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Industrial Organization
Answer Key to Assignment # 3

1. *In a linear Hotelling town there are* 100 *potential costumers that are uniformly distributed on a unit mile. Each consumer has a willingness to pay for pizza of* \$30, *and would buy only one pizza per week. It cost a resident* \$10 *per mile to travel. Assume two pizza stores are considering opening shops on opposite ends of the street. After opening, each store would have a marginal cost of* \$5 *per pizza, an there is no fixed cost for opening a store.*

   (a) *What are the equilibrium prices each store will charge for pizza? What would their profits be?*
   Each pizza store will charge $p = t + c = 10 + 5 = 15$, where t is the transportation cost and c is the marginal cost. At these prices, the stores would split the market. All customers would purchase because the farthest any customer has to travel is 0.5 miles, incurring a \$5 transportation cost. So, the highest effective price is \$20 (because it is $p + tx = 15 + 10(0.5)$), well below the consumers willingness to pay. Thus $q_1 = q_2 = 50$. So the profit per week for each firm would be $\pi_1 = \pi_2 = (15 - 5) * 50 = \$500$.

   (b) *Intuitively, would both stores be happy with their price and location choice, or would one of them want to change their price/location? In fact, what would happen to the firms' prices if they were located right next to each other?*
   Both firms have an incentive to change their location. Keeping prices fixed, if one firm moves towards the other, it would expand its market share (simply draw the two firms effective prices before and after one of the firms moves to see this). Thus, when both firms change \$15 for their pizzas, neither one is happy with their location. However, the closer and closer firms locate, the less their products differentiated, there will be fierce price competition. In the limit, if they are located right next to each other, prices will go down to marginal cost ($p = c$).

   (c) *Suppose the firms decide to merge (i.e., they become a monopolist). What would the firms incentive to merge be (assume that the firms would serve the entire market after the merger)?*
   We need to compare the profits before and after the merger. Before the merger (part (a)), both firms get $\pi_1 + \pi_2 = 2 * 500 = \$1000$. Let's calculate now the profits post merger. First, notice that, if the firms merge, they no longer have to worry about price competition. Since the maximum a consumer is willing to pay is \$30 and the maximum transportation cost a consumer will pay is \$5 (this is the transportation cost paid by the marginal consumer, located at the $z = \frac{1}{2}$), then the merged firm can charge \$25 and still sell to all the consumers. In this case the profits per week are $\Pi_M = (25 - 5) * 100 = \$2000$
   Therefore, before the merger both firms together got $\pi_1 + \pi_2 = \$1000$, and after the merger the new firm gets $\Pi_M = \$2000$. The difference (\$1000) is the total incentive to merge.

2. *Cabral, problem 10.1.: First-time subscribers to the Economist pay a lower rate than repeat subscribers. Is this price discrimination? Of what type?*

   This is an example of third-degree price discrimination. The market is segmented into new subscribers and repeat subscribers. New subscribers, know the product less well and are thus likely to be more price sensitive. Moreover, the fact that they have not subscribed in the past indicates that they are likely to be willing to pay less than current subscribers. It is therefore optimal to set a lower price for new subscribers.

3. *Cabral, problem 10.3.: Cement in Belgium is sold at a uniform delivered price throughout the country, that is, the same price is set for each customer, including transportation costs, regardless of where the customer is located. The same is practice is also found in the sale of plasterboard in the United Kingdom. Are these cases of price discrimination?*

   Yes, these are cases of price discrimination. Consider the total price being paid by each customer, P, as being composed of the price actually charged and the transportation cost; $P = p_i + t_i$. Since locations are different, transportation costs are different, thus, each consumer is charged a price p_i that depends on his or her location. This is a clear example of geographic price discrimination.

4. *Cabral, problem 10.4.: A restaurant in London has recently removed prices from its menu: each consumer is asked to pay what he or she thinks the meal was worth. Is this a case of price discrimination?*

   It is likely that each consumer will pay a price that reflects his or her willingness to pay. In that sense, this is a situation of close to perfect price discrimination.

