Product Differentiation
Firms seek to be unique along some dimension that is valued by consumers. If the firm/product is unique in some respect, the firm can command a price greater than cost.

• **Horizontal product differentiation:** Consumers have different preferences along one dimension of a good.

• **Vertical product differentiation:** Consumers have the same ordinal preferences, but not the same cardinal preferences. –e.g. all consumers prefer better fuel efficiency, but their willingness to pay will differ.

Three Approach’s

Exogenous Product Differentiation


Horizontal Product Differentiation


Vertical Product Differentiation

**Exogenous Product Differentiation**


**Nash Equilibrium in Prices**

Assume 2 firms, with marginal and fixed costs zero. Assume exogenously imposed product differentiation

\[ q_i = a - bp_1 + \theta p_2 \]
\[ q_2 = a + \theta p_1 - bp_2 \]

The profit function for firm 1 is
\[ \pi_1 = p_1(a - bp_1 + \theta p_2) \]
F.O.C.:
\[ \frac{\partial \pi_1}{\partial p_1} = a - 2bp_1 + \theta p_2 = 0 \]

Best-response
\[ p_1^*(p_2) = \frac{a + \theta p_2}{2b} \]

Similarly:
\[ p_2^*(p_1) = \frac{a + \theta p_1}{2b} \]

Thus, \[ p_i = \frac{a + \theta p_j}{2b} \]
\[ p_1^* = a / (2b - \theta) = p_2^* \]
\[ p_i^* = a / (2b - \theta) \]
\[ q_i^* = ab / (2b - \theta) \]
\[ \pi_i^* = a^2b / (2b - \theta)^2 \]
Nash Equilibrium in Quantities (Cournot)
Find $p_1 = f(q_1, q_2)$ and $p_2 = g(q_1, q_2)$?

Inverse Demand function:

$$
\begin{align*}
p_1 &= \frac{a(1 + \theta)}{b - \theta^2} - \left( \frac{1}{b - \theta^2} \right)(q_1) - \left( \frac{\theta b}{b - \theta^2} \right)(q_2)
\end{align*}
$$

To simplify, rewrite as ,

$p_1 = \alpha - \beta q_1 - \gamma q_2$

Similarly,

$p_2 = \alpha - \gamma q_1 - \beta q_2$ where $\beta > 0$, and $\beta^2 > \gamma^2$

Max $\pi_i(q_1, q_2) = (\alpha - \beta q_i - \gamma q_j)q_i$ where $i, j = 1, 2$ and $i \neq j$

F. O. C.

$\frac{\partial \pi_i}{\partial q_i} = 0 = \alpha - 2\beta q_i - \gamma q_j$

Best-response

$q_i^* = R_i(q_j) = (\alpha - \gamma q_j) / 2\beta$

Solve for the equilibrium

$q_i^* = \alpha / (2\beta + \gamma)$

$p_i^* = \alpha\beta / (2\beta + \gamma)$

$\pi_i^* = \alpha^2 \beta / (2\beta + \gamma)^2$
How do Bertrand and Cournot compare now?

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<tr>
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<th>Bertrand</th>
<th>Cournot</th>
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<tr>
<td>$p^*_{i}$</td>
<td>$a / (2b - \theta )$</td>
<td>$\alpha \beta / (2\beta + \gamma)$</td>
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In order to compare them, we need to express \{a, b, \theta\} in terms of \{\alpha, \beta, \gamma\}.

Re-writing the Bertrand solution:

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<tr>
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<td>$\alpha^2 \beta / (2\beta + \gamma)^2$</td>
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Let’s focus on the difference in price.
When $\gamma = \beta$, Bertrand for homogeneous goods gives, $p^b_{i}= 0$ and $p^c_{i}= \alpha /3$
If goods independent, when $\gamma = 0$, monopoly outcomes, $p^b_{i}= p^c_{i}= \alpha /2$

$$p^c_{i} - p^b_{i} = \frac{\alpha}{4\beta^2}$$

In general:

$$p^c_{i} - p^b_{i} = \frac{\alpha}{4\beta^2 - 1}$$

We said $\beta^2 > \gamma^2$, $\alpha >0$, so $p^c_{i} > p^b_{i}$ but has $(\beta/\gamma)^2 \rightarrow \infty$ $p^c_{i}$ converges on $p^b_{i}$.

