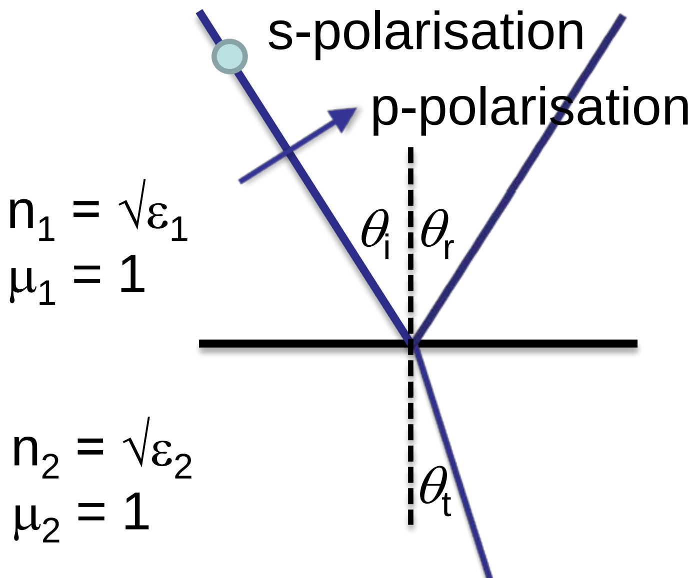
Fresnel Equations

Consider reflection and transmission of light at dielectric/dielectric boundary

Calculate reflection and transmission coefficients, R and T, as a function of incident light polarisation and angle of incidence using EM boundary conditions



s-polarisation **E** perpendicular to plane of incidence

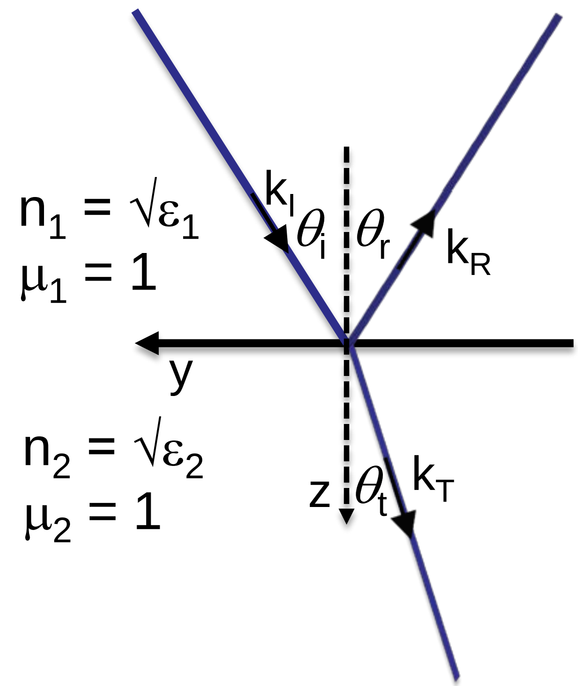
p-polarisation **E** parallel to plane of incidence

Fresnel Equations

Snell's Law

Boundary conditions apply across the entire, flat interface (z = 0)

Incident, reflected and transmitted waves are like



$\mathbf{E}_I = (\mathbf{e}_y \cos\theta_i + \mathbf{e}_z \sin\theta_i)\ E_{oI}\ e^{i(\omega t - k_I . r)}$

$\mathbf{E}_R = (-\mathbf{e}_y \cos\theta_r + \mathbf{e}_z \sin\theta_r)\ E_{oR}\ e^{i(\omega t - k_R . r)}$

$\mathbf{E}_T = (\mathbf{e}_y \cos\theta_t + \mathbf{e}_z \sin\theta_t)\ E_{oT}\ e^{i(\omega t - k_T . r)}$

To satisfy BC $(\mathbf{k}_I \, . \, \mathbf{r})_{z=0} = (\mathbf{k}_R \, . \, \mathbf{r})_{z=0} = (\mathbf{k}_T \, . \, \mathbf{r})_{z=0}$

(1) wave vectors lie in single plane

(2) projection of wave vectors on xy plane is same

From (1) $\theta_i = \theta_r$

From (2) $k_I \sin\theta_i = k_R \sin\theta_r = k_T \sin\theta_t$

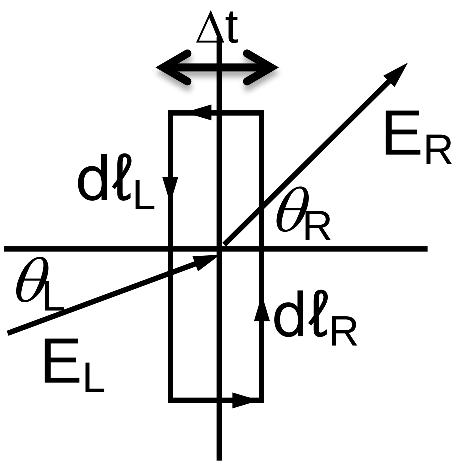
$k_I = k_R = \frac{\omega}{c}\sqrt{\mu_1\varepsilon_1}$ $k_T = \frac{\omega}{c}\sqrt{\mu_2\varepsilon_2}$

$k_I \sin\theta_i = k_T \sin\theta_t$ becomes $\sin\theta_i / \sin\theta_t = \sqrt{\mu_2\varepsilon_2} / \sqrt{\mu_1\varepsilon_1}$

