UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

Hilary 2008

SF Engineers
SF MEMS
SF MSISS
Foundation Scholarship

Course 2E1/2E2

Wednesday, March 12
Exam Hall
09.30 — 12.30

Dr. S.A. Frolov/ Dr. D. Zaitsev

Answer sections A and B in SEPARATE answer books. Credit will be given for the best THREE questions answered from section A and best THREE questions answered from section B.

Log tables are available from invigilators if required.

Non-programmable calculators are permitted for this examination, — please indicate the make and model of your calculator on each answer book used.
SECTION A

1. Consider the function \( f(x, y, z) = \sqrt{x + 2y - 2z}; \) at a point \( P = (0, 2, 0). \)
   
   (a) Find a unit vector in the direction in which \( f \) increases most rapidly at \( P. \)
   
   (b) Find the rate of change of \( f \) at \( P \) in that direction.
   
   Show the details of your work.

2. Use double integration to find the volume of the solid in the first octant bounded by the paraboloid \( z = 2x^2 + 6y^2, \) below by the plane \( z = 0, \) and laterally by \( y = x^2 \) and \( y = x. \) Sketch the projection of the solid onto the \( xy \)-plane. Show the details of your work.

3. Show that the integral is independent of the path, and use the Fundamental Theorem of Line Integrals to find its value. Show the details of your work.
   
   \[
   \int_{(-1,2)}^{(0,1)} (2x - y + 2) \, dx - (x + 4y + 1) \, dy.
   \]

4. Use the Divergence Theorem to find the flux of \( \mathbf{F} \) across the surface \( \sigma \) with outward orientation.

   \[
   \mathbf{F}(x, y, z) = (3x + 2z^2) \mathbf{i} - (2y + 3x^3) \mathbf{j} + (5x + y^3) \mathbf{k},
   \]

   and \( \sigma \) is the sphere \( x^2 + y^2 + z^2 = 4. \) Show the details of your work.
SECTION B

5. Find all vectors \((a, b)\) whose image under rotation through the angle \(\frac{\pi}{2}\) about the origin is \((b, a)\).

6. For all \(\lambda\) determine the span of the vectors

\[ v_1 = (\lambda, 1, 1), \quad v_2 = (1, \lambda, 1), \quad v_3 = (\lambda, 1, 1). \]

7. For each \(a\), find bases and dimensions for the row, column and null spaces of the matrix:

\[
\begin{pmatrix}
a & 1 & a \\
1 & a & 1
\end{pmatrix}
\]

8. Use the Gram-Schmidt process to transform the basis

\[ u_1 = (1, 0, 0), \quad u_2 = (0, 1, 0), \quad u_3 = (1, 1, 1) \]

into an orthogonal one with respect to the inner product

\[ \langle (u_1, u_2, u_3), (v_1, v_2, v_3) \rangle = (u_1 - u_2)(v_1 - v_2) + u_2v_2 + u_3v_3. \]