Commodity futures (Sharpe)

May: buy 5000b. July wheat, F=\$4.40 per b.

Initial margin: e.g. 5%×5,000×\$4.40 = \$1,100

(i) F & spot price S converge at delivery date.

If > \$4.40, 'long' has profit if contract held to delivery

= loss of corresponding 'short'.

(ii) before delivery, as F changes, the ± on open positions credited/debited to margin A/Cs daily.

e.g. day 2: F=\$4.43: credit long 3¢×5,000

(iii) If margin A/C falls below maintenance margin: then a *margin call* occurs. If margin A/C > requirements, then may withdraw excess.

(iv) Suppose F = \$4.50 in June.

Net credit since May: 5,000×10c. = \$500

-*Either* close out, by cash settlement: i.e. sell 5,000b. July wheat, and, close margin A/C including ± interest and profit of \$500.

-*Or* withdraw all/part of \$500, and continue.

(v) At delivery all open positions must be closed

- *Either* sell 5,000b. July wheat futures at delivery (cash settlement)

- *Or* take delivery of 5,000 b. at settlement price.

Hedging interest rates (1)

Debt to roll over in 1 period: use short hedge.

Debt instrument: discount bond, which matures at par (100) in 1 period's time, when you will redeem it & sell another. Futures contracts are written on these bonds, which happen to deliver in one period from now.

Initially, S=92, F=94. Sell one contract: locks in $R=\frac{6}{94}\times 100\%$ on loan starting at the delivery date.

Some possibilities: **at delivery,**

(a) S=F=91. Close at $\pi=3$. Sell bond for cash 91,

or

(b) S=F=95. Close at $\pi=-1$. Sell bond for cash 95.

In all cases, in effect, price of 94 locked in;

alternatively, make delivery$\rightarrow$same result.

Hedging

A perfect hedge is unlikely

- at delivery, F=S for delivery grade;

- but (a) the position being hedged might not be in this grade (*quality*; also, for commodities, *location*) ['Cross hedging'];

- (b) futures contracts relate to standardized quantities;

- & (c) delivery dates of futures contracts may differ from those of actuals activities;

- (a), (b) & (c) → 'basis risk' (see later);

- 'marking to market' & initial margin → ± interest.

<u>Hedging interest rates (2)</u>

As (1), except delivery date assumed much later than 'roll over'. *Basis* F–S=94–92=2 initially.

Sell 1 contract, & close out on rollover date. If e.g. F=97 then, then loss = 3.

If basis = 2 (as initially), then S=95 at that point.

Sell bond for 95, meet margin–call of 3. Net receipts 92.

–Generally, net receipts= $S_1-(F_1-F_0)= F_0-(F_1-S_1)$

<u>Iff the basis remains constant</u> **at (F_0-S_0) then net receipts = S_0**

Hedging to minimize risk

The optimal hedge-ratio h* of futures contracts to 'actuals' exposure minimizes the variance of the change in the value V of the hedged position over the life of the hedge (Hull, ch. 3.4).

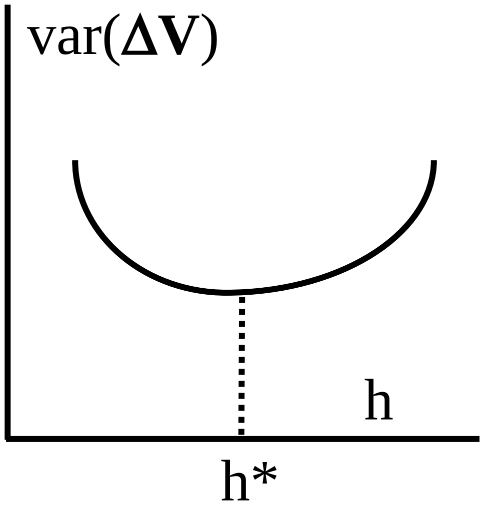
$\Delta V = \Delta S - h\Delta F$; and

$\mathrm{var}(\Delta V) = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$, so

$$\frac{\partial \mathrm{var}}{\partial h} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F$$

$$= 0 \text{ (for a min.)}$$

So $$h^* = \rho\,\frac{\sigma_S}{\sigma_F}$$



If $\rho=1$ & $\sigma_S=\sigma_F$:   then $h^* = 1$.

If $\rho\leq 1$ & $\sigma_S\leq\sigma_F$ with at least one strict: $h^*<1$:

- interpretation: hedged & unhedged positions are different assets, and each earns its position in an optimal portfolio.

Arbitrage (ignore 'margin' issues)

A riskless discount bond:

S=95, Maturity after 1 period. F=106

R_F=10% p.p. Assertion: F too high, S too low.

- borrow 95 for 1 period @ R_F

- buy 1 bond

- sell 1 futures contract.

After 1 period, repay 95×1.1	**= –104.50**
+ deliver on the futures contract	**= S_1**
+ profit on futures	**= (106–S_1)**

(equivalently cash-settle the futures contract at profit 106–S_1 and sell the bond 'cash' for S_1)

Profit 1.50 arises because $S(1+R_F) < F$

i.e. cost of carry $R_F S < F-S$ basis

To exclude arbitrage, need "=".

Problems with cash-and-carry arbitrage

(1) Initial margin + marking to market $\rightarrow$ interest $\pm$.

(2) Dividends: cost of carry becomes $R_F S - FV(\text{dividend})$.

(3) Bid-ask spreads, transactions costs, taxes.

(4) Arbs not the only transactors.