Boundary conditions on **E**

E fields at matter/vacuum interface

Boundary conditions on **E** from Faraday's Law $\oint_C \mathbf{E}.d\boldsymbol{\ell} = -\frac{d}{dt}\int_S \mathbf{B}.d\mathbf{S}$



$\oint_C \mathbf{E}.d\boldsymbol{\ell} = E_L.d\ell_L + E_R.d\ell_R$ (as $\Delta t \rightarrow 0$)

$\int_S \mathbf{B}.d\mathbf{S} \rightarrow 0$ (as $\Delta t \rightarrow 0$)

$-E_L \sin\theta_L d\ell_L + E_R \sin\theta_R d\ell_R = 0$

$E_L \sin\theta_L = E_R \sin\theta_R$

$E_{||L} = E_{||R}$ $E_{||}$ continuous

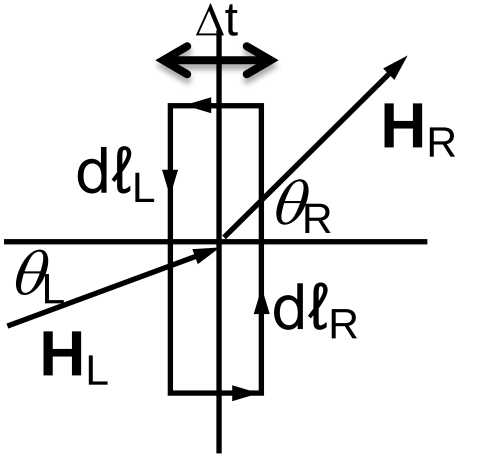
Boundary conditions on **H**

H fields at matter/vacuum interface

Boundary conditions on **H** from Ampère's Law $\nabla \times \mathbf{H} = \mathbf{j}_{free} + \partial \mathbf{D}/\partial t$

$$\int (\nabla \times \mathbf{H}).d\mathbf{S} = \int \left(\mathbf{j}_{free} + \frac{\partial \mathbf{D}}{\partial t} \right).d\mathbf{S} = \oint \mathbf{H}.d\boldsymbol{\ell}$$

$\mathbf{D}$, $\partial \mathbf{D}/\partial t$ are everywhere finite, so as as $\Delta t \rightarrow 0$, $\int \frac{\partial \mathbf{D}}{\partial t}.d\mathbf{S} \rightarrow 0$



For materials of finite conductivity, $\mathbf{j}_{free}$ is finite,

so $\int \mathbf{j}_{free}.d\mathbf{S} \rightarrow 0$ as $\Delta t \rightarrow 0$

For materials of infinite conductivity, $\mathbf{j}_{free}$ is infinite,

so $\int \mathbf{j}_{free}.d\mathbf{S} \rightarrow j_{free,surface}\, d\ell$ as $\Delta t \rightarrow 0$

$j_{free,surface}$ is surface current per unit length

Boundary conditions on **H**

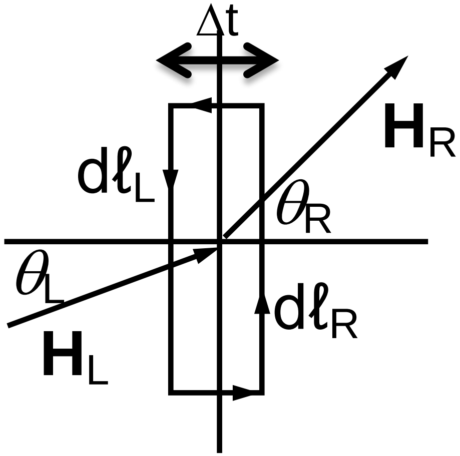
$$\oint \mathbf{H}.d\boldsymbol{\ell} = j_{free,surface}\, d\ell$$

$$-H_L \sin\theta_L d\ell_L + H_R \sin\theta_R d\ell_R = j_{free,surface}\, d\ell$$