The difference in price cost margins modelled is smaller when we relax price competition with product differentiation.
Horizontal Product Differentiation


Linear Cities

Hotelling (1929) Umbrellas
1. Consumers uniformly distributed with density 1 along a defined line segment $l$
2. Duopoly
3. Consumers incur transport cost $t$ per unit of distance $d$ traveled to the seller
4. $p^* = p + td$, consumers buy from the seller with lowest $p^*$
5. Let one seller be located at A, the other at B

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A   B
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• Slope of the umbrellas given by $t$ (exogenous)

• Height of the umbrella stem given by $p$ (seller at location B has lower $p$ than seller at A)

• Total cost to the consumer from buying from a particular seller = $p + td$

• Consumers buy from seller with lowest $p + td$

• Equation of the indifferent consumer

$$P_A + td_A = p_B + td_B$$

All consumers located to the left of the indifferent consumer will buy from seller and to the right of the indifferent consumer will buy from seller at B
Hotelling (1929) – “Simple Location Model”

Stage 1

Location Choice

(a,b)

Stage 2

NE Prices

(given location a,b)

No Nash Equilibrium in Pure Strategies
No price Competition in Stage 2

**Stage 1:** Firms A and B Choose location along a Spectrum

*The Principal of Minimal Differentiation – Focus on the Catchment Area*

Payoff for A(B): line segment covered

*A and B are back-to-back, but not in centre:*

![Diagram showing A and B back-to-back but not in centre](image)

*Incentive for A to jump in front of B.....*

*A and B are back-to-back in centre:*

![Diagram showing A and B back-to-back in centre](image)

*Neither firm has an incentive to move location, given the location of the other*

**The Midpoint is a Nash Equilibrium in Locations**

*Downs Theory of Modern Voting*
D’Aspremont et al (1979)  
Assumes Quadratic Transport Costs

Stage 2  
Given locations, find a NE in prices

\[ l - a - b > \]

\[ q_1 = a + d_1 \quad \text{and} \quad q_2 = b + d_2 \]

“indifferent consumer”

\[ p_1 + t d_1^2 = p_2 + t d_2^2 \]
With quadratic costs there is a NE in prices

\[ p_1 + td_1^2 = p_2 + td_2^2 \]
\[ d_1 + d_2 = l - a - b \]

solving for market share, we get

\[ q_1^* = a + d_1 = a + \frac{l-a-b}{2} + \frac{(p_2-p_1)}{2(t(l-a-b))} \]
\[ q_2^* = b + d_2 = b + \frac{l-a-b}{2} + \frac{(p_1-p_2)}{2(t(l-a-b))} \]

Now solve for a Nash equilibrium in prices:

\[ \max_{p_1} \pi_1 = p_1 q_1 \quad \text{s.t.} \quad p_2 \]

\[ \frac{\partial \pi_1}{\partial p_1} = 0 \quad \Rightarrow \quad p_1 = R(p_2) \]
The NE in prices solves as:

\[ p_1^* = t(l - a - b)(1 + \frac{a-b}{3}) \]

\[ p_2^* = t(l - a - b)(1 + \frac{b-a}{3}) \]

Thus, prices depend on locations:

\[ p_1^*(a,b) \] and \[ p_2^*(a,b) \]

**Stage 1:**

*Choose locations*

\[ \pi_1(a,b) \] and \[ \pi_2(a,b) \]

Solve for a Nash equilibrium in locations…

Results indicate that

\[ \frac{\partial \pi_1}{\partial a} < 0 \quad \text{and} \quad \frac{\partial \pi_2}{\partial b} < 0 \]

thus, bigger \( a \) will reduce firm 1’s profit

incentive to minimise \( a \) and move toward end

likewise for firm b

so find NE in locations occurs at \( a = b = 0 \), where firms locate

at the extreme points of the market
Assumption underlying the location models: distribution of consumers was uniform along the line

Non-uniform distribution of consumers:

**Trade-off:** how much competition there is in a certain niche against the available niches

**d’Aspremont et al:**
1. price competition strong
2. concentration of consumers is uniform

⇒ optimal location is at the extremes

“locate in a niche”

**Hotelling:**
1. no price competition
2. concentration of consumers is uniform

⇒ optimal location is at the midpoint

“locate where the demand is”

**General Predictions:**
Could be anywhere along the linear city…. Optimal location trades off the intensity of price competition with market coverage…..
Salop (1979) Circular Road Model

Stage 2:

1. N Sellers are located symmetrically around the circle
2. Circumference is normalised to $= 1$
3. Distance between each seller is thus $1/N$

Given number of firms $N$, find NE in prices
Representative Seller's profit function, where TC = 0:

- sales = 2d
- indifferent consumer:
  \[ p + td = \bar{p} + t[1/N - d] \]

so
\[ 2d = \frac{\bar{p} - p}{t} + \frac{1}{N} \]

\[ \pi = p \cdot 2d = p \left\{ \frac{1}{N} + \frac{\bar{p} - p}{t} \right\} \]

- seller maximises it's profit by choosing p, given \( \bar{p} \),

\[ \frac{\partial \pi}{\partial p} = \frac{1}{N} + \frac{\bar{p} - p}{t} - \frac{p}{t} = 0 \]

- S.N.E. \( \Rightarrow p = \bar{p} \)

\[ \Rightarrow p_i^* = \frac{t}{N} \]

as \( t \to 0 \) \( \Rightarrow p \to MC \)

as \( N \to \infty \) \( \Rightarrow p \to MC \)
Summary of **Short Run** relationship Price cost mark-ups and the number of firms in the market, i.e. the P(N) function, for Horizontally Differentiated goods

P, for any given N, depends on the ‘intensity of competition’ Product Differentiation Relaxes the intensity of price competition
Stage 1:

Enter with sunk cost $\sigma$?

