Openness and Inflation∗

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Abstract: This paper develops a two country general equilibrium model to analyze the optimal rate of inflation under discretion. Once agents’ welfare is the sole policy objective it is possible to show that openness and inflation no longer have a simple inverse relationship. Because the terms of trade are related to monopoly markups, a greater degree of openness may lead the policy maker to exploit the short run Phillips curve more aggressively, even if involves a smaller short run benefit. Inflation can then be higher in a more open economy.

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1. Introduction

Does inflation rise or fall as an economy becomes more open? One way to approach this question is through the time inconsistency problem in monetary policy. In an ad-hoc setting, when monetary policy is set with discretion, inflation is lower in an open economy because deteriorations in the terms of trade increase the costs associated with surprise monetary expansions. As an economy becomes more open, it is more exposed to movements in the terms of trade, and inflation falls.\(^1\) I determine the optimal rate of inflation under discretion in a two country general equilibrium model. When the maximization of agents’ welfare is the sole policy objective it is possible to provide a simple explanation as to why inflation may rise or fall as an economy becomes more open.

An important part of determining the rate of inflation under discretion rests on the introduction of the costs and benefits of expansionary monetary policy. Previous work studying the welfare implications of monetary policy in open economies, such as Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001) and Benigno (2002), does not incorporate an explicit welfare cost of current inflation. As a result, it is not possible to generate a time consistent (discretionary) equilibrium.\(^2\) This paper introduces a relatively simple welfare cost by assuming beginning-of-period real money balances directly enter the utility function. This brings about the existence of a time consistent equilibrium making it possible to study the relationship between openness and inflation in detail.

In the analysis presented below, households are required to make a money holding decision before production and consumption decisions.\(^3\) Inflation reduces household’s purchasing power and the functional form of utility generates a convex welfare cost. Labor is supplied in a monopolistic market and there are one period nominal wage rigidities. Because there is an inefficiently low level of output surprise inflation has welfare benefits. The costs and benefits of inflation are introduced in this way for

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\(^2\)Benigno (2002) can generate a time consistent equilibrium as a special case. The result relies on specific values for the structural parameters of the model.

\(^3\)As Nicolini (1998) emphasizes, this assumption is entirely consistent with the worker-shopper argument used to motivate both CIA and MIUF models. It can also be thought of as implying a precautionary demand for money. This approach has been adopted in Danthine and Donaldson (1988), Neiss (1999), and most recently, Persson et al. (2006).
two reasons. First, even in an open economy, it is possible to derive an explicit expression for inflation without linear approximations. Second, such assumptions create a trade-off that captures the basic mechanism present in ad-hoc models, without sacrificing internal consistency.

Openness alters inflation via two mechanisms in the model I develop. There is a standard channel which operates through the Phillips curve and the terms of trade. When the economy is more open, the slope of the Phillips curve is steeper, increasing the inflation cost and reducing the output gain from a surprise monetary expansion. However, domestic and foreign monopoly markups determine the long run terms of trade. This creates a second channel, which distorts the openness-inflation relationship. For example, from the domestic economy’s perspective, a reduction in the relative markup across the two economies improves the terms of trade. In this case, there is a larger incentive for the government to inflate because a one unit increase in output translates into a bigger increase in consumption and a larger utility gain. A greater degree of openness may then lead the policy maker to exploit the short run Phillips curve more aggressively, even if it involves a smaller short run benefit. In a rational expectations equilibrium, inflation will rise as the economy becomes more open economy, in contrast to the standard result.

There is a related literature that analyzes discretionary monetary policy in a dynamic general equilibrium framework. In Albanesi et al. (2003), inflation is costly because households use previously accumulated cash to buy a subset of goods, consistent with Svensson’s (1985) cash-in-advance timing change. Higher realized inflation forces households to substitute towards non-cash goods, which lowers welfare. Inflation also raises output because some prices are fixed. This reduces the monopoly distortion and improves welfare, generating a policy trade-off. Arseneau (2004) analyzes a two country model assuming private agents are subject to a cash-in-advance constraint. Following Ireland (1997), he assumes an upper bound to the rate of money growth, and this generates a discretionary outcome. Both Albanesi et al. and Arseneau show that there can be multiple discretionary Markov equilibria.4 This paper also studies a Markov game. It shows that there is either an interior solution or no solution, depending on an interplay between openness, size, and a country specific monopoly markup.

4Multiple discretionary equilibria also arise in Armenter and Bodenstein’s (2005) analysis of a model with financially constrained firms.
The linear-quadratic framework of Woodford (2003) is also used to study discretionary monetary policy. In this case, inflation is costly because there is a staggered pricing structure. As only a fraction of firms are allowed to re-optimize their price in response to a shock, the costs of expansionary policy (or rather positive levels of inflation) derive from relative price distortions across goods. Benigno and Benigno (2003) and Pappa (2004) extend this framework to a two country setting and incorporate a terms of trade distortion. The linear-quadratic approach - both in the closed and open economy - also assumes the benefits of expansionary monetary policy occur because there is monopolistic competition.

There is also a large empirical literature on openness and inflation. Romer (1993) and Lane (1997) study average inflation from 1973 to 1988. Their estimates indicate that more open economies have lower inflation. Temple (2002) uses sacrifice ratio data to test if the Phillips curve is steeper in more open economies but finds little evidence. Daniels et al. (2005) suggest that the sacrifice ratio-openness relationship could be positive, once central bank independence is controlled for. Finally, during the 1990s the sensitivity of inflation to the domestic output gap has declined - a flattening of the Phillips curve - in the United States and other advanced countries (Borio and Filardo, 2007), but without clear evidence that this is a reflection of increasing trade integration (Ball, 2006). Neither inflation nor the sacrifice ratio appear to have a clear relationship with openness in the data, and any existing relationships have weakened over time. This paper provides one explanation for why this may be the case. The terms of trade are related to monopoly markups, and these alter the incentives of the government when deciding on monetary policy.

The remainder of the paper is organized as follows. In section two I describe the model economy. In section three I present the long run solution to the model. Section four computes the Markov rate of inflation and looks in detail at it’s relationship with openness. Section five concludes.

2. Model Economy

\footnote{Clarida et al. (1999) also discuss this at length. King and Wolman (2004) provide a critique of the linear-quadratic approach in the context of discretionary monetary policy.}

\footnote{Temple (2002) also argues that the time inconsistency problem does not explain the openness-inflation relationship, and that alternative explanations should be sought. However, using a co-integration analysis for the US over the period 1960-1997, Ireland (1999) finds evidence to support the time inconsistency approach.}
The world economy is populated with a continuum of agents of unit mass. The population in the segment \([0, n]\) belong to the domestic economy and the population in the segment \((n, 1]\) belong to the foreign economy. Households consume domestic and foreign goods, supply a differentiated labor type, hold real money balances and nominal bonds. Firms produce a single specialized output using labor. Each government controls the money supply through lump-sum transfers. Consumption, output and the nominal price of the domestic output are denoted with \(h\)-subscripts. Foreign consumption, output and prices are denoted with \(f\)-subscripts. Asterisks denote foreign economy variables.