$$H_R \sin\theta_R d\ell = H_L \sin\theta_L d\ell + j_{free,surface}\, d\ell$$

$H_{||R} = H_{||L} + j_{free,surface}$ Infinite conductivity at interface

$H_{||R} = H_{||L}$ Finite conductivity at interface



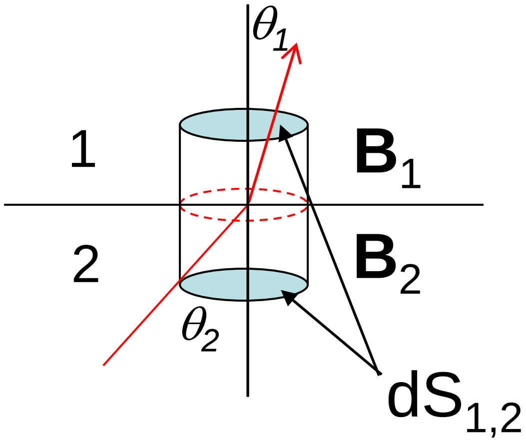
Boundary conditions on **B**

B field at matter/vacuum interface

$$\nabla.\mathbf{B} = 0 \Rightarrow \int_S \mathbf{B}.d\mathbf{S} = 0$$

$$B_1 \cos\theta_1 \, dS - B_2 \cos\theta_2 \, dS = 0$$

$$\Rightarrow B_{1\perp} = B_{2\perp}$$



Boundary conditions on **D**

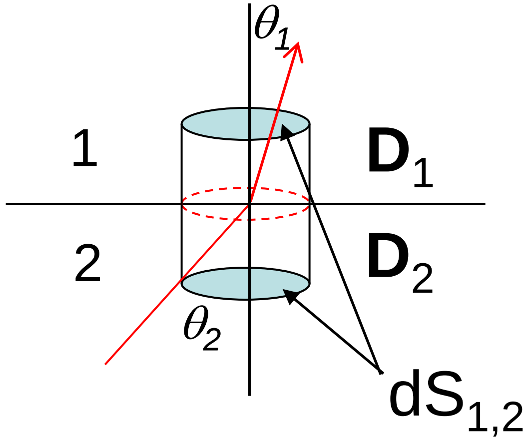
D field at matter/vacuum interface

$\nabla.\mathbf{D} = \rho_{free}$

$\int \mathbf{D}.d\mathbf{S} = \int \rho_{free}\, dv$

$\int \mathbf{D}.d\mathbf{S} = 0$ — No free charges at interface

$\int \mathbf{D}.d\mathbf{S} = \int \nabla.\mathbf{D}\, dv = \sigma_{free}\, dS$ — Free charge density σ_{free} at interface



$\int \mathbf{D}_1 .\, d\mathbf{S}_1 + \int \mathbf{D}_2 . d\mathbf{S}_2 = \int \rho_{free}\, dv$

$D_{\perp 1}\, dS - D_{\perp 2}\, dS = \sigma_{free}\, dS \qquad dS_1 = dS_2 = dS$

$D_{\perp 1} = D_{\perp 2}$ — No interface free charges

$D_{\perp 1} - D_{\perp 2} = \sigma_{free}$ — Interface free charges

Boundary conditions summary

$E_{\|\|L} = E_{\|\|R}$	$E_{\|\|}$ continuous	
$B_{\perp 1} = B_{\perp 2}$	$B_{\perp}$ continuous	
$D_{\perp 1} = D_{\perp 2}$	$D_{\perp}$ continuous	No interface free charges
$D_{\perp 1} - D_{\perp 2} = \sigma_{free}$		Interface free charges
$H_{\|\|R} = H_{\|\|L}$	$H_{\|\|}$ continuous	Finite conductivity at interface
$H_{\|\|R} = H_{\|\|L} + j_{free,surface}$		Infinite conductivity at interface

Fresnel Equations

Reflection coefficient R and Transmission coefficient T

$\nabla^2\mathbf{E} - \mu_o\varepsilon_o\varepsilon\,\partial^2\mathbf{E}/\partial t^2 = 0$

$\mathbf{E}(\mathbf{r}, t) = E_o\,\mathbf{e}_x\,\mathrm{Re}\{e^{i(\omega t - \mathbf{k}.\mathbf{r})}\}$ $\qquad$ $k = \omega\sqrt{(\mu_o\mu\varepsilon_o\varepsilon)}$

$\nabla \times \mathbf{E} = -i\,\mathbf{k} \times \mathbf{E}$ take curl of plane wave $\mathbf{E}$

$\nabla \times \mathbf{E} = -\,\partial\mathbf{B}/\partial t$ Faraday's law

$-\,\partial\mathbf{B}/\partial t = -i\omega\,\mathbf{B}$ time harmonic, plane wave $\mathbf{B}$

$-i\omega\,\mathbf{B} = -i\,\mathbf{k} \times \mathbf{E}$

$\mathbf{B} = \mathbf{k} \times \mathbf{E} / \omega = k\,\mathbf{e}_k \times \mathbf{E} / \omega = \omega\sqrt{(\mu_o\mu\varepsilon_o\varepsilon)}\;\mathbf{e}_k \times \mathbf{E} / \omega = \sqrt{(\mu\varepsilon)}\;\mathbf{e}_k \times \mathbf{E} / c$

Fresnel Equations

$\mathbf{B} = \mathbf{k} \times \mathbf{E} / \omega = k\, \mathbf{e}_k \times \mathbf{E} / \omega = \omega\sqrt{(\mu_o \mu \varepsilon_o \varepsilon)}\ \mathbf{e}_k \times \mathbf{E} / \omega = \sqrt{(\mu\varepsilon)}\ \mathbf{e}_k \times \mathbf{E} / c$

$\mathbf{N} = \mathbf{E} \times \mathbf{H} = \mathbf{E} \times \mathbf{B} / \mu_o \mu = \mathbf{E} \times (\sqrt{(\mu_o \mu \varepsilon_o \varepsilon)}\ \mathbf{e}_k \times \mathbf{E}) / \mu_o \mu$

$N = E^2 \sqrt{(\varepsilon_o \varepsilon / \mu_o \mu)}$

R = reflected energy / incident energy $= E_R^2 \sqrt{(\varepsilon_o \varepsilon_1 / \mu_o \mu_1)} / E_I^2 \sqrt{(\varepsilon_o \varepsilon_1 / \mu_o \mu_1)}$

$= E_R^2 / E_I^2$

T = transmitted energy / incident energy $= E_T^2 \sqrt{(\varepsilon_o \varepsilon_2 / \mu_o \mu_2)} / E_I^2 \sqrt{(\varepsilon_o \varepsilon_1 / \mu_o \mu_1)}$

$= E_T^2 / E_I^2\ n_2 / n_1$ (if $\mu_1 = \mu_2 = 1$)

