Homework Problems II for PY2T10

1. The magnetic moment, **m**, of a current density, **j**(**r'**), is defined to be

$$\mathbf{m} = \frac{1}{2}\int \mathbf{r'} \times \mathbf{j}(\mathbf{r'})\, d\mathbf{r'}$$

For a circular current loop of radius, R, the current density is $\mathbf{j} = \frac{I}{a}\hat{\theta}$,

where I is the current, a is the cross-sectional area of a thin wire carrying the current and $\hat{\theta}$ is a unit vector along the wire parallel to the current. Show that the magnetic moment for this current loop is IA**z**, where **z** is a unit vector perpendicular to the current loop and A is the area of the loop, πR^2. *Hint:* Use cylindrical coordinates in this problem.

2. The potential energy, U, for a magnetic moment in a magnetic field is U = -**m**.**B** and the torque, **N**, on a magnetic moment in a magnetic field is **N** = **m** x **B**.

An electron orbits a nucleus in a circle with a radius of 0.1 nm at an angular frequency of 10^{15} rad s^{-1}. It is in a **B** field of strength 1 Tesla.

(a) Evaluate the change in potential energy of the atom when the magnetic moment of the electron changes from being exactly parallel to the **B** field to being exactly anti-parallel.

(b) Evaluate the torque on the atom (magnitude and direction) when the magnetic moment lies in the xz plane, inclined at an angle of 60^{o} to the z axis and the magnetic field is along the z axis.

(c) The magnetic moment and angular momentum vectors for the atom above are anti-parallel for a negative (electron) charge. The equation of motion relating torque and angular momentum is d**L**/dt = **N**. Describe the motions of the magnetic moment and angular momentum vectors by considering the directions of **N** and d**L**/dt or d**m**/dt. *Hint:* Think about the change in the tangential velocity vector, **v**, in circular motion over a short time, Δt, and how this relates to the acceleration of the body towards the centre of motion.

3. Show that

$$\mathbf{j}(\mathbf{r'}) \times \frac{\mathbf{r} - \mathbf{r'}}{|\mathbf{r} - \mathbf{r'}|^3} = \nabla \times \frac{\mathbf{j}(\mathbf{r'})}{|\mathbf{r} - \mathbf{r'}|}$$

using

$$\frac{\mathbf{r} - \mathbf{r'}}{|\mathbf{r} - \mathbf{r'}|^3} = -\nabla \frac{1}{|\mathbf{r} - \mathbf{r'}|}$$

and the identity $\nabla \times (\psi \mathbf{A}) = \psi \nabla \times \mathbf{A} + \nabla\psi \times \mathbf{A}$ where ψ is a scalar field and **A** is a vector field. *Hint:* ∇ acts on functions of **r** only.