2.1. Domestic Firms

Domestic firms maximize profits, \(\varphi_t(j) = P_{h,t}y_t - \int_0^n W_t(j)l_t(j) \, dj\), choosing amongst differentiated labor types, \(l_t(j)\), subject to a constant elasticity of substitution production function,

\[
y_t = \left[\frac{1}{n} \int_0^n l_t(j)^{\Phi} \, dj\right]^{\alpha/\Phi}\]

Above, \(W_t(j)\) is the \(j\)th individuals nominal wage, \(P_{h,t}\) is the GDP deflator, \(y_t\) is domestic output, \(\alpha < 1\) measures the returns to scale in production, and \(1/(1 - \Phi)\) is the elasticity of substitution between labor types.

Solving the cost minimization problem of firm \(j\), conditional labor demand is the following.

\[
l_t(j) = \left(\frac{W_t}{W_t(j)}\right)^{1/(1-\Phi)} y_t^{1/\alpha}\]

where \(W_t \equiv \left[\frac{1}{n} \int_0^n W_t(j)^{\Phi/(\Phi-1)} \, dj\right]^{(\Phi-1)/\Phi}\) is the wage index. The solution to firm \(j\)'s profit maximization problem gives final labor demand,

\[
y_t = \left(\frac{W_t}{\alpha P_{h,t}}\right)^{\alpha/(\alpha-1)}\]

In equilibrium, all households set the same wage, \(W_t(j) = W_t\), and the production function is \(y_t = l_t^\alpha\).

2.2. Domestic Households

Suppressing the \(j\) index, domestic households have following utility function.

\[
U_0 = \sum_{t=0}^{\infty} \beta^t \left[u(C_t) + v(m_t) - \phi l_t\right]
\]
Here, $C_t$ is a consumption index of domestic and foreign goods and $m_t \equiv M_t/P_t$ denotes real money balances, where $P_t$ is the consumer price index (CPI). The utility from total consumption is $u(C_t)$ and the utility from real money balances is $v(m_t)$. Both are increasing and strictly concave. The disutility from labor is linear.\footnote{We could allow for a concave function over labor, but this is not key to the results. Below I also impose $\phi = \alpha$.} Finally, $\beta \in (0,1)$ is the subjective rate of discount.

The consumption index is defined in the following way.

$$C_t = C_{h,t}^{1-\theta} C_{f,t}^\theta / \theta^\theta (1 - \theta)^{1-\theta}$$  (5)

where $\theta = (1 - n) \gamma$ is a measure of the overall share of foreign goods in the domestic consumption basket, or the degree of consumption home-bias in the domestic economy. Consumption home-bias is a composite of two parameters; the relative size of the domestic economy, $n$, and the extent of openness, $\gamma$. As $n \to 0$, the domestic economy is approximately small. When $\gamma = 0$ the domestic economy is completely closed.

Households maximize utility choosing nominal bond and domestic nominal money holdings, consumption, and a desired wage rate, subject to the constraint,

$$B_t + M_{t+1} = \vartheta_t + W_t d_t - P_{h,t} C_{h,t} - P_{f,t} C_{f,t} + T_t + B_{t-1} (1 + i_{t-1}) + M_t$$  (6)

and conditional labor demand (2), where $B_t$ is an international bond denominated in domestic currency paying a net nominal rate of interest $i_t$. Household also receive lump-sum transfers, $T_t$. The following conditions hold for all $t \geq 0$,

$$P_{t+1} u'(C_{t+1}) = \beta (1 + i_t) P_t u'(C_t)$$  (7)

$$w_t = \alpha / \Phi u'(C_t)$$  (8)

$$v'(m_{t+1}) / u'(C_{t+1}) = i_t$$  (9)

where primes denote the derivative of a function and $w_t \equiv W_t/P_t$ is the real wage. Equation (7) is the standard consumption Euler equation. Equation (8) describes labor supply and includes the CPI. It is now also clear that $\Phi$ is a monopoly markup. When wages are set in advance, labor is demand determined because agents are always willing to supply more labor given the existence of monopoly
profits. Equation (9) expresses the demand for money. This expression is non-standard and reflects the assumption that money holdings are effectively chosen in period $t-1$, and so $M_t$ is predetermined in period $t$.

The optimal allocation of expenditure between domestic and foreign goods is,

$$C_{h,t} = (1 - \theta) P_t C_t / P_{h,t} \quad \text{and} \quad C_{f,t} = \theta P_t C_t / P_{f,t}$$

(10)

where $P_t \equiv P_{h,t}^{1-\theta} P_{f,t}^\theta$.

2.3. Domestic Government

The domestic government’s budget constraint is given by the following.

$$T_t = M_{t+1} - M_t$$

(11)

Now all the constraints have been introduced it is clear how the beginning-of-period real money balances assumption works. Because period $t$ nominal money holdings are predetermined, a monetary expansion in period $t$ is not an increase in $M_t$, rather $M_{t+1}$ increases. This produces the increase in $P_t$. Surprise inflation in period $t$, that is, $P_t/P_{t-1}$, reduces real money balances, $M_t/P_t$, which has a welfare cost.

2.4. Foreign Economy

The foreign economy is identical to the domestic economy. Foreign firms have access to the same technology as in (1) and foreign households have the same form of utility as in (4). The consumption index in the foreign economy is defined in the following way.

$$C^*_t = C_{h,t}^{\theta^*} C_{f,t}^{1-\theta^*} / \theta^{\theta^*} (1 - \theta^*)^{1-\theta^*}$$

(12)

where $\theta^* = n\gamma$ is a measure of the overall share of domestic goods in the foreign consumption basket and is a composite of the relative size of the economy and the extent of openness. Note that $\theta^* + \theta = \gamma$.

When $\gamma = 1$, consumption baskets in both economies are identical, and PPP holds. More generally, when $\gamma < 1$ there is consumption home-bias in each economy. The optimal allocation of expenditure between domestic and foreign goods by foreign households is,

$$C_{h,t}^* = \theta^* P_t^* C_t^* / P_{h,t}^* \quad \text{and} \quad C_{f,t}^* = (1 - \theta^*) P_t^* C_t^* / P_{f,t}^*$$

(13)
where $P_t^* = P_{h,t}^* P_{f,t}^{*1-\theta^*}$. 

