Lecture # 6 - Input-Output Analysis

- Important for production planning.
- It is a way to represent the production in an economy
- It assumes:
  - There are n interlinked industries
  - Each industry produces one single good.
  - Each industry uses a fixed-proportion technological process
- Idea: Suppose we produce glass
  - Our output can be sold directly to consumers (e.g. to home-owners), or can be used as an input for other industries (e.g cars)
  - We can used inputs from other industries to produce our good (e.g. machinery, which is made of steel)
- So there is interindustry dependence
  - Our output should be consistent to the input requirements of the economy.

- Simplest case: Suppose we divide the economy into 3 sectors:

  1. Agriculture
  2. Manufacturing
  3. Services

- .The three industries each use inputs from two sources:

  1. Domestically produced commodities form the three industries
  2. Other inputs, such as imports, labour, and capital.

- The outputs of the industries have two broad uses or destinations:

  1. Inputs to production of the three industries (intermediate inputs)
  2. Final demand (Consumption, Investment, Government expenditure, Exports)

- All this can be summarised in a so-called input-output table (in billions of euros):

		Outputs				
		Agriculture	*Manufactures*	*Services*	*Final demand*	*Total*
Inputs	*Agriculture*	30	40	0	30	100
	Manufactures	10	200	50	140	400
	Services	20	80	200	200	500
	Other sources	40	80	250	230	600
	Total	100	400	500	600	1600

- Take for example manufacturing:
  - Its output is worth €400 bln, which is allocated as follows:
    - €10 bln is used by the agricultural sector
    - €200 bln as intermediate goods for the manufacturing sector
    - €50 bln. is used by the services sector.
    - €140 bln. is the final demand (consumption, investment, government expenditure & exports
  - In order to produce, the manufacturing sector uses inputs worth of €400, of which
    - €40 bln comes from the agricultural sector,
    - €200 bln from the manufacturing sector (intermediate inputs),
    - €80 bln from the services sector,
    - €80 bln from other sources, including imports, labour and capital
  - Note that sector outputs equal sector inputs and that the economy-wide value of inputs equals the value of outputs at €1600 bln.
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- Define the following 2 vectors
  - $b = \begin{bmatrix} 30 \\ 140 \\ 200 \end{bmatrix}$ is the vector of final demands for output of the industry sectors
  - $x = \begin{bmatrix} 100 \\ 400 \\ 500 \end{bmatrix}$ is the vector of total ouput of the industry sectors

- As said above, one (critical) assumption is that each sector produces according to fixed-proportion technological coefficients (also called *input-output coefficients*)
- Example: agriculture uses €20 bln from the services sector. Given that the value of its total inputs is €100 bln, then services represent $20/100 = 0.20$ of its total inputs.
  - The Leontieff assumption is that, whatever the value of the inputs used by agriculture, 0.20 (or 20%) comes from services.
- As such, we can calculate the input-output coefficients by dividing each element in the input-output table by its column total:

		Outputs				
		Agriculture	*Manufactures*	*Services*	*Final demand*	*Total*
Inputs	*Agriculture*	$\frac{3}{10}$	$\frac{1}{10}$	0	$\frac{1}{20}$	$\frac{1}{16}$
	Manufactures	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{7}{30}$	$\frac{1}{4}$
	Services	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{5}{16}$
	Other sources	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{23}{60}$	$\frac{3}{8}$
	Total	1	1	1	1	1

- In matrix notation:
  - $A = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} & 0 \\ \frac{1}{10} & \frac{1}{2} & \frac{1}{10} \\ \frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$ is the matrix of inter-industry coefficients

- One important consequence of the input-output analysis is that we can express the vector of total demand (x) as a function of the final demand (b) and the matrix of inter-industry coefficients (A):

$$x = Ax + b$$

(Verify it)

- Then:

$$\begin{aligned} x - Ax &= b \\ (I - A)\,x &= b \end{aligned}$$

- If $(I - A)$ has an inverse:

$$\begin{aligned} (I - A)^{-1}(I - A)\,x &= (I - A)^{-1}\,b \\ x &= (I - A)^{-1}\,b \end{aligned}$$

- The matrix $(I - A)^{-1}$ is known as the input-output inverse, or the Leontieff inverse
  - Let's find it!!!
  - $(I - A)^{-1} = \begin{bmatrix} 1.49 & 0.32 & 0.05 \\ 0.42 & 2.23 & 0.37 \\ 0.64 & 0.85 & 1.81 \end{bmatrix}$
  - Show that $x = (I - A)^{-1}\,b$

Application:

- Suppose there is a change in final demand, from b_1 to b_2. What is the change in total demand?
  - For example, suppose there is an increase in the demand for agriculture products in the US (increase in exports)
  - Then $x_2 = (I - A)^{-1} b_2$
- In general, if:
  - $x_1 = (I - A)^{-1} b_1$
  - $x_2 = (I - A)^{-1} b_2$
  - Then $\Delta x = x_2 - x_1 = (I - A)^{-1} b_2 - (I - A)^{-1} b_1$
  - So $\Delta x = (I - A)^{-1} \Delta b$, where $\Delta b = b_2 - b_1$

Derivatives of Functions of One Variable

Rate of Change and the Derivative

- Suppose you have a function $y = f(x)$
- What happens to y if there is a change in x?
  - Examples:
    - What happens to the equilibrium solution of Y if there is a change in the marignal propensity to consume?
    - What if all of a sudden, all consumers are willing to pay an extra €100 for an iPod (rightward shift in demand)
- Initial state: $x_0 \Longrightarrow y_0 = f(x_0)$
- Suppose x changes to a new value $x_0 + \Delta x$
  - Δx : denotes the change in x
- Then the value of the function $y = f(x)$ changes from $f(x_0)$ to $f(x_0 + \Delta x)$
- **Difference quotient**: represents the change of y per unit of change in x

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

  - The difference quotient is a function of x_0 and Δx

- **Derivative**: the rate of change of y when Δx is very small:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f\left(x_0 + \Delta x\right) - f\left(x_0\right)}{\Delta x}$$

  - It reads: "the limit of the rate of change of y as Δx approaches 0"
  - The derivative is a function (if you want, a "*derived*" function)
    - The original function is called the "*primitive*" function
  - The derivative is a function ONLY of x_0
  - The rate measured by the derivative is the **instantaneous** rate of change
  - Notation:

$$\frac{dy}{dx} \equiv f'\left(x\right) \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

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- Example: Suppose $y = f\left(x\right) = 7x - 3$
  - $f\left(x_0\right) = 7x_0 - 3$
  - $f\left(x_0 + \Delta x\right) = 7\left(x_0 + \Delta x\right) - 3$
  - $\frac{\Delta y}{\Delta x} = \frac{7(x_0+\Delta x)-3-(7x_0-3)}{\Delta x} = 7\frac{\Delta x}{\Delta x} = 7$
  - So $\frac{dy}{dx} \equiv f'\left(x\right) \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2$

- Example: Suppose $y = f\left(x\right) = x^2 + 5$
  - $f\left(x_0\right) = \left(x_0\right)^2 + 5$
  - $f\left(x_0 + \Delta x\right) = \left(x_0 + \Delta x\right)^2 + 5$
  - $\frac{\Delta y}{\Delta x} = \frac{(x_0+\Delta x)^2+5-\left(x_0^2+5\right)}{\Delta x} = \frac{2x_0\Delta x+(\Delta x)^2}{\Delta x} = 2x_0 + \Delta x$
  - So $\frac{dy}{dx} \equiv f'\left(x\right) \equiv \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x_0$
  - Since we pick x_0 arbitrarily, we can drop the subscript.