Equilibrium profits solve as:

$$x_i = 2 \frac{d}{N} = \frac{1}{N}$$ \hspace{1em} i.e. where $\bar{p} = p$

$$x_i = 2 \frac{ds}{N} = \frac{s}{N}$$ \hspace{1em} where $s$ is market size

$$\pi_i = p_i x_i = \frac{t}{N} \cdot \frac{s}{N} = t \frac{s}{N^2}$$

Last firm enters where \textit{expost} entry profit $= \sigma$

$$\pi_i = t \frac{s}{N^2} = \sigma$$

Thus, solving for equilibrium number of firms:

$$N^* = \sqrt{t \frac{s}{\sigma}}$$
Summary of Long Run relationship between market concentration \((1/N)\) and market size \(S\) relative to exogenous sunk costs \(\sigma\), for Horizontally Differentiated Goods

\[
C = \frac{1}{N}
\]

Greater product differentiation induces more entry (so less concentration) for any given \(s/\sigma\)

- For a given \(S/\sigma\), the equilibrium level of concentration increases with the intensity of competition
- For a given intensity of competition, the equilibrium level of concentration falls with an increase in Market Size relative to sunk costs
**Vertical Product Differentiation**


Assume

1. A number of firms offer distinct substitute goods which vary in quality

2. Consumer buy 1 unit or zero

3. Zero costs

4. label goods \(k = 1, \ldots, n\), firm \(k\) sells product \(k\) at \(p_k\)

5. Continuum of consumers of different incomes uniformly distributed, with a density of one, along a line segment \(a\) to \(b\).

6. Utility \(0 < u_0 < u_1 < \ldots < u_n\)

\[ U(t, k) = u_k(t - p_k) \]

\[ U(t, 0) = u_0 t \]

7. Define \(C_k = u_k/(u_k - u_{k-1}) > 1\)

Equation of the Indifferent Consumer

\[ u_k(t_k - p_k) = u_{k-1}(t_{k-1} - p_{k-1}) \]

or

\[ t_k = p_{k-1}(1 - C_k) + p_k C_k \]

NOTE: \( \frac{\partial t_k}{\partial p_k} = C_k \) and \( \frac{\partial t_k}{\partial p_{k-1}} = 1 - C_k \)
Market Share:

Consider good n: if $t_n > a$ so more than one good survives

$$\pi_n = p_n(b - t_n)$$

F.O.C

$$\frac{\partial \pi_n}{\partial p_n} = b - t_n - p_n \frac{\partial t_n}{\partial p_n} = 0$$

$$b - t_n - p_n C_n = 0$$

Note $p_n . C_n = t_n - p_{n-1} (1 - C_n)$

$$b - t_n - (t_n - p_{n-1} (1 - C_n)) = 0$$

$$b - 2t_n + p_{n-1} (1 - C_n) < 0$$

∴ $b - 2t_n > 0$

∴ $t_n < b/2$
Now assumes that \( a < \frac{b}{2} \)

\[ \pi_k = p_k(t_{k+1} - t_k) \]

F.O.C

\[ \frac{\partial \pi_k}{\partial p_k} = t_{k+1} - t_k - p_k \left[ (C_{k+1} - 1) + C_k \right] = 0 \]

Note \( p_k \cdot C_k = t_k - p_{k-1} \cdot (1 - C_k) \)

\[ t_{k+1} - 2t_k - p_k \left( C_{k+1} - 1 \right) - p_{k-1} (C_k - 1) > 0 \]

\[
\begin{align*}
\therefore & \quad t_{k+1} - 2t_k > 0 \\
\therefore & \quad t_k < t_{k+1}/2
\end{align*}
\]

It follows that,

\[ t_n < \frac{b}{2}, \quad t_{n-1} < \frac{b}{4}, \quad t_{n-2} < \frac{b}{8} \ldots \quad \text{etc for any } a > 0 \]

There exists a bound independent of product qualities and consumer density to the number of firms which can survive with positive prices at a NE in prices.
Number of firms depends on lower bound to income, $a$.

1. Can have lots of firms, $a \to 0$ and $b \to \infty$, but 2-3 firms will dominate.

2. A firm that sets price equal to zero will win all the market.

3. Pattern of Market shares will be independent of the density of consumers (Market Size).