Self Assessment Solutions

Linear Economic Models

1. Demand and supply in a market are described by the equations

\[
\begin{align*}
Q_d &= 66 - 3P \\
Q_s &= -4 + 2P
\end{align*}
\]

(i) Solve algebraically to find equilibrium \( P \) and \( Q \)

In equilibrium \( Q_d = Q_s \)

\[
\begin{align*}
66 - 3P &= -4 + 2P \\
-5P &= -70 \\
P &= 14
\end{align*}
\]

\( Q_d = Q_s = 66 - 3P = 66 - 3(14) = 24 \)

(ii) How would a per unit sales tax \( t \) affect this equilibrium and comment on how the tax is shared between producers and consumers

Sales tax reduces suppliers price by \( t \) \( (P-t) \)

Supply curve becomes: \( Q_s = -4 + 2(P-t) \)

In equilibrium \( Q_d = Q_s \)

\[
\begin{align*}
66 - 3P &= -4 + 2(P-t) \\
66 - 3P &= -4 + 2P - 2t \\
-5P &= -2t - 66 \\
P &= 14 + \frac{2}{5}t
\end{align*}
\]

\[ Q_d = Q_s = 66 - 3P = 66 - 3(14 + \frac{2}{5}t) = 24 - \frac{6}{5}t \]

Equilibrium price increases by \( \frac{2}{5}t \) of the tax. This implies that the supplier absorbs \( \frac{3}{5}t \) of the tax and receives a price \( P - \frac{3}{5}t \) for its goods. The consumer pays \( \frac{2}{5}t \) of the tax. Equilibrium quantity falls by \( \frac{6}{5}t \).

(iii) What is the equilibrium \( P \) and \( Q \) if the per unit tax is \( t=5 \)

\[ t = 5, \quad Q_s = -4 + 2(P-5) = -4 + 2P - 10 = -14 + 2P \]

In equilibrium \( Q_d = Q_s \)

\[
\begin{align*}
66 - 3P &= -14 + 2P \\
-5P &= -66 \\
P &= 16 \text{ (i.e. } 14 + \frac{2}{5}t) \text{)}
\end{align*}
\]

\[ Q_d = Q_s = 66 - 3P = 66 - 3(16) = 18 \text{ (i.e. } 24 - \frac{6}{5}t) \]

(iv) Illustrate the pre-tax equilibrium and the post-tax equilibrium on a graph

\[
\begin{align*}
Q_d &= 66 - 3P \\
Q_s &= -4 + 2P
\end{align*}
\]
Let $P = 0$
$Q_d = 66$
$P = 22 - Q_d/3$ (Inverse Demand)
Let $Q_d = 0$
$P = 22$

Let $P = 22$
$Q_s = -4 + 2(22) = -4 + 44 = 40$
$P = 2 + Q_s/2$ (Inverse Supply)
Let $Q_s = 0$
$P = 2$

$Q_s = -14 + 2P$
Let $P = 22$
$Q_s = -14 + 2(22) = -14 + 44 = 30$
$P = 7 + Q_s/2$
Let $Q_s = 0$
$P = 7$

Fill in equilibrium before tax, equilibrium after tax, amount paid by consumer, amount paid by producer.

2. The demand and supply functions of a good are given by
$Q_d = 110 - 5P$
$Q_s = 6P$
where $P$, $Q_d$ and $Q_s$ denote price, quantity demanded and quantity supplied respectively.

(i) Find the inverse demand and supply functions

$Q_d = 110 - 5P$
$5P = 110 - Q_d$
$P = 110 - Q_d/5$

$Q_s = 6P$
$P = Q_s/6$

(ii) Find the equilibrium price and quantity
Solve simultaneously:
$Q_d = 110 - 5P$
$Q_s = 6P$

At equilibrium $Q_d = Q_s$
Collect the terms
-5P - 6P = -110
11P = 110
P = 110/11
P = 10

Solve for Q*
Qd = Qs = 6P = 6(10) = 60 = Q*

3. Demand and supply in a market are described by the equations
   \[ Q_d = 120 - 8P \]
   \[ Q_s = -6 + 4P \]
   a. Solve algebraically to find equilibrium P and Q

   Qd = Qs
   120-8P = -6+4P
   -8P-4P = -6-120
   -12P = -126
   12P = 126
   \[ P^* = 10.5 \]

   Qd = Qs = 120-8P = 120-8(10.5) = 120-84 = 36 = Q*

   b. How would a per unit sales tax t affect this equilibrium and comment on how the tax is shared between producers and consumers?