2.5. Equilibrium

In the world economy, $y_t$ and $y_t^*$ measure domestic and foreign per-capita output levels, and $ny_t$ and $(1 - n)y_t^*$ correspond to the aggregate levels of output. The domestic resource constraint is, $ny_t = \int_0^n C_{h,t}^d(j) dj + \int_n^1 C_{h,t}^* (j) dj$, so that,

$$P_{h,t}^* y_t = (1 - \theta) P_t C_t + \left( \frac{1 - n}{n} \right) \theta^* P_t^* C_t^* s_t$$  \tag{14}

In the foreign economy, $(1 - n)y_t^* = \int_0^n C_{f,t}^d(j) dj + \int_n^1 C_{f,t}^* (j) dj$, and,

$$P_{f,t}^* y_t^* = \left( \frac{n}{1 - n} \right) \theta P_t C_t + (1 - \theta^*) P_t^* C_t^* s_t$$  \tag{15}

Labor markets clear in periods $t \geq 1$ because agents have perfect foresight. Finally, the national budget constraints are,

$$B_t - B_{t-1} (1 + i_{t-1}) = P_{h,t} y_t - P_t C_t \text{ and } \left[ B_t^* - B_{t-1}^* (1 + i_{t-1}) \right] / s_t = P_{f,t} y_t^* - P_t^* C_t^*$$  \tag{16}

The foreign currency return on the international bond at time $t$ is $(1 + i_t) s_t / s_{t+1}$. The end-of-period bond level is equal to output minus the rate of absorption plus interest from claims on bonds. The international bond is in zero net supply and from here on I also assume a zero initial net foreign asset position.

3. Model Solution

I begin by demonstrating that the current account is zero when the money growth rate is constant for periods $t \geq 1$. To do this, I solve for the level of the nominal exchange rate. The time path of the exchange rate is governed by a UIP condition, which introduces the domestic and foreign nominal interest rates. Nominal interest rates depend on real interest rates and the real interest rates depend on the real side of the economy. Once I establish the zero current account result, I present the long run solution of the model. I then derive the global and non-cooperative solutions to the social planner’s problem.

3.1. Exchange Rate and Current Account


I define the inverse terms of trade as $\rho_t \equiv P_{f,t}/P_{h,t}$, and rewrite the domestic and foreign resource constraints as functions of output, consumption, and the terms of trade.

$$y_t = (1 - \theta) C_t \rho_t^\theta + \left(\frac{1 - n}{n}\right) \theta^* C_t^* \rho_t^1 - \theta^*$$ and $$y_t^* = \left(\frac{n}{1 - n}\right) \theta C_t \rho_t^{\theta - 1} + (1 - \theta^*) C_t^* \rho_t^{-\theta^*}$$

(17)

In periods $t \geq 1$, labor markets clear, and there is a second set of relations in the same endogenous variables.

$$y_t = \left[u' (C_t) \Phi \rho_t^\theta\right]^{\alpha/(1 - \alpha)}$$ and $$y_t^* = \left[u' (C_t^*) \Phi^* \rho_t^{\theta^*}\right]^{\alpha/(1 - \alpha)}$$

(18)

Equations (17) and (18) solve for the terms of trade as an implicit function of domestic and foreign consumption levels. Combining the domestic and foreign consumption Euler equations with the UIP condition produces a difference equation in domestic and foreign consumption and the terms of trade. Since (17) and (18) pin-down the terms of trade for a given level of consumption, the difference equation is self-contained. From this we can conclude $C_t = C_{t+1}$ and $C_t^* = C_{t+1}^*$ for $t \geq 1$, and as a result, $1 + r_t = 1 + r^*_t = 1/\beta$ and $\rho_t = \rho_{t+1}$ for $t \geq 1$. Output is also a function of consumption, so $y_t = y_{t+1}$ and $y_t^* = y_{t+1}^*$ for $t \geq 1$. The interpretation of these conditions is that the real side of the domestic and foreign economies reach the long run in period $t = 1$. I denote these long run solutions $C, C^*, y, y^*$ and $\rho$. This argument does not pin-down the short run real interest rates, $r_0$ and $r_0^*$, because they depend on the rigidity in labor markets and under such conditions (18) fails to hold.

I use the money demand functions and consumption Euler equations to determine the $t \geq 0$ nominal interest rates. In the domestic economy, for example, this leads to the following condition, which holds for all $t \geq 0$.

$$m_{t+2} \left[1 + v' (m_{t+2}) / u' (C_{t+2})\right] \frac{u' (C_{t+2})}{u' (C_{t+1})} = m_{t+1} / \beta \mu_{t+1}$$

(19)

where $\mu_{t+1} \equiv M_{t+1}/M_{t+2}$ is defined as the inverse money growth rate. I now assume the money growth rate is constant in periods $t \geq 1$.\(^8\) Since $\mu_{t+1} = \mu$ and $C_{t+1} = C$ for $t \geq 0$, equation (19) is a self-contained difference equation in real balances. As $m_{t+1}$ is non-predetermined, we need a saddle path condition to hold such that real balances jump to their steady state value. Differentiating (19)

\(^8\)This will always be the case in the discretionary equilibrium I analyze in section 4.
and evaluating the resulting expression at the steady state,

$$\frac{dm_{t+2}}{dm_{t+1}} = \frac{1 + i}{1 + i \left[ 1 + m \nu''(m) / \nu'(m) \right]}$$

From this condition, we conclude $m_{t+1} = m$ for $t \geq 0$. Returning to money demand, $i_t = v'(m_{t+1}) / u'(C_{t+1})$, and so $1 + i_t = 1 + i = 1 / \beta \mu$ for $t \geq 0$. Therefore, we have pinned down the path of $m_{t+1}$ and $i_t$ for $t \geq 0$. A similar argument for the foreign economy rules out nominal exchange rate dynamics and determines the evolution of the exchange rate as $s_{t+1} / s_t = \mu / \mu^*$ for $t \geq 0$.

To find the initial nominal exchange rate, $s_0$, I solve the domestic national budget constraint forward.

$$0 = \sum_{t=0}^{\infty} \left[ \theta P_t^* C_t^* s_t - \theta P_t C_t \right] / [(1 + i_0) \ldots (1 + i_{t-1})]$$

Ponzi games are ruled out so that $\lim_{t \to \infty} B_t / [(1 + i_{t-1}) \ldots (1 + i_0)] = 0$. I focus exclusively on situations in which the current account is zero. From the above condition, this must be consistent with $s_t = P_t C_t / P_t^* C_t^*$ for all $t$, which can only be an equilibrium in two cases. I relegate the details of this to the Appendix because it has already received significant discussion in the literature. However, when PPP holds; that is, when $\theta + \theta^* = 1$, I assume $-Cu''(C) / u'(C) = \sigma$. When there is home-bias in consumption, the restriction $\sigma = 1$ is required to generate a zero current account.