Some deviation likely, but once *basis and cost of carry* get too far apart, arbitraging ('programme trading') begins, until:

1. Long underlying+short futures+borrow=0

2. Short underlying+long futures+lending=0

e.g. If index futures reach large enough discount v. index: sell stock, buy futures and buy T-bills.

From 1. short index futures=short stock+lending, & 2., long index futures=long stock+borrowing. Implications for portfolio rebalancing?

Portfolio insurance

- We know that, for some m>1,

1 stock L. + m calls S. = riskless

- Also, by put-call parity,

m stock L. + m calls S.+ m puts L. = riskless

Subtracting the first from the second,

→L. stock +L. puts = L. riskless bond

or L. puts = S. stock + L. bonds:

i.e. synthetic puts.

***Problem*: short stock requirement.**

***Solution:* use short futures. We can synthesize puts on the stock index, *provided cash-and-carry arbitrage is respected*. See problem.**

Futures prices and E(S) **(Hull 5.14)**

Suppose that you speculate on a rise in the price S of an asset, taking a long futures position, while putting an amount PV(F) into the riskless asset.

At delivery, cash-settle, with profit = $S_1 - F$

(equivalently, take delivery & sell for cash)

+ matured value of bond $\underline{+F}$

$= S_1$

Initially $E(PV) = -F/(1+R_F) + E(S_1)/(1+k)$ where k is risk-adjusted. If $E(PV)=0$, then:

$E(S_1)/(1+k)=F/(1+R_F)$ or $E(S_1)=F(1+k)/(1+R_F)$

If S_1 is uncorrelated with the market, the speculation has no systematic risk, $k=R_F$, & $E(S_1)=F$. If S_1 is positively correlated with the market, $k>R_F$, and $E(S_1)>F$.

Derivatives: extensions

European options on assets paying a continuous dividend yield at the rate q (Hull, ch. 16.3)

Replace S with Se^{-qT} in Black & Scholes.

Logic: suppose that the underlying cash-flows grow at the cts. rate g, with/without dividends.

With dividends, $S_T = Se^{(g-q)T} = (Se^{-qT})e^{gT}$

i.e. buying dividend-paying stock for S is like buying non-dividend-paying stock for (Se^{-qT}).

Applications: (Hull ch. 16, 17)

Options on futures;

Currency options;

also: Interest-rate caps (Hull, ch. 28.2).

Options on futures

Recall: $F=S(1+R_F)^T$ or $F = Se^{R_F T}$ in cts. time.

In B&S, replace S with $Fe^{-R_F T}$:

Black's model: Hull, ch. 17.8.

European currency options

Currency is like a stock with a known dividend yield: the foreign riskless rate.

Puts & calls are options to buy/sell €1 at an exercise price $E. Other variables: $S, $F (fwd), $p, $c, $R_\$$. In B&S, replace S with $Se^{-R_€ T}$.

Put-call parity:

Buy a call, write a put, at same E, & sell €1 fwd.

Value at expiry

	S>E	S<E
Call	**S–E**	**0**
Put	**0**	**–(E–S)**
Fwd	**F–S**	**F–S**
Total	**F–E**	**F–E**

Riskless return if F–E>0, so arbitrage $\Rightarrow$

$$c - p = \frac{F - E}{1 + R_{\$}}$$

*** This is *put-call parity***

*** *What if* F–E<0? =0? *what if* c–p $\neq \frac{F-E}{1+R_{\$}}$?**

*** What assumptions have I made?**

Strategies involving currency options

L. call+S. put+S. Fwd = riskless bond;

S. call+L. put+L. fwd = riskless bond.

Any one instrument (S.or L.) may be synthesized from the other three.

[Q. - using p.c.p. for options on stock, how could you synthesize a short-stock position?]

NB: put on $ = call on €; call on $ = put on €.

Caps **(Hull, ch. 28)**

I face n successive quarterly interest payments on a loan of size L. R_k = 3-mo Libor at stage k, and R= min(R_k,R_X) where R_X is the cap rate. At t=k+1, the writer pays: 0.25L.max(R_k–R_X,0). This is a European call option on a future (or rather forward) interest rate.

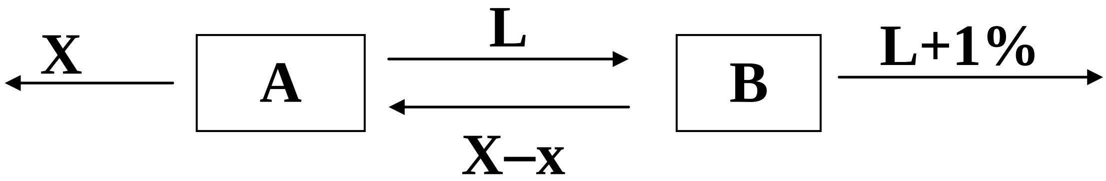
Interest rate swaps **(Hull, ch. 7)**

	Borrowing rates	
Type	**A's**	**B's**
Fixed	**X**	**X+1.2%**
Floating	**L(ibor)+0.3%**	**L+1%= L+0.3%+0.7%**

Both types cheaper for A; 0.7%<1.2% so B has a comparative advantage in floating. B wants fixed rate, A wants floating.

A borrows €1m at X (fixed), B borrows €1m at L+1% (floating).

A pays B periodically at Libor on €1m. B pays A at X–x, where x is an agreed margin.



A's net cost is L+x, a floating rate.

B's net cost is X+1% –x, a fixed rate.

B gains if x>0. Need x<0.3% for A to gain also.

Total gain: (0.3%–x)+(1.2%–1%+x)=1.2%–0.7%