5. *Cabral, problem 10.8.: Coca-Cola recently announced that it is developing a "smart" vending machine. Such machines are able to change prices according to the outside temperature. Suppose for the purposes of this problem that the temperature can be either "High" or "Low." On days of "High" temperature, demand is given by $Q = 280 - 2p$, where Q is number of cans of Coke sold during the day and p is the price per can measured in cents. On days of "Low" temperature, demand is only $Q = 160 - 2p$. There is an equal number days with "High" and "Low" temperature. The marginal cost of a can of Coke is 20 cents.*

   (a) *Suppose that Coca-Cola indeed installs a "smart" vending machine, and thus is able to charge different prices for Coke on "Hot" and "Cold" days. What price should Coca-Cola charge on a "Hot" day? What price should Coca-Cola charge on a "Cold" day?*

   On a Hot day, $Q = 280 - 2p$, or $p = 140 - \frac{1}{2}Q$. Marginal revenue is $MR = 140 - Q$. Equating to marginal cost (20) and solving, we get $Q^* = 120$ and $p^* = 80$. On a Cold day, $Q = 160 - 2p$, or $p = 80 - \frac{1}{2}Q$. Marginal revenue is $MR = 80 - Q$. Equating to marginal cost (20) and solving, we get $Q^* = 60$ and $p^* = 50$.

   (b) *Alternatively, suppose that Coca-Cola continues to use its normal vending machines, which must be programmed with a fixed price, independent of the weather. Assuming that Coca-Cola is risk neutral, what is the optimal price for a can of Coke?*

   Observe from part (a) that even on a Hot day the optimal price is no greater than 80 cents. So, we can restrict our attention to prices of 80 cents or less. In this price range, the expected demand is given by $Q = 0.5(280-2p)+0.5(160-2p) = 220-2p$. Solving for p gives $p = 110-\frac{1}{2}Q$. The marginal revenue associated with this expected demand curve is given by $MR = 110 - Q$. Equating this marginal revenue to marginal cost, we get $Q^* = 90$. and $p^* = 65$.

   (c) *What are Coca-Cola's profits under constant and weather-variable prices? How much would Coca-Cola be willing to pay to enable its vending machine to vary prices with the weather, i.e., to have a "smart" vending machine?*

Under price discrimination, from part (a), profits on a Hot day are $(80 - 20)120 = \$72$, and profits on a Cold day are $(50-20)60 = \$18$. Expected profits per day are therefore $(\$72+\$18) = 2 = \$45$. Under uniform pricing, expected profits per day are $(65 - 20)90 = \$40.50$. It follows that Coca-Cola should be willing to pay up to an extra $\$4.50$ per day for a "smart" vending machine.

6. *A monopolist faces the inverse demand curve $P = z\,(36 - Q)$, where P is price, Q is total output and z is the quality og product sold, which can take on only two values. The monopolist can choose to market a low-quality product for which $z = 1$. Alternatively, the monopolist can choose to market a high-quality product for which $z = 2$. Marginal cost is independent of quality and is constant at zero. Fixed, cost, however, depends on the product design and increases with the quality chose. Specifically, fixed cost is equal to $65z^2$.*

   (a) *Find the monopolist's profits if it maximises profits and chooses a low-quality design.*

   For $z = 1$, profits for this firms are given by:

$$\begin{aligned} \Pi &= PQ - VC(Q) - FC \\ &= (1)(36 - Q)(Q) - 0 - 65(1)^2 \\ &= 36Q - Q^2 - 65 \end{aligned}$$

   The first order condition yields:

$$\begin{aligned} \frac{d\Pi}{dQ} &= 36 - 2Q = 0 \\ &\rightarrow Q = 18 \\ &\rightarrow P = 36 - (18) = 18 \end{aligned}$$

   Profit is then give by:

$$\begin{aligned} \Pi &= PQ - VC(Q) - FC \\ &= (18)(18) - 65 \\ &= 324 - 65 = 259 \end{aligned}$$

   (b) *Find the monopolist's profits if it maximises profits and chooses a high-quality design.*

   For $z = 2$, profits for this firms are given by:

$$\begin{aligned} \Pi &= PQ - VC(Q) - FC \\ &= (2)(36 - Q)(Q) - 0 - 65(2)^2 \\ &= 72Q - 2Q^2 - 260 \end{aligned}$$

   The first order condition yields:

$$\begin{aligned} \frac{d\Pi}{dQ} &= 72 - 4Q = 0 \\ &\rightarrow Q = 18 \\ &\rightarrow P = (2)\,(36 - (18)) = 36 \end{aligned}$$

Profit is then give by:

$$\begin{aligned}\Pi &= PQ - VC(Q) - FC \\ &= (36)(18) - 260 \\ &= 648 - 260 = 388\end{aligned}$$

(c) *Comparing your answers to (a) and (b), what quality choice should the monopolist make?*
The monopolist will go with high quality.