Using the domestic and foreign resource constraints I write the respective zero current account conditions in the following way.

$$y_t = \rho_t^0 C_t \text{ and } y_t^* = \rho_t^* C_t^*$$

The zero current account conditions are central in obtaining a closed-form solution to the model.

### 3.2. Long Run Solution and Social Planner Problems

The long run solution for output, consumption, and the terms of trade depend on the form of $u(C_t)$. There are two cases. First, when PPP does not hold ($\theta + \theta^* = \lambda$) and $-Cu''(C) / u'(C) = 1$. Second, when PPP holds ($\gamma = 1$) and $-Cu''(C) / u'(C) = \sigma > 0$. In the Appendix I show that for both cases the key step is to recognize that the ratio of the zero current account conditions given in (21) imply $\rho_t = y_t / y_t^*$. In turn, domestic and foreign consumption levels are a weighted average of domestic
and foreign output; \( C_t = y_t^{1-\theta} y_0^{\theta} \) and \( C_t^* = y_t^{\theta} y_t^{1-\theta} \). Combining these conditions with (18) I solve for long run variables as a function of the domestic and foreign monopoly markups. The following table presents the long run solutions for domestic output, consumption, and the terms of trade, where \( \Phi_R = \Phi/\Phi^* \) is defined as the relative monopoly markup.

[Table 1 Here]

Column 1 is the consumption home-bias case and column 2 is the PPP case. When there is consumption home-bias we can explicitly consider how changes in the relative size and openness of the domestic economy impact per-capita variables. Because households do not accumulate net foreign assets, output has a very simple form, and long run labor supply is independent of size and openness. Consumption depends on size, openness, and the relative markup. When PPP holds, the relative markup affects per-capita output, but only when the intertemporal elasticity of substitution in consumption differs from unity. Finally, the terms of trade are determined by the relative markup. This solution has the same form in both cases.

As output is below the competitive level it might appear that the presence of monopolistic competition gives the government an incentive to undertake expansionary monetary policy. But this decision depends on the optimal level of output, and since households have access to two traded goods, it does not necessarily follow that the level of output the social planner chooses coincides with the perfect competition level of output (which we find as \( \Phi \) and \( \Phi^* \to 1 \)). Following Cole and Obstfeld (1991), we can derive a solution to the social planners problem, from a global perspective, by using population weighted averages of the domestic and foreign utility functions. Dropping \( t \) subscripts, the global planner chooses \( C_h, C_f, C_h^* \) and \( C_f^* \), given the domestic and foreign resource constraints, and the consumption indexes. It is straightforward to show that the solution to the global planners problem coincides with the perfect competition level of output, where consumption equals output in both economies.

As the monetary policy problem I consider is non-cooperative, it is also relevant to solve the planner’s problem from the domestic perspective. In other words, to determine what each planner will do, given the other planner’s choice. In this case, we cannot rule out that the level of output targeted by wage
setters (the long run monopolistic level of output) will fall below that targeted by the government.\footnote{Faia and Monacelli (2006) undertake a similar analysis in the context of optimal monetary policy.} This point is important because differences between these levels of output create the incentives that induce the government to inflate the economy when monetary policy is set with discretion. In the domestic economy, for example, the planner maximizes domestic utility, choosing $C_h$, $C_f$ and $\rho$, subject to the economy’s resource constraint, the allocation of expenditure across each good, and balanced trade. Using these constraints, the domestic planner’s problem can be simplified to the following. It chooses $y$ to maximize,

$$U = u \left( y^{1-\theta} y^\theta \right) - \alpha y^{1/\alpha}$$

where $y^*$ is taken as given. If we consider the consumption home-bias case, where $u(C) = \ln (C)$, the domestic planner’s outcome is $y = (1 - \theta)^\alpha$, the global planner chooses $y = 1$, and the long run monopolistic level of output is $y = \Phi^\alpha$. Therefore when the planner does not cooperate, output is always below the efficient level, and may be below the long run monopolistic level, depending on the relative size of the economy, openness, and the domestic monopoly markup. The terms of trade depend explicitly on the extent of consumption home-bias in both economies, whereas the global solution implies that the terms of trade are unity. The reason for the divergence between these solutions follows similar lines to an optimal tariff argument. If a country is large enough in world markets it can impose a tariff on imports to alter the terms of trade, which increases welfare. The tariff reduces the overall volume of trade in the world and generates production and consumption costs, but by improving the terms of trade a moderate tariff can produce benefits that outweigh these costs. Here, each economy has power over the price level because it exports a specialized output. In a decentralized economy no individual private agent is able to affect the terms of trade despite monopolistic power over the wage rate.\footnote{Although an optimal tariff argument is unusual in this context it is an application of the beggar-thy-neighbor argument stressed in the analysis of Tille (2001).}

4. Optimal Monetary Policy

I now consider optimal monetary policy. I start by deriving the domestic government’s reaction function and determining the rational expectations equilibrium. I then show how domestic inflation
varies with openness and how the openness-inflation relationship depends on the relative markup across the two economies.

4.1. Government’s Optimization Problem and Rational Expectations Equilibrium

I study a Markov game with four players; two governments and two sets of private agents (wage setters). Each government controls the money growth rate in its economy. The choice of money growth is independent of past policy decisions. When deciding on the \( t = 0 \) money growth rate, each government also takes future policy choices as given.\(^{11}\) In the domestic economy, for example, the government chooses \( \mu_0 \equiv M_0/M_1 \) with \( \{ \mu_t \}_{t=1}^{\infty} = \mu \) given. Rather than use \( \mu_0 \) as the domestic policy variable, I express the government’s optimization problem in terms of period \( t = 0 \) real money balances. The real money balances variable, \( m_0 \), is directly linked to \( \mu_0 \) and has a natural interpretation for both the government and private agents. At \( t = 0 \), because \( M_0 \) is predetermined, \( \mu_0 \) is determined by choosing \( M_1 \). However, this also determines \( P_0 \), and therefore, \( m_0 = M_0/P_0 \). Now consider the following expression that relates real money balances to nominal money growth.

\[
m_0 = \mu_0 \pi_1 m_1 \tag{22}
\]

The variable \( \pi_1 \equiv P_1/P_0 \) is the discretionary rate of (gross) inflation in period \( t = 1 \). In this, and all other future periods, inflation is constant at \( \pi \), and equal to the constant rate of money growth, \( 1/\mu \). When \( \{ \mu_t \}_{t=1}^{\infty} = \mu \), we also know \( m_1 = m \), and from (22) we conclude, \( m_0 = (\mu_0/\mu) m \). That is, the period \( t = 0 \) choice of real money balances is equivalent to the choice of money growth.

