

## Tutorial Problems for PY2T10 (2013/14)

1). GP 3.1 An electron beam has a circular cross-section of radius,  $R$ , and the charge density within the beam has the constant value  $\rho$ . Solve Gauss' Law in its integral form to find the electric field inside and outside the beam. Then solve Poisson's equation

$$\nabla^2 \phi(r) = -\frac{\rho}{\epsilon_0} \quad \text{inside } R$$

$$\nabla^2 \phi(r) = 0 \quad \text{outside } R$$

for the electrostatic potential inside and outside the beam. *Hints:* Think about the symmetry of the problem. Use the field deduced from Gauss' law to find the form of the potential which solves Poisson's equation in each region. In cylindrical coordinates the radial part of the Laplacian operator is

$$(\nabla^2)_r = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right)$$

2). SF Ann 2013 The magnetic induction  $\mathbf{B}$  field inside a wire of radius,  $R$ , carrying a steady current is

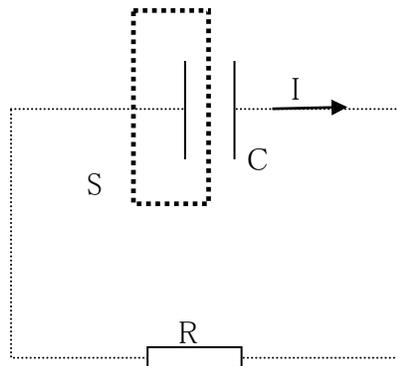
$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I_0}{2\pi R^2} (-y, x, 0)$$

Suppose that the voltage source causing the current is switched off and that the current decays as

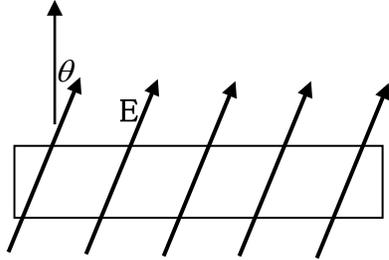
$$I(t) = I_0 e^{-\alpha t}$$

If the  $\mathbf{B}$  field decays with time in the same way, what  $\mathbf{E}$  field does Faraday's Law predict inside the wire? *Hint:* The  $\mathbf{E}$  field should be radially symmetric inside the wire since the decaying  $\mathbf{B}$  field is radially symmetric.

3. Positive charge leaves one plate of a parallel plate capacitor which is discharging through a resistor. At a certain time the rate of change is  $I$  Amps. Calculate the displacement current flowing at this instant through a surface  $S$  which encloses one plate of the capacitor. Show that the magnitude of the displacement current is equal to the conduction current  $I$ .



4). A slab of dielectric of relative permittivity  $\epsilon$  is placed in a uniform external field  $\mathbf{E}$  whose field lines make an angle  $\theta$  with a normal to the surface of the slab. What is the density of polarisation charge on the surface of the slab? *Hint*: Consider the *total* field inside the slab, which is the externally applied field plus the depolarising field from surface charges.



5). A small sphere of radius  $R$  is uniformly charged throughout its volume with charge density,  $\rho_c$ , and mass density,  $\rho_m$ . It is rotating with constant angular velocity,  $\omega$ . The magnetic moment,  $\mathbf{m}$ , is given in terms of the current density,  $\mathbf{j}$ , by

$$\mathbf{m} = \frac{1}{2} \int_{\text{all space}} \mathbf{r} \times \mathbf{j}(\mathbf{r}) \, d\mathbf{r}$$

(a) Show that the magnetic moment is directed along the axis of rotation and has magnitude  $4 \rho_c \omega R^5/15$ . *Hints*:  $\mathbf{j} = \rho_c \mathbf{v}$ , components of  $\mathbf{m}$  perpendicular to the rotation axis cancel by symmetry, the integral of  $\sin^3\theta$  from 0 to  $\pi$  is  $4/3$ .

(b) The angular momentum,  $\mathbf{L}$ , of the rotating mass is parallel to the magnetic moment and has magnitude  $8 \rho_m \omega R^5/15$ . Show that the magnetic moment and angular momentum vectors are related by

$$\mathbf{m} = Q / 2M \mathbf{L}.$$

6). GP 11.8 The electrical conductivity of sea water is about  $4 (\Omega\text{m})^{-1}$ . What is the skin depth for low frequency radio waves of wavelength 3000 m?