7. *General Foods is a monopolist and knows that its market for Bran Flakes contains two types of consumers. Type A consumers have indirect utility functions $V_A = 20z$, while type B consumers have indirect utility functions $V_B = 10z$. In each case, z is a measure of product quality, which can be chosen from the interval $[1, 2]$. There are N consumers in the market, of which General Foods knows that a fraction λ is of type A, and the remainder from type B. For simplicity, assume that all costs are zero.*

   (a) *Suppose that General Foods can tell the different consumer types apart and so can charge them different prices for the same quality of breakfast cereal. What is the profit-maximising strategy for General Foods?*
   Since both types have increasing willingness to pay as quality rises, the firm will sell maximum quality $z_A = z_B = 2$ to each type. Price to type A consumers is 40 while the price to type B consumers is 20.

   (b) *Suppose now that General Foods does not know what type of consumer is which. Show how its profit-maximising strategy is determined by λ.*
   From the participation constraint of type B consumers, we get $p_B = 10z_B$. From the incentive compatibility constraint of type A consumers, we get $p_A = 20z_A - 10z_B$. Then the firm's total profit is $\Pi = N\left[\lambda p_A + (1-\lambda)\, p_B\right] = N\left[20\lambda z_A + 10\,(1-2\lambda)\, z_B\right]$. It is easy to see that the firm will set z_A as high as possible, so $z_A = 2$. The quality for type B consumers will depend on the value of λ.

   - If $\lambda \leq \frac{1}{2}$ (so that there more low-type consumers), then profit is increasing in z_B. As such, the firm will set $z_B = 2$. In other words, General Foods will offer a unique product, of the highest quality, at price $p = 20$
   - If $\lambda \geq \frac{1}{2}$ (so that there more high-type consumers), then profit is decreasing in z_B. As such, the firm would set $z_b = 1$. At this point the firm even has to decide whether to offer a low-quality good. If the firm only sells the high-quality product, it can set a price as high as $p = 40$, getting profits $\Pi = 40N\lambda$. If the firm sells both products, then it will charge a price $p_B = 10$ for the low-quality good, but the highest possible price they can charge for the high quality good is $p_A = 30$ (in order to satisfy type A's incentive compatibility constraint). As a result, profits will be $\Pi = N\left[30\lambda + 10\,(1-\lambda)\right] = (10 + 20\lambda)\, N$. Hence, they will only offer the high quality good.

8. *In a two-period economy, one consumer wishes to buy a TV set in period 1. The copnsumer lives for two periods, and is willing to pay a maximum price of 100 euros per period of TV usage. In period 2, two consumers (who live in period 2 only) are born. each of the newly-born consumers is willing to pay a maximum of 50 euros for using a TV in period 2. Suppose that in this market there is only one firm producing TV sets, that TV sets are durable, and that production is costless.*

(a) *Calculate the prices the monopoly charges for TV sets in perios 1 and 2.*

We solve for the monopolys profit maximising prices starting from the second period. The second period outcome may depend on two cases:

- **First-period consumer does not buy in period 1**: Clearly, in this case, the second period profit maximising price is $P_2 = 50$, yielding a profit level of $\Pi_2 = 3 \times 50 = 150$.
- **First-period consumer buys in period 1**: In this case the second period profit maximising price is again $P_2 = 50$, yielding a profit level of $\Pi_2 = 2 \times 50 = 100$.

Altogether, the second-period price is independent of the action of the first period buyer in the first period. Therefore, the maximum price the monopoly can charge the first-period buyer in the first period is $P_1 = 150$.