The real money balances variable can also be interpreted as playing the role of current inflation, \( \pi_0 \). Likewise, the expected level of real money balances, \( m^e_0 \), can be interpreted as expected inflation, \( \pi^e_0 \). The variable \( m^e_0 \) implicitly incorporates the nominal wage set by private agents. At the beginning of period \( t = 0 \), when private agents decide on the nominal wage, they make a forecast of the price level, \( P^e_0 \). In doing so, they target the real wage, \( w^e_0 = W_0/P^e_0 \), consistent with the long run monopolistic level of output, \( y \). This forecast is equivalent to forecasting real balances, because \( m^e_0 \equiv M_0/P^e_0 \) is the expected price level normalized by the beginning-of-period (and predetermined) nominal money supply.

\(^{11}\)These policy choices are assumed to be determined in a future stage game.
When domestic monetary policy is unexpectedly expansionary, the CPI is higher than expected, and real money balances and the real wage are lower than expected \((m_0 < m_0^* \text{ and } w_0 < w_0^*)\).\(^{12}\)

Private agents in each economy play against their respective government, and the governments play against one another. The domestic and foreign governments pick real money balances, \(m_0\) and \(m_0^*\), and domestic and foreign private agents pick expected real money balances, \(m_0^e\) and \(m_0^e^*\). When, for example, the domestic government picks a value for \(m_0\), it takes \(\{m_0^*, m_0^e, m_0^e^*\}\) as given, and in that sense, the choice of \(m_0\) determines a best response, given other player's strategies. Assuming \(\theta + \theta^* = \gamma\) and \(Cu''(C)/u'(C) = -1\), the domestic government's constrained optimization problem can be expressed in the following way. It chooses \(m_0\) to maximize,

\[
U_0 = \ln (C_0) + v(m_0) - \alpha l_0 + \sum_{t=1}^{\infty} \beta^t [\ln (C) + v(m) - \alpha l]\] (23)

such that,

\[
C_0 = y_0^{1-\theta} y_0^* \theta\] (24)

\[
y_0 = l_0^\alpha\] (25)

\[
y_0 = \bar{y} \left( \frac{m_0^e}{m_0} \right) \omega [1-\alpha (1-\theta)] \left( \frac{m_0^e^*}{m_0^e} \right) \omega^\theta \alpha\] (26)

\[
y_0^* = \bar{y}^* \left( \frac{m_0^e^*}{m_0} \right) \omega [1-\alpha (1-\theta)] \left( \frac{m_0^e}{m_0^e^*} \right) \omega^\theta^* \alpha\] (27)

where \(\omega \equiv \alpha / (1-\alpha) [1-\alpha (1-\gamma)]\). Equation (24) is the ratio of the current account conditions combined with the CPI. Equation (25) is the aggregate production function. Equations (26) and (27) are the domestic and foreign economy Phillips curves, which are derived by introducing expectations into the final labor demand conditions (cf. equation (3) for the domestic economy) and scaling actual and expected prices by the beginning-of-period nominal money supply.

The solution to the domestic government's problem describes a reaction function where real money balances are an implicit function of expected real money balances and foreign policy.\(^{13}\)

\[
m_0 \cdot v'(m_0) = \omega \left\{ [(1 - \theta) - \alpha (1 - \gamma)] - [1 - \alpha (1 - \theta^*)] l_0 \right\} = R(m_0; m_0^*)\] (28)

\(^{12}\)Again, it is worth noting that private agents decide on the nominal wage, \(w_0\), at the beginning of period 0. They decide on money holdings, \(M_0\), at the end of period \(-1\). It is after this that the government makes it policy decision.

\(^{13}\)We can also derive a similar expression for the foreign government.
For any forecast of real balances by domestic and foreign private agents and foreign government policy we can use (28) to map out the response of the domestic government. From the domestic government’s reaction function, as expected real balances rise, the government raises real balances. However, to generate a rational expectations equilibrium, it must be the case that the domestic government does not overreact. Using (28), it is possible to show the following.

\[
\frac{\partial \ln (m_0)}{\partial \ln (m_0^n)} = \frac{\alpha I_0}{\alpha I_0 - R(m_0; m_0^n) \cdot [1 + m_0 v''(m_0) / v'(m_0)] \{(\omega / \alpha) [1 - \alpha (1 - \theta^*)]\}}^2
\]  

(29)

This expression must lie between zero and one. Consider the domestic government’s problem in ln \((m_0)\) and ln \((m_0^n)\) space. We know that the private agent’s reaction function is a 45-degree line because the Phillips curve is vertical in the long run. In this case, if \(\partial \ln (m_0) / \partial \ln (m_0^n) > 1\), we say the government overreacts to the shock. If \(\partial \ln (m_0) / \partial \ln (m_0^n) = 1\), we get instability because the government and public reaction functions coincide. When \(R(m_0; m_0^n) > 0\), these conditions will only be satisfied for \(m_0 v''(m_0) / v'(m_0) < -1\), where the marginal cost to inflating is increasing in the discretionary rate of inflation.\(^{14}\) Similarly, in \(-\ln (m_0)\) and ln \((y_0)\) space, the government’s reaction function is positive when \(m_0 v''(m_0) / v'(m_0) < -1\) and increasingly steep as \(m_0 v''(m_0) / v'(m_0) \to -1\). The Phillips curves are always positively sloped and the set of indifference curves, which can be derived by holding utility constant, using the constraint (24), and taking the derivative of (23) with respect to \(m_0\), become flat as \(R(m_0; m_0^n)\) approaches zero.

[Figure 1 Here]

The short run Phillips curve in figure 1 only allows for tangency points on the positively sloped part of the government’s set of indifference curves. From the domestic perspective, the long run level of output is below the optimal level, denoted \(\tilde{y}\). It is also clear why the convex cost of holding money (or rather of inflation) is required for the equilibrium to be well defined. If \(v(m_0)\) is logarithmic, the governments set of indifference curves are linear displacements of one another, and a rational expectations equilibrium only occurs as a special case. This corresponds to \(\partial \ln (m_0) / \partial \ln (m_0^n) = 1\), in (29), and is similar to Benigno’s (2002) analysis of the discretionary equilibrium. It is also consistent with an ad-hoc model when the loss function is linear in inflation and quadratic in output.

