2E1 ENGINEERING MATHEMATICS III [5 credits]

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Module organisation
The module runs for the first half (12 weeks) of the academic year and comprises of three lectures and one tutorial per week (total of 43 hours contact time).

Module description, aims and contribution to programme
- to introduce and illustrate the fundamental ideas and methods of Linear Algebra and Fourier Analysis;
- to introduce the concept of n-dimensional vectors and show their role and importance in practice;
- to show the interrelations between linear systems, linear transformations and their matrices;
- to promote mathematical confidence and sensibility;
- to enable the students to apply their knowledge to new situations.

Learning outcomes
Upon completion of this module, students will be able to:
1. Pass effectively between linear systems, linear transformations and their matrices.
2. Analyse a system of vectors for linear dependence and for being a basis.
3. Calculate dimension of a subspace.
4. Calculate the rank and nullity of a matrix and understand their importance.
5. Construct a basis for row, column, and null spaces of a matrix.
6. Calculate eigenvalues and eigenvectors of matrices.
7. Apply the Gram-Schmidt process to transform a given basis into orthogonal one.
8. Apply methods of general and particular solutions to ordinary differential equations.
9. Calculate the Fourier series of a given function and analyse its behaviour.
10. Apply Fourier series to solving ordinary differential equations.
11. Calculate the Fourier transformation.

Module content
- Euclidean n-space and n-vectors;
- Linear transformations and their matrices; subspaces; linear combinations of vectors;
- Subspaces spanned by a set of vectors; linear independence of a set of vectors;
- Basis and dimension; standard basis in n-space; coordinates of vectors relative to a basis;
- General and particular solutions for a linear system;
- Row, column and null space of a matrix, finding bases for them using elementary row operations, rank and nullity of a matrix;
- Inner products, lengths, distances and angles relative to them;
- Orthogonal and orthonormal bases relative to an inner product, orthogonal projections to subspaces, Gram-Schmidt Process;
- Best approximation by the least squares method;
- Eigenvalues and eigenvectors of square matrices;
- Fourier series for periodic functions, Euler formulas for the Fourier coefficients, even and odd functions, Fourier cosine and Fourier sine series for them, Fourier integral and Fourier transform.

**Teaching strategies**
The teaching strategy is a mixture of lectures, independent and team-based homework and tutorials. The lectures present the material in traditional form, including motivation, theory and uses. The most critical phenomena and typical mistakes are emphasised. The exercises are assigned weekly and aimed to stimulate students to actively use and revise the learned material. As an important by-product, students learn how to express their way of solving problems clearly in written form. This process is controlled by grading the student solutions and discussing them in the tutorials.

**Assessment**
Assessment is by means of tutorial assignments and a two-hour end-of-year written examination. The overall grade is calculated using the maximum of 90% end-of-year examination + 10% assessment.

**Required textbook**
*Elementary Linear Algebra (with applications)*, Anton and Rorres, Chapters 4 - 7
*Advanced Engineering Mathematics*, Kreyszig, Chapter 10

**Further information**
[http://www.maths.tcd.ie/~zaitsev/2E02-2011/2E02.html](http://www.maths.tcd.ie/~zaitsev/2E02-2011/2E02.html)