(b) *Answer the previous question assuming that in the first period a consumer who lives two periods is willing to pay no more than 20 euros per period for TV usage.*

The second period outcome may depend on two cases:

- **First-period consumer does not buy in period 1**: In this case, the monopoly has two choices: (i) charging $P_2 = 20$, and sell to all three consumers, thereby earning a second period profit of $\Pi_2 = 3 \times 20 = 60$; or, (ii) charging $P_2 = 50$, and selling only to the second period consumers, thereby earning a second period profit of $\Pi_2 = 2 \times 50 = 100$.
- **First-period consumer buys in period 1**: In this case the second period profit maximising price is again $P_2 = 50$, yielding a profit level of $\Pi_2 = 2 \times 50 = 100$.

Altogether, the second period price is independent of the actions of the first period buyer. Now, in order to attract the first-period buyer to purchase in period 1, the monopoly should set $P_1 = 40$, thereby extracting all surplus from all consumers.

9. *Cabral, problem 15.2.: In less than one year after the deregulation of the German telecommunications market at the start of 1998, domestic long-distance rates have fallen by more than 70%. Deutsche Telekom, the former monopolist, accompanied some of these rate drops by increases in monthly fees and local calls. MobilCom, one of the main competitors, fears it may be unable to match the price reductions. Following the announcement of a price reduction by Deutche Telekom at the end of 1998, shares of MobilCom fell by 7%. Two other competitors, O.tel.o and Mannesmann Arcor, said they would match the price cuts. VIAG Interkom, however, accused Telekom of "competition-distorting behavior," claiming the company is exploiting its (still remaining) monopoly power in the local market to subsidize its long-distance business. Is this a case of predatory pricing? Present arguments in favor and against such assertion.*

   One could indeed argue that this is a case of predatory pricing. If Deutsche Telekon has monopoly in local markets, it likely has financial resouces strong enough to afford losing money in the long distance market by pricing below marginal cost. However, since there are two other competitors that matched Deutsche Telekom's prices, one can argue that there exists technology with marginal cost less than the low-price charged. Evidently, other explanations can also invoked, namely low-cost signaling and reputation for toughness.

10. *Cabral, problem 15.3.: "The combined output of two merging firms decreases as a result of the merger." True or false?*

    If the merger implies little or no cost efficiencies (namely at the level of marginal cost), we would expect the combined output of the merging firms to decline. If however the merger reduces the marginal cost of the combined firm significantly, then it is possible that the combined output increases as a result of the merger.

11. *Kikkoman is the dominant supplier in the market of say sauce, but it faces continuous entry threats. Suppose that Kikkoman incurs a cost $C(q_1) = 6q_1$. Kikkoman faces potential entry by Red Zen, which produces a perfect substitute for Kikkoman's product. However, Red Zen's production costs are given by $C(q_2) = 100 + 12q_2$, so that $MC_2 = 12$. The demand for soy sauce is given by $D(P) = 120 - P$.*

   (a) *Suppose initially that the incumbent, Kikkoman, can credibly commit to produce some output, after which Red Zen will choose its own output*

      i. *Find Red Zen's best response function*
      Red Zen's profit function is $\pi_2 = P \cdot q_2 - C(q_2) = (120 - q_1 - q_2)q_2 - 100 - 12q_2$. Marginal revenue is $MR_2 = 120 - q_1 - 2q_2$, while marginal cost is $MC_2 = 12$. Since $MR_2 = MC_2$, we obtain the entrant's best response function:

      $$q_2^* = BR(q_1) = 54 - \frac{1}{2}q_1.$$

      ii. *If Kikkoman accommodates entry, find the incumbent's profit-maximizing quantity and its resulting profits.*
      Kikkoman will act as a Stackelberg leader and will choose its quantity knowing what Red Zen's best response will be. Thus, Kikkoman's effective demand is $P = 120 - q_1 - q_2^* = 120 - q_1 - \left(54 - \frac{1}{2}q_1\right) = 66 - \frac{1}{2}q_1$, so marginal revenue is $MR_1 = 66 - q_1$, while marginal cost is $MC_1 = 6$. This gives $q_1 = 60$, making profits $\pi_1 = 1800$