\(^{14}\)We need both conditions to be satisfied to generate a discretionary equilibrium. I provide an interpretation for \(R(m_0; m_0^n) > 0\) when discussing inflation.
Imposes \( m_0 = m_0^* = m \) on (28), I solve for the rational expectations equilibrium.

\[
R(m) = \omega \{[(1 - \theta) - \alpha (1 - \gamma)] - \Phi [1 - \alpha (1 - \theta^*)]\}
\]  

(30)

where \( l_0 = l = \Phi \). Neither foreign policy nor the foreign monopoly markup enter this expression. A similar result holds in ad-hoc models. In Canzoneri and Henderson (1991), the discretionary rate of inflation is zero because, in each economy, wage setters and the government target the same level of employment. In Rogoff (1985), wage setters target a lower level of employment than the government, which creates an incentive for each government to unleash a surprise monetary expansion. In the case I study, there is a target level of output for wage setter, at \( y = \Phi^\alpha \), and assuming consumption home-bias, a government target for output, at \( y = (1 - \theta)^\alpha \). However, there is also an optimal level of output, \( \tilde{y} \). These levels of output are determined by the underlying structural parameters of the model and, in particular, monopoly markups, size and openness.

4.2. Openness and Inflation

To solve for the optimal rate of inflation note that in equilibrium inflation is equal to the money growth rate such that \( \mu \pi = 1 \). From the money demand function this implies \( \nu'(m) = (\pi - \beta) / \beta C \). Rearranging, inflation is split between the Friedman rule rate of inflation and a bias term representing the incentives that drive the government to inflate the economy.

\[
\pi = \beta [1 + C \nu'(m)]
\]  

(31)

The Friedman rule rate of inflation in this economy is \( \pi = \beta \) and is the outcome when the government has access to a commitment technology. The intuition for this result is that there is a wedge between the private and social marginal cost of holding money and when \( i > 0 \) this generates an inefficiency. If there were no opportunity cost to holding money the inefficiency would disappear, but this requires inflation equal the inverse of the real interest rate, which is given by \( \beta \). In turn, the inflation bias is split between consumption and real money balances. The consumption-real money balances split is important because the consumption term plays a role in overturning many of the results in the ad-hoc literature, whilst the real balances part captures the basic mechanism behind this approach. Below I adopt the following functional form.

\[
\nu(m) = \chi m^{1-\epsilon} / (1 - \epsilon)
\]  

(32)
where $\chi$ is the weight placed on real balances in utility and $\epsilon$ captures the elasticity of substitution between consumption and real money balances. This parameter also controls the convexity of the inflation cost. To generate a discretionary equilibrium, $\nu'(m) + m\nu''(m) < 0$ requires $\epsilon > 1$. As $\epsilon \to 1$, the cost becomes linear such that it is optimal for the government to exploit the short run Phillips curve more aggressively.

### 4.2.1. Consumption Home-Bias

Using (30)-(32) along with (24), I can write an explicit expression for inflation.

$$\pi = \beta \left\{ 1 + y^{1-\theta} y^\theta \chi^{1/(1-\epsilon)} [R(m)]^{\epsilon/\epsilon - 1} \right\}$$

(33)

where $R(m)$ and the long run level of per-capita domestic and foreign output have already been determined as functions of the underlying parameters of the model. The interpretation of $R(m) > 0$ is that, all else equal, as $R(m) \to 0$, inflation tends to the Friedman rule and there is no inflation bias. In a closed economy ($\gamma = 0$) and when $\Phi = 1$, there is no inflation bias because wage setters target the same level of output as the government. In an open economy ($0 < \gamma \leq 1$), as $\Phi \to 1$ there may not be an equilibrium because we cannot rule out the government targeting a level of output below the socially optimal level.

Taking the derivative of (33) with respect to $\gamma$ produces the following.

$$\frac{\partial \pi}{\partial \gamma} = (\pi - \beta) \left\{ \frac{\partial \ln [R(m)]}{\partial \ln (\gamma)} \frac{\epsilon/\gamma}{\epsilon - 1} - \alpha (1 - n) \ln (\Phi_R) \right\}$$

(34)

Openness affects inflation through two channels. One channel relies on $R(m)$, which is related to the Phillips curve. The second relies on the relative monopoly markup, $\Phi_R$, and is related to consumption. To understand why there are two channels, recall that the determination of inflation rests on two standard distortions. Both economies use their own differentiated labor types as an input to production, and this creates an internal monopoly distortion in supply. Both economies also produce a specialized output traded on world markets, and this creates an external distortion via the terms of trade. In ad-hoc models, there is no direct link between the terms of trade and the underlying structure of the economy. However, in a general equilibrium setting, the long run terms of trade are a function of the relative monopoly markup. Specifically, from table 1, $\alpha (1 - n) \ln (\Phi_R)$ can be rewritten as
\((1 - n) \ln (\rho)\). The second channel identified in equation (34) is therefore a direct result of analyzing the discretionary equilibrium in a general equilibrium model. In this case it is possible to determine all the factors that affect the relationship between openness and inflation.

To understand how the first channel works, I take the derivative of (30) with respect to \(\gamma\).

\[
\frac{\partial R(m)}{\partial \gamma} = \omega \{n(1 - \Phi \alpha) - [1 + R(m)](1 - \alpha)\} \tag{35}
\]

The first term in the right-hand side of (35) is positive and the second negative. However, as \(n \to 0\), the domestic economy becomes small, and the first term drops out, so we can attribute this to a size effect. As \(n \to 0\), I also assume \((1 - \gamma) > \Phi\), which should be interpreted as before: \(\Phi\) reflects the market outcome and \(1 - \gamma\) reflects the domestic social planner’s outcome. This means we need only impose that the government target a higher level of output than wage setters for \(i > 0\).\(^{15}\) For \(0 < n \leq 1\), the size effect implies that \(\partial R(m)/\partial \gamma\) may be positive. However, the second term dominates, so that holding consumption constant, the standard story holds. As the economy become more open, the slope of the Phillips curve become less favorable, and a monetary expansion leads to a greater change in the terms of trade. The increase in output associated with the surprise monetary expansion produces a smaller utility gain, reducing the incentive to inflate.

The second channel works in the following way. Consider the case when \(\Phi_R < 1\) and recall that \(\rho\) is the inverse terms of trade. When the relative markup is low, the terms of trade are favorable for the domestic economy. A one unit gain in output will then lead to a relatively large change in consumption and the government will create a larger surprise change in the money supply. As the economy becomes more open, it is more exposed to movements in the terms of trade, and inflation rises with openness. That is, when \(\Phi_R < 1\), there is an extra incentive for the government to the exploit the short run Phillips curve, even if it involves a smaller short run benefit.\(^{16}\)

The two main results presented here; that there may be no inflation bias, and that inflation may rise with openness, initially appear surprising. However, there is a natural link with the existing theoretical

\(^{15}\)As \(n \to 0\), it is easy to verify that \(i^* > 0\), as \(R(m^*) = (1 - \Phi^*)[1 - \alpha (1 - \gamma)]\).

