PY4P05 Electromagnetic Interactions II 12 Lectures

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Course text

Electromagnetism, 2nd Edn. Grant and Phillips (Wiley)

Topics and reading in Grant and Phillips

Review Ch. 1, 2, 4, 5, 6

1). Maxwell's Equations in Vacuum and Matter	Ch. 10, 11 (pp348-360, 364-391)
2). Reflection and Transmission of Electromagnetic Waves	Ch. 11 (pp 391-404)
3). Waveguides	Ch. 12 (pp 408-430)
4). Generation of Electromagnetic Radiation	Ch 13 (pp 432-450)

Units in Electromagnetism
Classical Electrodynamics, J. D. Jackson

Basic law is Coulomb's Law

$F_1 = k_1 qq'/r^2$ (1)

Constant of proportionality k_1, magnitude and dimensions are determined by the definition of the unit of charge. Since F_1 has dimensions of MLT^{-2}, k_1qq' must have dimensions of ML^3T^{-2}.
The electric field is defined as the force per unit charge.

$E = k_1 q/r^2$ (2)

Force per unit length $dF_2/d\ell$ between two parallel current carrying wires (I and I') separated by distance d

$dF_2/d\ell = 2k_2 I\, I'/d$ (3)

From (1), (3) k_1/k_2 has dimension L^2T^{-2} (velocity squared) $k_1/k_2 = c^2$
Magnetic induction is derived from Ampère's law.

$B=2k_2\, \alpha I/d$

	k_1	k_2	α
MKSA	$1/4\pi\varepsilon_o=10^{-7}\ c^2$	$\mu_o/4\pi=10^{-7}$	1
Gaussian	1	c^{-2} (T^2L^{-2})	c (TL^{-1})

Physical quantity	Symbol	MKSA unit
Charge	q	1 coulomb
Charge density	ρ	1 coulomb m^{-3}
Current	I	1 ampère (amp.)
Current Density	j	1 amp m^{-2}
Electric field	E	1 volt m^{-1}
Potential	ϕ	1 volt
Polarisation	P	1 coulomb m^{-2}
Displacement	D	1 coulomb m^{-2}
Capacitance	C	1 farad
Magnetic flux	ϕ, F	1 weber
Magnetic induction	B	1 tesla
Magnetic field	H	1 amp m^{-1}
Magnetisation	M	1 amp m^{-1}
Inductance	I	1 henry

PY4P05 Electromagnetic Interactions II Tutorial Problems

Electromagnetic waves

1) GP 11.1 The root mean square of the displacement current density in a linearly polarised monochromatic plane wave in free space is 10^{-5} Am^{-2}. The frequency is 10^8 Hz. Obtain values for the electric and magnetic fields in the wave.

2) GP 11.3 A laser is a device that emits a parallel beam of monochromatic light. The intensity may be assumed constant across the beam. If the power is 1 W and the beam has a diameter of 1 mm, calculate the maximum amplitude of the magnetic field **B** in the beam in free space.

3) GP 11.8 The electrical conductivity of sea water is about 4 $(\Omega m)^{-1}$. What is the skin depth for low frequency radio waves of wavelength 3000 m?

4) GP 11.10 A monochromatic plane wave in free space is incident normally on the plane surface of a medium of refractive index equal to 2. If the amplitude of the electric field in the incident wave is 10 Vm^{-1}, what is its value inside the medium?

5) GP 11.14 The interstellar medium is a dilute plasma containing free electrons and ions with density 3 x $10^{4}m^{-3}$. A pulsar which is at a distance of 10^{19} m from Earth emits a short pulse of EM radiation containing frequency components covering the spectrum from visible light down to radiofrequencies. Find the difference in arrival times at Earth between radiation corresponding to red light and radiofrequency radiation at 100 MHz.

Waveguides

6) GP12.2 Determine the maximum and minimum widths of a waveguide of square cross section if it is to transmit waves of free-space wavelength λ in the TE_{01} mode only.

7) GP 12.4 A source maintains electric fields of constant amplitude in the TE_{01} mode in a rectangular wave guide. Discuss how the power received at the far end of the guide varies as the frequency of the radiation is reduced. If the guide is terminated such that there is no reflected wave, what is the energy density in the guide?

8) GP 12.7 A rectangular cavity has dimensions a = 2 cm, b = 3 cm, d = 4 cm. How many resonances are there within the frequency range 5 x 10^9 Hz to 10^{10} Hz?

9) GP 12.8 Show that the assumption that a TEM wave can be propagated within the interior of a hollow metal pipe leads to contradictions with electromagnetic theory. This constitutes a proof that there can be no TEM waves within such a wave guide.

10) GP 12.10 A rectangular cavity with dimensions a, a, $2a$ in the x, y and z directions, respectively, has walls of very high conductivity and is excited in a mode for which the electric field components are

$E_x = E_o \sin(\pi y/a) \sin(\pi z/a) e^{i\omega t}$

$E_y = E_z = 0$

What are the components of the magnetic field, **B**, and what is the angular frequency of the resonance?

Generation of Electromagnetic waves

11) GP 13.1 A half-wave dipole transmitter in the centre of a city is required to give signals to cars travelling in the city within a radius of 10 km. If the minimum field amplitude required is 0.02 Vm^{-1} what must the mean power of the transmitter be? Why would the frequency chosen be likely to be around 100 MHz?

12) GP 13.3 An antenna consists of four vertical half-wave dipoles separated from each other by one wavelength in the East-West direction. Calculate and sketch the horizontal polar diagram of the array when the dipoles are fed in phase with equal currents.

13) GP 13.4 Why is an antenna much shorter than a wavelength inefficient as a radiator of radio waves? If a commercial radio station transmitting at a wavelength of 500 m did not wish to string a cable 250 m long well above the ground to form a half-wave dipole antenna, what might it do to make an efficient aerial system?

14) GP 13.Electric quadrupole radiation is produced when the currents in two very small dipoles, placed side by side very close to each other, oscillate 180^o out of phase. Show that the electric radiation field produced is proportional to the cube of the angular frequency. Hence show that the probability per unit time for the decay of an excited microscopic energy level via quadrupole radiation is proportional to the fifth power of the photon energy.