$R = E_R^2 / E_I^2$

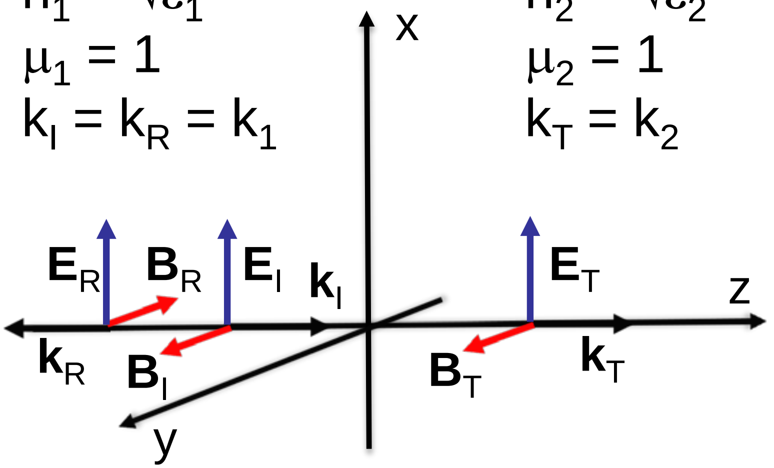
$T = E_T^2 / E_I^2\ n_2 / n_1$

Fresnel Equations

Normal Incidence

$n_1 = \sqrt{\varepsilon_1}$
$\mu_1 = 1$
$k_I = k_R = k_1$

$n_2 = \sqrt{\varepsilon_2}$
$\mu_2 = 1$
$k_T = k_2$



Fields

$\mathbf{E}_I = \mathbf{e}_x E_{oI} e^{i(\omega t - k_1 z)}$

$\mathbf{B}_I = \mathbf{e}_y B_{oI} e^{i(\omega t - k_1 z)}$

$\mathbf{E}_R = \mathbf{e}_x E_{oR} e^{i(\omega t + k_1 z)}$

$\mathbf{B}_R = -\mathbf{e}_y B_{oR} e^{i(\omega t + k_1 z)}$

$\mathbf{E}_T = \mathbf{e}_x E_{oT} e^{i(\omega t - k_2 z)}$

$\mathbf{B}_T = \mathbf{e}_y B_{oT} e^{i(\omega t - k_2 z)}$

Boundary conditions

$E_{||1} = E_{||2}$ $\qquad E_{oI} + E_{oR} = E_{oT}$ $\qquad B_\perp = D_\perp = 0$ (normal incidence)

$H_{||1} = H_{||2}$ $\qquad (B_{oI} - B_{oR}) / \mu_1\mu_o = B_{oT} / \mu_2\mu_o$ $\qquad \mathbf{B} = \mu\mu_o \mathbf{H}$ $\qquad \mu_1 = \mu_2 = 1$

Fresnel Equations

$B_{oI} = n_1 E_{oI} / c$ $B_{oR} = n_1 E_{oR} / c$ $B_{oT} = n_2 E_{oT} / c$

$n_1 (E_{oI} - E_{oR}) = n_2 E_{oT}$ from $B_{oI} - B_{oR} = B_{oT}$ when $\mu_1 = \mu_2 = 1$

$E_{oI} + E_{oR} = E_{oT}$

$E_{oT} = E_{oI} + E_{oR} = n_1(E_{oI} - E_{oR}) / n_2$ Eliminate E_{oT}

$E_{oR} (n_1 + n_2) = E_{oI} (n_1 - n_2)$

$E_{oR} / E_{oI} = (n_1 - n_2) / (n_1 + n_2)$ $E_{oR} / E_{oI} < 0$ if $n_1 < n_2$) => π change of phase

Fresnel Equations

$n_1 (E_{oI} - E_{oR}) = n_2 E_{oT}$ Eliminate E_{oR}

$E_{oI} + E_{oR} = E_{oT}$

$E_{oR} = E_{oT} - E_{oI} = E_{oI} - n_2 E_{oT} / n_1$

$E_{oT} (n_1 + n_2) = 2n_1 E_{oI}$

$E_{oT} / E_{oI} = 2n_1 / (n_1 + n_2)$

$(E_{oR} / E_{oI})^2 + (E_{oT} / E_{oI})^2 = (n_1 - n_2)^2 / (n_1 + n_2)^2 + 4n_1^2 / (n_1 + n_2)^2 \neq 1!$

Fresnel Equations

Reflectivity

$R_\perp = (E_{oR} / E_{oI})^2 = (n_1 - n_2)^2 / (n_1 + n_2)^2$

Transmittivity

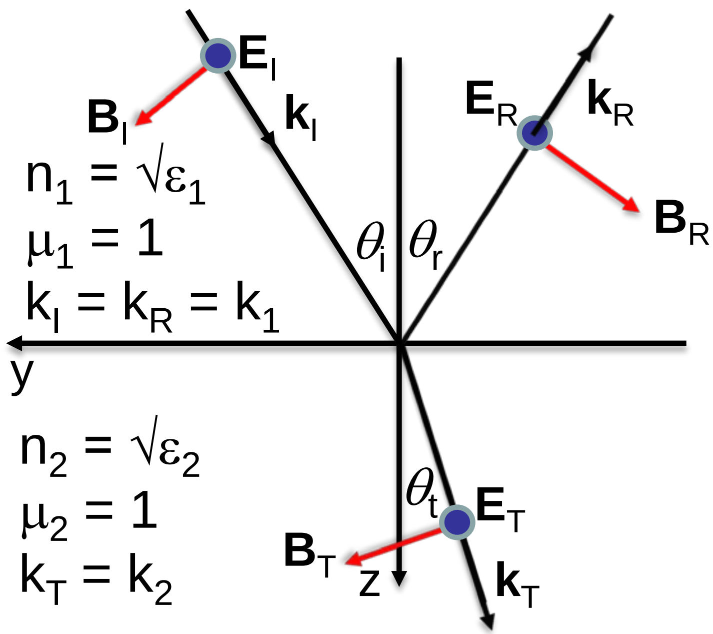
$T_\perp = (E_{oT} / E_{oI})^2 \sqrt{(\mu_2\varepsilon_2)} / \sqrt{(\mu_1\varepsilon_1)} = 4n_1^2 / (n_1 + n_2)^2 (n_2 / n_1) = 4n_1n_2 / (n_1 + n_2)^2$