\(^{16}\)One possible alternative interpretation is to think of \(\Phi\) and \(\Phi^*\) as proxies for the extent of unionization. See Bowdler and Nunziata (2007) for a recent discussion.
literature. Corsetti and Pesenti (2001) analyze the welfare implications of interdependence when there are exogenous changes in the money supply. They note that domestic market failures, i.e. monopoly markups, may not give rise to an inflation bias when looking at optimal policy because any bias in inflation depends on the relation of openness to other variables. In the discretionary equilibrium I study, the bias in inflation depends on an interplay between openness, size, and a country specific monopoly markup. When I consider how the bias changes with openness, monopoly markups in both countries play an important role. The analysis in Corsetti and Pesenti stops short of analyzing optimal policy. However, the analysis presented there, and the channels identified here, both only arise in a general equilibrium setting where the utility function of the representative agent is used as the metric for policy decisions.

4.2.2. PPP and CRRA Utility

I now depart from the assumption that the logarithm of consumption enters utility and assume $-Cu''(C)/u'(C) = \sigma$. For the zero current account condition to hold consumption baskets need to be identical across countries; that is, $\gamma = 1$.\footnote{One obvious drawback of the PPP restriction is that it is not possible to study openness in the sense above. However, Obstfeld and Rogoff (1998) interpret $n$ as a measure of openness in their analysis.} This case is of interest a number of reasons. First, in my analysis, $\sigma \neq 1$ introduces a role for the relative markup in the reaction function of the government, in addition to the relative markup that determines inflation through money demand. Second, the intertemporal elasticity of substitution in consumption plays an important role when analyzing the spillover effects of exogenous money shocks, generating greater scope for strategic interaction between economies. Finally, although the inflation cost and interest elasticity of money demand are controlled through the parameter $\epsilon$, the ratio $\epsilon/\sigma$ now determines the consumption elasticity.

I perform the same analysis as in section 4.1 to determine inflation. The standard argument now corresponds to $\partial \pi / \partial n > 0$. As the domestic economy becomes relatively large in the global economy - and so the foreign good accounts for less of the domestic economy’s consumption basket - equilibrium inflation is higher. Taking the derivative of inflation with respect to $n$ produces the following.

$$
\frac{\partial \pi}{\partial n} = (\pi - \beta) \left\{ \frac{\partial \ln [R(m)]}{\partial \ln (n)} \frac{\epsilon}{n} - 1 + \frac{\sigma \alpha}{1 - \alpha (1 - \sigma)} \ln (\Phi_R) \right\}
$$

(36)
Equation (36) has the same features as (34) in that there is one channel operating through the Phillips curve and another channel operating through the relative markup. The first term in the square brackets should be positive if the standard argument holds. However, the derivative of the government’s reaction function with respect to \( n \) is the following.

\[
\frac{\partial R(m)}{\partial n} = \omega \left\{ \left[ (y^n \cdot y^{1-n})^{1-\sigma} - \alpha y^{1/\alpha} \right] + \xi (1 - \sigma) \ln (\Phi_R) \right\}
\]

where \( \xi \equiv R(m) (1 - \alpha) / [1 - \alpha (1 - \sigma)] > 0 \) and domestic and foreign output levels are given in table 1. The sign of \( \partial R(m)/\partial n \) is only unambiguous when \( \sigma = 1 \), as then \( \partial R(m)/\partial n = \omega (1 - \alpha \Phi) > 0 \). When \( \sigma \neq 1 \), the Phillips curve channel can exert a positive or negative effect on inflation through a second relative markup channel, and through what is essentially a wedge between consumption and output. Of greatest interest is determining if the new markup channel works in the same direction - and if so why - as the markup channel in (36).

The main result is that the new markup channel is positive (negative) when the relative markup and the intertemporal elasticity of substitution in consumption \((1/\sigma)\) are greater (less) than one. When \( \sigma > 1 \) domestic and foreign goods are substitutes in utility (the intertemporal elasticity being less than the intratemporal elasticity, which is set at one) and when \( \sigma < 1 \) they are complements. When \( \Phi_R < 1 \) the domestic terms of trade are favorable and when \( \Phi_R < 1 \) they are unfavorable. Consider the case when \( \Phi_R \) and \( \sigma < 1 \). As the economy becomes larger, the foreign good constitutes a smaller fraction of the overall consumption basket, the Phillips curve trade-off improves, and there is an incentive to inflate. However, there is an interaction effect. As the domestic and foreign goods are complements, the marginal utility from the domestic (foreign) good increases with the consumption of the foreign (domestic) good. In this case, if the terms of trade are favorable, the incentive to inflate is smaller, because the utility gain is lower. Therefore, although the Phillips curve is steeper, it effectively exerts a weaker influence over inflation than in the previous case when there was consumption home-bias, but \( \sigma = 1 \). In terms of inflation, the two markup channels work in the same direction when domestic and foreign goods are complements and in the opposite direction when they are substitutes. Although it is not possible to tie down the reaction of inflation further, this result suggests that the relative markup channel is an important part of the openness-inflation story. Alternative specifications of the utility function provide insights into the policy trade-offs that arise between the internal and external
distortions present in an open economy.

5. Conclusion

This paper develops a two country general equilibrium model to analyze the optimal rate of inflation under discretion. It makes two main contributions. First, it is one a small number of papers that analyzes discretionary monetary policy in an open economy using a dynamic general equilibrium framework. Second, it provides a strong case for the argument that the openness-inflation relationship is dependent on the underlying structure of the economy. To repeat the argument. The standard mechanism suggests that inflation depends on openness because surprise monetary expansions affect the terms of trade. The more open the economy, the larger this effect, and the more costly it is to inflate. Inflation is lower in a more open economy. In a general equilibrium model, the relative monopoly markup also affects the terms of trade. The result is that the openness-inflation relationship depends on a relative markup channel. In the first case I study, the relative markup can distort the terms of trade to the extent that inflation may rise with openness, despite a less favorable Phillips curve trade-off. In the second case, the relative markup channel affects the government’s reaction function directly, and it’s influence depends on whether or not the two traded goods are complements or substitutes in utility.
References


Appendix

Here I establish the conditions under which $B_t = 0$, given $B_{-1} = 0$. I also derive the long run solutions presented in table 1.