   (b) *Alternatively, the incumbent can attempt to deter entry by engaging in "limit pricing". In fact, it would set a quantity so that the entrant will not be able to make a profit.*

      i. *Show that $q_1 = 88$ is the quantity that results in the limit price, and find that price and the incumbent's associated profit.*
      Since Kikkoman knows Red Zen's best response function, it can find the quantity that will make Red Zen achieve zero profits. Red Zen's profits are $\pi_2 = (120 - q_1 - q_2)q_2 - 100 - 12q_2$. Substituing $q_2 = 54 - \frac{1}{2}q_1$ into π_2, we get:

      $$\pi_2 = \left(108 - q_1 - \left(54 - \frac{1}{2}q_1\right)\right)\left(54 - \frac{1}{2}q_1\right) - 100 = 0$$

      This reduces to $\left(54 - \frac{1}{2}q_1\right)^2 = 100 \Rightarrow q_1 = 88$.
      When $q_1 = 88$, Red Zen is indifferent between entry and non-entry, so $q_2 = 0$. Then, $P = 120 - 88 = 32$ and $\pi_1 = (32 - 6)88 = 2288$.

      ii. *Will the incumbent prefer to deter entry or accommodate entry?. Explain.*
      The incumbent will prefer to deter entry. By limit pricing, Kikkoman receives higher profits with detering entry ($\pi_1 = 2288$) than by acommodating entry ($\pi_1 = 1800$).

12. *Consider a Cournot industry composed of 3 firms, facing a demand $D(P) = 150 - P$. Initially, the three firms are identical and have the same marginal cost \$30. As such, the Cournot-Nash equilibrium is for each firm to produce 30 units at a price of \$60. Suppose that two of those firms decide to merge, and that, as a result, the merged firm will realize a savings in its variable cost. In other words, post-merger marginal cost would be equal to \$c < 30.*

(a) *Calculate the post-merger equilibrium output level of the merged firm and the non-merged firm, and compute the corresponding price and profits.*

Now we have an asymmetric Cournot duopoly. Without loss of generality, suppose firm 1 & 2 merge. The best response function for the merged firm is:

$$q_m = \frac{150 - c}{2} - \frac{q_3}{2}$$

The other firm's best response function is:

$$q_3 = \frac{150 - 30}{2} - \frac{q_m}{2} = 60 - \frac{q_m}{2}$$

Solving we get:

$$q_m = \frac{150 - c}{2} - \frac{1}{2}\left(60 - \frac{q_m}{2}\right) \Rightarrow q_m = 60 - \frac{2}{3}c$$

and

$$q_3 = 30 + \frac{1}{3}c$$

Then, $P = 150 - \left(60 - \frac{2}{3}c\right) - \left(30 + \frac{1}{3}c\right) = 60 + \frac{1}{3}c$. Profits for the merged firm are $\pi_m = \left(60 + \frac{1}{3}c - c\right)\left(60 - \frac{2}{3}c\right) = \left(60 - \frac{2}{3}c\right)^2 = 3600 - 80c + \frac{4}{9}c^2$ and profits for the nonmerged firm are $\pi_3 = \left(60 + \frac{1}{3}c - 30\right)\left(30 + \frac{1}{3}c\right) = \left(30 + \frac{1}{3}c\right)^2 = 900 + 20c + \frac{1}{9}c^2$.

(b) *Compare your results in part (a) with the (pre-merger) Cournot-Nash equilibrium and determine:*

i. *How large should the savings be for the merger to be profitable?*
The merger is profitable if

$$\begin{aligned} \pi_m &\geq \pi_1 + \pi_2. \\ 3600 - 80c + \frac{4}{9}c^2 &\geq 2(900) \\ 1800 - 80c + \frac{4}{9}c^2 &\geq 0 \end{aligned}$$

Solving, we get $c \leq 90 - 45\sqrt{2} \approx 26.3$. In other words, the new marginal cost has to reduce up to aproximately $c = 26$ in order for the merger to be profitable for the merger firms.

ii. *How large should the savings be for the merger to benefit consumers?*
The merger will benefit consumers if

$$\begin{aligned} P_m &\leq P \\ 60 + \frac{1}{3}c &\leq 60 \end{aligned}$$

It is very easy to see that the merger will never benefit consumers