   Supply price becomes P-t
   Supply function becomes Qs = -6+4(P-t)
   Solve for equilibrium
   Qd = Qs
   120-8P = -6+4(P-t)
   120-8P = -6+4P-4t
   -8P-4P = -120-6-4t
   -12P = -126-4t
   12P = 126+4t
   P = 10.5+4t/12
   P = 10.5+t/3

   Qd = Qs = 120-8(10.5+t/3) = Q*
   Q* = 120-84-8t/3
   Q* = 36-8/3t

   The impact of the tax will therefore be to increase equilibrium price by 1/3 and reduce equilibrium quantity by 8/3. Since 1/3 of tax is passed on to the consumer the supplier pays 2/3 of the tax.

   c. What is the equilibrium P and Q if the per unit tax is 4.5
\[ P = 10.5 + \frac{t}{3} \]
\[ P = 10.5 + \frac{4.5}{3} \]
\[ P = 10.5 + 1.5 \]
\[ P = 12 \]

Supplier gets \(10.5 - \frac{2}{3}t = 10.5 - 3 = 7.5\)

\[ Q = 36 - \frac{8}{3}t \]
\[ Q = 36 - \frac{8}{3}(4.5) \]
\[ Q = 36 - 12 \]
\[ Q = 24 \]

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4. At a price of €15, and an average income of €40, the demand for CDs was 36. When the price increased to €20, with income remaining unchanged at €40, the demand for CDs fell to 21. When income rose to €60, at the original price €15, demand rose to 40.

i) Find the linear function which describes this demand behaviour

General Form: \[ Q_d = a + bP + cY \]

<table>
<thead>
<tr>
<th>Price (P)</th>
<th>Demand (Qd)</th>
<th>Income (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>60</td>
</tr>
<tr>
<td>15</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

\[ Eq1 \quad 36 = a + 15b + 40c \]
\[ Eq2 \quad 40 = a + 15b + 60c \]
\[ Eq3 \quad 21 = a + 20b + 40c \]

Solve Simultaneously

\[ Eq1 \quad 36 = a + 15b + 40c \]
\[ Eq2 \quad 40 = a + 15b + 60c \]

STEP 1
\[ a = 36 - 15b - 40c \]
\[ a = 40 - 15b - 60c \]

STEP 2
\[ 36 - 15b - 40c = 40 - 15b - 60c \]

STEP 3
\[ -15b + 15b - 40c + 60c = 40 - 36 \]
\[ 20c = 4 \]
\[ c = \frac{4}{20} = \frac{1}{5} \]

STEP 4
\[ Eq1 \quad 36 = a + 15b + 40\left(\frac{1}{5}\right) \]
\[ 36 = a + 15b + 8 \]
\[ 36 - 8 = a + 15b \]
\[ 28 = a + 15b \]

\[ Eq3 \quad 21 = a + 20b + 40\left(\frac{1}{5}\right) \]
\[ 21 - 8 = a + 20b \]
\[ 13 = a + 20b \]

**STEP 1**

Eq1' \[ 28 = a + 15b \]
Eq2' \[ 13 = a + 20b \]

\[ a = 28 - 15b \]
\[ a = 13 - 20b \]

**STEP 2**

\[ 28 - 15b = 13 - 20b \]

**STEP 3**

\[ -15b + 20b = 13 - 28 \]
\[ 5b = -15 \]
\[ b = -3 \]

**STEP 4**

\[ a = 28 - 15b \]
\[ a = 28 - 15(-3) \]
\[ a = 28 + 45 \]
\[ a = 73 \]

**General Form**

\[ Q_d = a + bP + cY \]
\[ Q_d = 73 - 3P + \frac{1}{5}Y \]

ii) **Given the supply function** \( Q_s = -7 + 2P \) **find the equations which describe fully the comparative statics of the model.**

\[ Q_d = 73 - 3P + \frac{1}{5}Y \]
\[ Q_s = -7 + 2P \]

In equilibrium \( Q_d = Q_s \)

\[ 73 - 3P + \frac{1}{5}Y = -7 + 2P \]
\[ -3P - 2P = -7 - 73 - \frac{1}{5}Y \]
\[ 5P = 80 + \frac{1}{5}Y \]
\[ P^* = 16 + \frac{1}{25}Y \]

\[ Q_d = Q_s = -7 + 2P = -7 + 2(16 + \frac{1}{25}Y) = -7 + 32 + \frac{2}{25}Y = 25 + \frac{2}{25}Y = Q^* \]

iii) **What would equilibrium price and quantity be if income was €50?**

\[ P^* = 16 + \frac{1}{25}Y = 16 + 1/25(50) = 16 + 2 = 18 \]
\[ Q^* = 25 + \frac{2}{25}Y = 25 + 2/25(50) = 25 + 4 = 29 \]