A.1 Zero Current Account

Since a zero current account in the domestic economy implies a zero current account in the foreign economy I use the domestic national budget constraint. Solving this forward, and ruling out Ponzi games, gives the national intertemporal budget constraint (NIBC).

\[
0 = \sum_{t=0}^{\infty} (P_{h,t}y_t - P_tC_t) / [(1 + i_0) \ldots (1 + i_{t-1})]
\]

In the text I show, independent of the current account, $1 + i_t = 1/\beta \mu$ for all $t \geq 0$. Using the resource constraints, this implies the following.

\[
0 = \sum_{t=0}^{\infty} (\beta \mu)^t \left( \frac{1-n}{n/\theta^*} P_t^* C_t^* s_t - \theta P_tC_t \right)
\]

The consumption Euler and UIP equations also imply,

\[
\frac{u'(C_t)}{P_t} = \frac{1}{\mu} \frac{u'(C_{t+1})}{P_{t+1}} \quad \text{and} \quad \frac{u'(C_t^*)}{P_t^*} = \frac{1}{\mu^*} \frac{u'(C_{t+1}^*)}{P_{t+1}^*}
\]

\[
s_t/s_{t+1} = \mu/\mu^*
\]

To understand the restrictions on preferences required to generate a zero current account consider the most general case, where PPP fails ($\theta^* + \theta = \gamma$) and $-C u''(C) / u'(C) = \sigma$. In this case, the nominal consumption Euler equations are, $P_tC_t = \mu P_{t+1}C_{t+1} (C_{t+1}/C_t)^{\sigma-1}$ and $P_t^* C_t^* = \mu^* P_{t+1}^* C_{t+1}^* \left( C_{t+1}^*/C_t^* \right)^{\sigma-1}$. Thus, we must have, $P_tC_t = (1/\mu)^t P_0 C_0 (C_0/C_t)^{\sigma-1}$, and, $P_t^* C_t^* = (1/\mu^*)^t P_0^* C_0^* (C_0^*/C_t^*)^{\sigma-1}$. The time path of the nominal exchange rate is, $s_t = (\mu^*/\mu)^t s_0$. Applying these conditions to the domestic NIBC,

\[
0 = \sum_{t=0}^{\infty} \beta^t \left( \frac{1-n}{n/\theta^*} P_0^* C_0^* s_0 - \theta P_0 C_0 (C_0/C_t)^{\sigma-1} \right)
\]

When $\sigma = 1$, the NIBC reduces to a static condition. As $n \theta = (1-n) \theta^*$, an immediate result is $s_0 = P_0 C_0/P_0^* C_0^*$. Using the resource constraint and the period zero budget constraint, $B_0 - B_{-1} (1 + i_{-1}^*) = P_{h,0}y_0 - P_0 C_0 = 0$. Since $B_{-1} (1 + i_{-1}^*) = 0$, then $B_0 = 0$. In this case, $B_0 = B_t$.
for all \( t \geq 1 \). That is, if the nominal interest rate is constant and at its steady state in all periods, and in the initial period the bond stock is assumed to be zero, in all future periods the bond stock will remain at zero. If we instead allow \( \sigma \neq 1 \), and impose \( \theta^* + \theta = 1 \), then \( C_t/C_{t+1} = C_t^*/C_{t+1}^* \), or rather, \( C_0/C_t = C_0^*/C_t^* \). The previous condition can then be written as,

\[
0 = \sum_{t=0}^{\infty} \left[ \beta^t (C_0/C_t)^{\alpha-1} \right] \left( \frac{1-n}{\theta^* s_0} - \theta P_0 C_0 \right)
\]

As \( C_0/C_t \neq 1 \), we conclude \( s_0 = P_0 C_0/P_0^* C_0^* \). We can use the same argument as before to establish the zero current account result, despite \( \sigma \neq 1 \), essentially because relative consumption is equalized across countries. If we do not impose \( C_0/C_t = C_0^*/C_t^* \), associated movements in the real exchange rate lead to a non-zero current account.

### A.2 Long Run Solutions

Given the zero current account conditions (\( P_{h,t} y_t = P_t C_t \) and \( P_{f,t} y_t^* = P_t^* C_t^* \)), it is straightforward to derive expressions for the long run levels of output and consumption. Dropping time subscripts, the ratio of current account conditions implies, \( \rho = \frac{y}{y^*} \). Re-substituting into each of the current account conditions produces, \( C = y^{1-\theta} y^{*\theta} \) and \( C^* = y^{\theta*} y^{1-\theta^*} \). Labor market clearing in each economy, along with the respective zero current account condition, implies the following.

\[
y^{1/\alpha} = Cu' (C) \Phi \quad \text{and} \quad y^{*1/\alpha} = C^* u' (C^*) \Phi^* \]

When \( \sigma = 1 \), \( Cu' (C) = 1 \), and the solution for output is trivial; \( y = \Phi^\alpha \) and \( y^* = \Phi^{*\alpha} \). This also implies \( \rho = (\Phi/\Phi^*)^\alpha \). Finally, as \( C = y^{1-\theta} y^{*\theta} \) we find \( C = (\Phi^{1-\theta} \Phi^{*\theta})^{\alpha} \) and \( C^* = (\Phi^{\theta*} \Phi^{1-\theta^*})^{\alpha} \). The first expression is reported in table 1, where \( \Phi_R \equiv \Phi/\Phi^* \).

When \( \gamma = 1 \), we allow \( \sigma \neq 1 \), and so, \( Cu' (C) = C^{1-\sigma} \). In this case, we solve for consumption first, then output. We now know \( C = C^* \) and \( C = y^n y^{1-n} \). Taken together, and using both domestic and foreign goods market clearing conditions, \( C = (\Phi^n \Phi^{*1-n})^{\frac{n}{1-n(1-\sigma)}} \), where we can again use the definition, \( \Phi_R \equiv \Phi/\Phi^* \). Now we can solve for individual country output levels. In the domestic economy, \( y = C^{\alpha(1-\sigma)} \Phi^\alpha \). Therefore, \( y = \Phi^\alpha (\Phi^n \Phi^{*1-n})^{\frac{n^2(1-\sigma)}{1-n(1-\sigma)}} \).
\[ \theta + \theta^* = \gamma \text{ and } \sigma = 1 \quad \theta + \theta^* = 1 \text{ and } \sigma \neq 1 \]

\[
\[
C = \left( \Phi \Phi_{R}^{1-\theta} \right)^{\alpha} \quad C = \left( \Phi \Phi_{R}^{n} \right) \frac{1}{1-(1-\sigma)\alpha}
\]

\[
y = \Phi^{\alpha} \quad y = \Phi \frac{\alpha}{1-(1-\sigma)\alpha} \Phi_{R} ^{\frac{(n-1)(1-\sigma)\alpha^2}{1-(1-\sigma)\alpha}}
\]

\[
\rho = \Phi_{R}^{\alpha} \quad \ldots \quad \ldots \quad \ldots
\]

Table 1: Long Run Solution for the Domestic Economy in Two Cases
\[
\ln(y) - \ln(m) - \ln(\tilde{y})
\]

Govt’s RF, \(R(m_0; m_0^*)\)

(Minus) Log Real Balances, \(-\ln(m_0)\)

Log Domestic Output, \(\ln(y_0)\)

SRPC
Govt’s IC

Figure 1: Government’s Optimization Problem