Energy conservation

$R_\perp + T_\perp = (n_1 - n_2)^2 / (n_1 + n_2)^2 + 4n_1n_2 / (n_1 + n_2)^2 = 1$

Fresnel Equations

Off-normal incidence, s-polarisation



Fields

$\mathbf{E}_I = \mathbf{e}_x E_{oI} e^{i(\omega t - k_1.r)}$

$\mathbf{B}_I = (\mathbf{e}_y \cos\theta_i + \mathbf{e}_z \sin\theta_i) B_{oI} e^{i(\omega t - k_1.r)}$

$\mathbf{E}_R = \mathbf{e}_x E_{oR} e^{i(\omega t + k_1.r)}$

$\mathbf{B}_R = (-\mathbf{e}_y \cos\theta_r + \mathbf{e}_z \sin\theta_r) B_{oR} e^{i(\omega t + k_1.r)}$

$\mathbf{E}_T = \mathbf{e}_x E_{oT} e^{i(\omega t - k_2.r)}$

$\mathbf{B}_T = (\mathbf{e}_y \cos\theta_t + \mathbf{e}_z \sin\theta_t) B_{oT} e^{i(\omega t - k_2.r)}$

Boundary conditions

$E_{||1} = E_{||2}$ $\qquad E_{oI} + E_{oR} = E_{oT}$

$H_{||1} = H_{||2}$ $\qquad (B_{oI} - B_{oR}) \cos\theta_i / \mu_1\mu_o = B_{oT} \cos\theta_t / \mu_2\mu_o \quad \mu_1 = \mu_2 = 1$

Fresnel Equations

$\mathbf{B} = \sqrt{(\mu\varepsilon)}\, \mathbf{k} \times \mathbf{E} / ck = n / (ck)\, \mathbf{k} \times \mathbf{E}$ in uniform dielectric

$B_{oI} = n_1 E_{oI} / c$ $\quad B_{oR} = n_1 E_{oR} / c$ $\quad B_{oT} = n_2 E_{oT} / c$

$n_1 (E_{oI} - E_{oR}) \cos\theta_i = n_2 E_{oT} \cos\theta_t$ from $(B_{oI} - B_{oR}) \cos\theta_i / \mu_1\mu_o = B_{oT} \cos\theta_t / \mu_2\mu_o$
with $\mu_1 = \mu_2 = 1$

$E_{oI} + E_{oR} = E_{oT}$ $\quad$ Eliminate E_{oT}

$E_{oT} = E_{oI} + E_{oR} = n_1(E_{oI} - E_{oR}) \cos\theta_i / (n_2 \cos\theta_t)$

$E_{oR} (n_1 \cos\theta_i + n_2 \cos\theta_t) = E_{oI} (n_1 \cos\theta_i - n_2 \cos\theta_t)$

$E_{oR} / E_{oI} = (n_1 \cos\theta_i - n_2 \cos\theta_t) / (n_1 \cos\theta_i + n_2 \cos\theta_t)$

Fresnel Equations

$n_1 \cos\theta_i (E_{oI} - E_{oR}) = n_2 \cos\theta_t E_{oT}$ Eliminate E_{oR}

$E_{oI} + E_{oR} = E_{oT}$

$E_{oR} = E_{oT} - E_{oI} = E_{oI} - n_2 \cos\theta_t E_{oT} / (n_1 \cos\theta_i)$

$E_{oT} (n_1 \cos\theta_i + n_2 \cos\theta_t) = 2n_1 \cos\theta_i E_{oI}$

$E_{oT} / E_{oI} = 2n_1 \cos\theta_i / (n_1 \cos\theta_i + n_2 \cos\theta_t)$

Reflectivity

$R_S = (E_{oR} / E_{oI})^2 = (n_1 \cos\theta_i - n_2 \cos\theta_t)^2 / (n_1 \cos\theta_i + n_2 \cos\theta_t)^2$

Fresnel Equations

Transmittivity

$$T_S = (E_{oT} / E_{oI})^2 \sqrt{(\mu_2\varepsilon_2)} \cos\theta_t / \sqrt{(\mu_1\varepsilon_1)} \cos\theta_i$$

$$= 4n_1^2 \cos^2\theta_i / (n_1\cos\theta_i + n_2\cos\theta_t)^2 (n_2\cos\theta_t / n_1\cos\theta_i)$$

