cutoff,01} = c\left(\frac{0^2}{4a^2} + \frac{1^2}{4x4.10{-4}}\right)^{1/2} \qquad = 7.5 \times 10^9 \text{ Hz}$$

$$\nu_{cutoff,01} = c\left(\frac{0^2}{4x1.10{-4}} + \frac{1^2}{4x4.10{-4}}\right)^{1/2} \qquad = 7.50 \times 10^9 \text{ Hz}$$

$$\nu_{cutoff,10} = c\left(\frac{1^2}{4x1.10{-4}} + \frac{0^2}{4x4.10{-4}}\right)^{1/2} \qquad = 1.50 \times 10^{10} \text{ Hz}$$

$$\nu_{cutoff,11} = c\left(\frac{1^2}{4x1.10{-4}} + \frac{1^2}{4x4.10{-4}}\right)^{1/2} \qquad = 1.68 \times 10^{10} \text{ Hz}$$