$$= 4n_1n_2 \cos\theta_i \cos\theta_t / (n_1\cos\theta_i + n_2\cos\theta_t)^2$$

Energy conservation

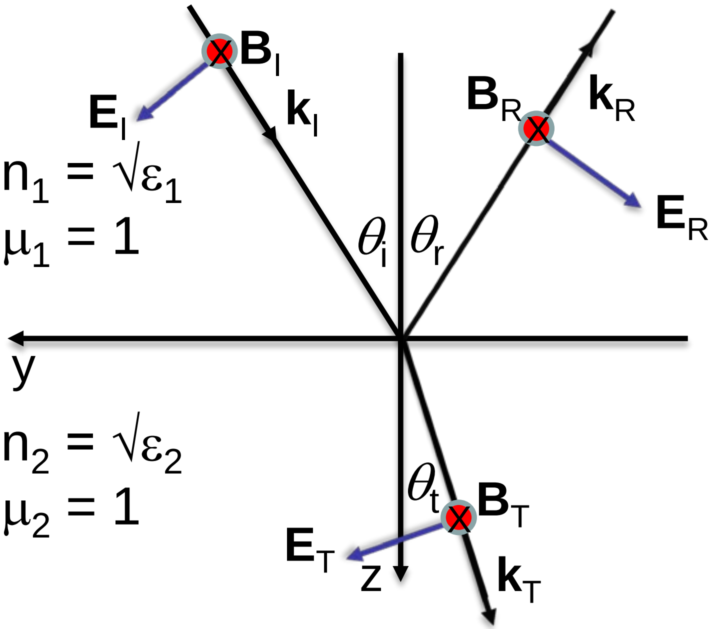
$$R+T = (n_1\cos\theta_i - n_2\cos\theta_t)^2 /(n_1\cos\theta_i + n_2\cos\theta_t)^2 + 4n_1n_2\cos^2\theta_i /(n_1\cos\theta_i + n_2\cos\theta_t)^2$$

$$= (n_1^2\cos^2\theta_i - 2n_1n_2\cos\theta_i \cos\theta_t + n_2^2\cos^2\theta_t + 4n_1n_2\cos\theta_i \cos\theta_t) /(n_1\cos\theta_i + n_2\cos\theta_t)^2$$

$$= 1$$

Fresnel Equations

Off-normal incidence, p-polarisation



$n_1 = \sqrt{\varepsilon_1}$
$\mu_1 = 1$

$n_2 = \sqrt{\varepsilon_2}$
$\mu_2 = 1$

Fields

$\mathbf{E}_I = (\mathbf{e}_y \cos\theta_i + \mathbf{e}_z \sin\theta_i)\, E_{oI}\, e^{i(\omega t - k_1.r)}$

$\mathbf{B}_I = -\mathbf{e}_x\, B_{oI}\, e^{i(\omega t - k_1.r)}$

$\mathbf{E}_R = (-\mathbf{e}_y \cos\theta_r + \mathbf{e}_z \sin\theta_r)\, E_{oR}\, e^{i(\omega t + k_1.r)}$

$\mathbf{B}_R = -\mathbf{e}_x\, B_{oR}\, e^{i(\omega t + k_1.r)}$

$\mathbf{E}_T = (\mathbf{e}_y \cos\theta_t + \mathbf{e}_z \sin\theta_t)\, E_{oT}\, e^{i(\omega t - k_2.r)}$

$\mathbf{B}_T = -\mathbf{e}_x\, B_{oT}\, e^{i(\omega t - k_2.r)}$

Boundary conditions

$E_{||1} = E_{||2}$ $(E_{oI} - E_{oR}) \cos\theta_i = E_{oT} \cos\theta_t$

$H_{||1} = H_{||2}$ $(B_{oI} + B_{oR}) / \mu_1\mu_o = B_{oT} / \mu_2\mu_o$ $\mu_1 = \mu_2 = 1$

Fresnel Equations

$\mathbf{B} = \sqrt{(\mu\varepsilon)}\, \mathbf{k} \times \mathbf{E} / ck = n / (ck)\, \mathbf{k} \times \mathbf{E}$ in uniform dielectric

$B_{oI} = n_1 E_{oI} / c$ $B_{oR} = n_1 E_{oR} / c$ $B_{oT} = n_2 E_{oT} / c$

$n_1 (E_{oI} + E_{oR}) = n_2 E_{oT}$ from $(B_{oI} + B_{oR}) / \mu_1\mu_o = B_{oT} / \mu_2\mu_o$ with $\mu_1 = \mu_2 = 1$

$(E_{oI} - E_{oR}) \cos\theta_i = E_{oT} \cos\theta_t$

$E_{oT} = (E_{oI} + E_{oR})\, n_1 / n_2 = (E_{oI} - E_{oR}) \cos\theta_i / \cos\theta_t$ Eliminate E_{oT}

$E_{oR} (n_1 / n_2 + \cos\theta_i / \cos\theta_t) = E_{oI} (- n_1 / n_2 + \cos\theta_i / \cos\theta_t)$

$$E_{oR} / E_{oI} = (- n_1 / n_2 + \cos\theta_i / \cos\theta_t) / (n_1 / n_2 + \cos\theta_i / \cos\theta_t)$$
$$= (n_2\cos\theta_i - n_1\cos\theta_t) / (n_2\cos\theta_i + n_1\cos\theta_t)$$

Fresnel Equations

Reflectivity

$R_P = (E_{oR} / E_{oI})^2 = (n_2\cos\theta_i - n_1\cos\theta_t)^2 / (n_2\cos\theta_i + n_1\cos\theta_t)^2$

$E_{oR} = E_{oT}\, n_2 / n_1 - E_{oI} = E_{oI} - E_{oT} \cos\theta_t / \cos\theta_i$ Eliminate E_{oR}

$E_{oT} (n_2 / n_1 + \cos\theta_t / \cos\theta_i) = 2E_{oI}$

$E_{oT} / E_{oI} = 2E_{oI} / (n_2 / n_1 + \cos\theta_t / \cos\theta_i)$

$E_{oT} / E_{oI} = 2 / (n_2 / n_1 + \cos\theta_t / \cos\theta_i) = 2n_1\cos\theta_i / (n_1\cos\theta_t + n_2\cos\theta_i)$

Fresnel Equations

Transmittivity

$T_P = (E_{oT} / E_{oI})^2 \sqrt{(\mu_2\varepsilon_2)} \cos\theta_t / \sqrt{(\mu_1\varepsilon_1)} \cos\theta_i$

$= 4n_1{}^2\cos^2\theta_i\, n_2\cos\theta_t / (n_1\cos\theta_t + n_2\cos\theta_i)^2\, n_1\cos\theta_i$

$= 4n_1\cos\theta_i\, n_2\cos\theta_t / (n_1\cos\theta_t + n_2\cos\theta_i)^2$

Energy conservation

$R_P + T_P = ((n_2\cos\theta_i - n_1\cos\theta_t)^2 + 4n_1\cos\theta_i\, n_2\cos\theta_t) / (n_2\cos\theta_i + n_1\cos\theta_t)^2 = 1$

Fresnel Equations

Normal incidence

$R_{\perp} = (n_1 - n_2)^2 / (n_1 + n_2)^2$

$T_{\perp} = 4\, n_1 n_2 / (n_1 + n_2)^2$

S-polarisation

$R_S = (n_1\cos\theta_i - n_2\cos\theta_t)^2 / (n_1\cos\theta_i + n_2\cos\theta_t)^2$

$T_S = 4 n_1 n_2 \cos\theta_i \cos\theta_t / (n_1\cos\theta_i + n_2\cos\theta_t)^2$

P-polarisation

$R_P = (n_2\cos\theta_i - n_1\cos\theta_t)^2 / (n_2\cos\theta_i + n_1\cos\theta_t)^2$

$T_P = 4 n_1 n_2 \cos\theta_i \cos\theta_t / (n_1\cos\theta_t + n_2\cos\theta_i)^2$

Energy conservation

R + T = 1 in each case

Fresnel Equations

Light polarisation by reflection - the Brewster angle

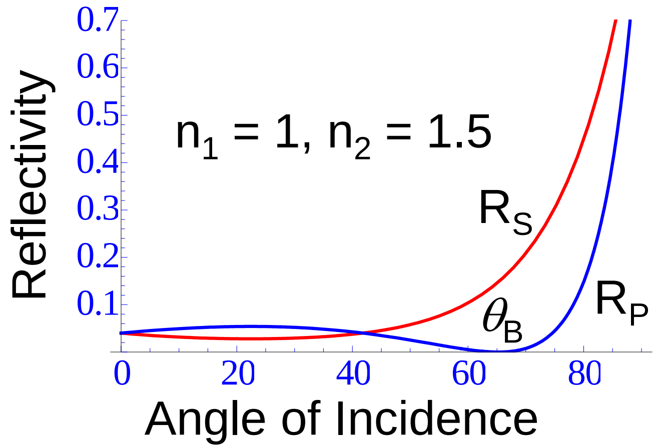
$R_S = (n_1\cos\theta_i - n_2\cos\theta_t)^2 / (n_1\cos\theta_i + n_2\cos\theta_t)^2$

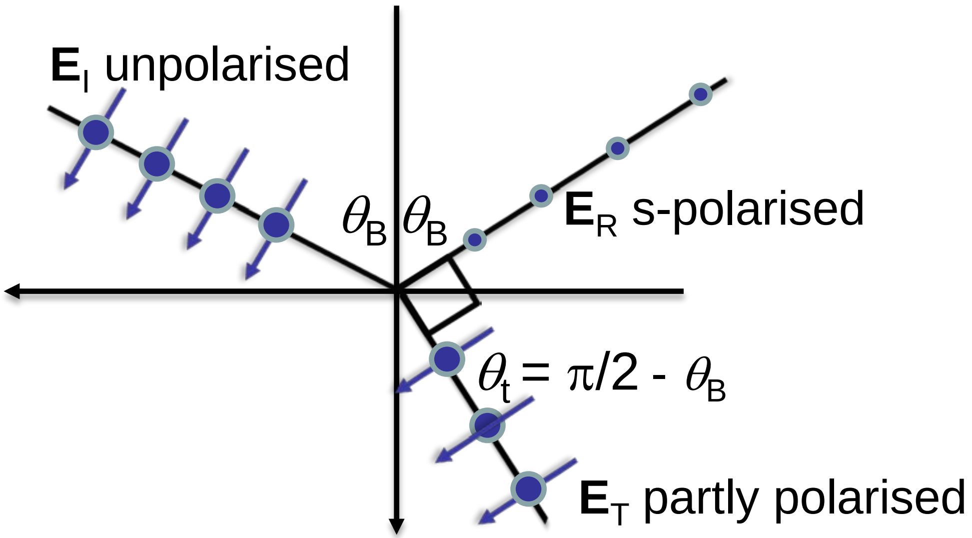
$R_P = (n_2\cos\theta_i - n_1\cos\theta_t)^2 / (n_2\cos\theta_i + n_1\cos\theta_t)^2$

If $n_1 < n_2$ (*e.g.* $n_1 = 1$, $n_2 > 1$), $\theta_i > \theta_t$ then $n_1\cos\theta_i < n_2\cos\theta_t$

Consequently $R_S \neq 0$ for *any* θ_i

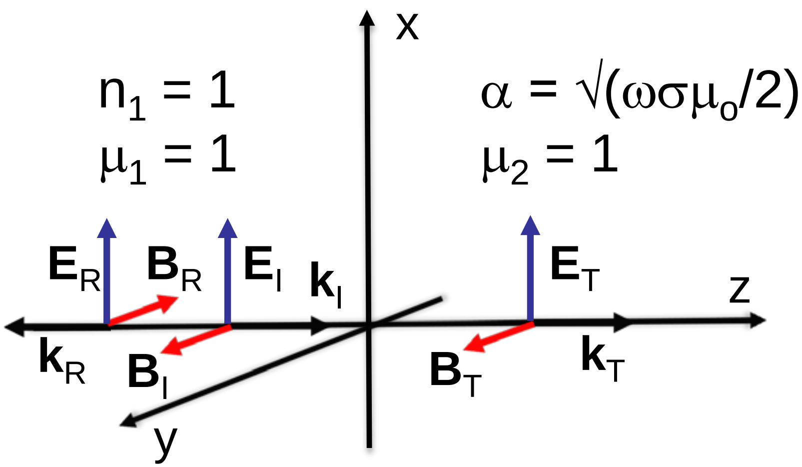
If $n_1 < n_2$, $n_2\cos\theta_i = n_1\cos\theta_t$ then $R_P = 0$ for $\theta_i = \theta_B$ **Brewster angle**





Fresnel Equations

Normal Incidence, metal-vacuum interfaces



Fields

$\mathbf{E}_I = \mathbf{e}_x E_{oI} e^{i(\omega t - k_1 z)}$

$\mathbf{B}_I = \mathbf{e}_y B_{oI} e^{i(\omega t - k_1 z)}$

$\mathbf{E}_R = \mathbf{e}_x E_{oR} e^{i(\omega t + k_1 z)}$

$\mathbf{B}_R = -\mathbf{e}_y B_{oR} e^{i(\omega t + k_1 z)}$

$\mathbf{E}_T = \mathbf{e}_x E_{oT} e^{i(\omega t - \alpha z)} e^{-\alpha z}$

$\mathbf{B}_T = \mathbf{e}_y B_{oT} e^{i(\omega t - \alpha z)} e^{-\alpha z}$

Boundary conditions

$E_{||1} = E_{||2}$ $\quad$ $E_{oI} + E_{oR} = E_{oT}$ $\quad$ $B_\perp = D_\perp = 0$ (normal incidence)

$H_{||1} = H_{||2}$ $\quad$ $(B_{oI} - B_{oR}) / \mu_1\mu_o = B_{oT} / \mu_2\mu_o$ $\quad$ $\mathbf{B} = \mu\mu_o \mathbf{H}$ $\quad$ $\mu_1 = \mu_2 = 1$

Fresnel Equations

$\mathbf{B} \neq \sqrt{(\mu\varepsilon)}\, \mathbf{k} \times \mathbf{E} / ck$ in lossy matter, use Faraday's law instead $\nabla \times \mathbf{E}_T = -\frac{\partial \mathbf{B}_T}{\partial t}$

$\nabla \times \mathbf{E}_T = -\,\mathbf{e}_y\, E_{oT}\, e^{i(\omega t - \alpha z)}\, e^{-\alpha z}\, \alpha(1 + i)$

$-\frac{\partial \mathbf{B}_T}{\partial t} = -i\omega\, \mathbf{e}_y\, B_{oT}\, e^{i(\omega t - \alpha z)}\, e^{-\alpha z}$

$E_{oT}\, \alpha(1 + i) = i\omega\, B_{oT}$

$B_{oI} = n_1 E_{oI} / c$ $\quad$ $B_{oR} = n_1 E_{oR} / c$ $\quad$ $B_{oT} = \alpha(1 - i)\, E_{oT} / \omega$

$H_{||1} = H_{||2}$ becomes $n_1 (E_{oI} - E_{oR}) / c = \alpha(1 - i)\, E_{oT} / \omega$ $\quad$ $\alpha = \sqrt{(\omega\sigma\mu_o/2)}$

$E_{||1} = E_{||2}$ becomes $E_{oI} + E_{oR} = E_{oT}$ $\quad$ set $n_1 = \mu_1 = \mu_2 = 1$

Fresnel Equations

$(E_{oI} - E_{oR}) / c = \alpha(1 - i)\, E_{oT} / \omega$

$E_{oI} + E_{oR} = E_{oT}$ Eliminate E_{oT}

$E_{oT} = E_{oI} + E_{oR} = (E_{oI} - E_{oR}) / a(1 - i)$ $a = \omega / \alpha c = \sqrt{(\sigma/2\varepsilon_o\omega)}$

$E_{oR}\,(a(1 - i) + 1) = E_{oI}\,(1 - a(1 - i))$

$E_{oR} / E_{oI} = (1 - a(1 - i)) / (1 + a(1 - i))$

$R_\perp = |E_{oR} / E_{oI}|^2 \approx 1 - 2 / a = 1 - 2\sqrt{(2\varepsilon_o\omega/\sigma)}$

For Cu metal, $\sigma = 6.7 \times 10^7\ (\Omega m)^{-1}$ For $\omega = 7 \times 10^{14}$ $R_\perp \approx 1 - 2.8 \times 10^{-2} = 0.97$