A Goldilocks Theory of Fiscal Policy*

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Abstract

Fiscal policy in advanced economies faces a "Goldilocks dilemma": Fiscal consolidation risks prolonged episodes at the zero lower bound (ZLB), while fiscal expansion raises sustainability concerns. This paper proposes a dynamic fiscal policy framework to study fiscal space subject to this trade-off. At the core of our analysis is a deficit-debt diagram, which we use to measure how much fiscal expansion is necessary to avoid the ZLB, when fiscal policy can run deficits indefinitely, and at what debt level the interest rate rises above the growth rate. Rising inequality and weak aggregate demand expand fiscal space, allowing greater indefinite deficits, while slowing growth tightens the ZLB constraint, requiring greater and greater debt levels. We characterize the effects of various tax policies on fiscal space and provide a cross-country comparison.

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1 Introduction

Advanced economies in recent years have been characterized by sustained proximity to the zero lower bound on nominal interest rates, also known as "secular stagnation" (Summers 2014), which has led some economists to argue that persistent fiscal deficits may be necessary to generate demand.[1] At the same time, government debt to GDP ratios are at historical highs, leading to concerns about fiscal sustainability going forward if deficits are not reduced.[2] The tension between these views has triggered an active debate on fiscal policy.

This paper proposes a tractable model of fiscal policy that encapsulates the trade-off highlighted in this debate. The model is based on two main features. The first is that the interest rate on government debt R increases in the level of debt. Our model achieves this simply by assuming that debt provides otherwise standard households with concave convenience benefits (e.g., Krishnamurthy and Vissing-Jorgensen 2012), but we show that the exact microfoundation is not crucial for our analysis. The second feature is a zero lower bound (ZLB) constraint on R, which allows for the possibility that persistently weak demand reduces output and inflation, and thus also the nominal growth rate G of the economy. The government's fiscal policy can stimulate demand by borrowing but must satisfy the government budget constraint at all times.

A key insight of the model is that fiscal policy can either be too conservative, pushing the economy toward the ZLB; or it can be too aggressive, pushing the economy towards non-sustainability. We analyze this trade-off in an intuitive *deficit-debt diagram*, in which a locus characterizes the feasible set of steady state combinations of the primary deficit (or surplus) and the level of debt. The diagram makes it easy to see how much fiscal expansion is necessary to avoid the ZLB, $R > 0$, and at what debt levels R rises above G.

A crucial property of the deficit-debt locus is that it is hump-shaped when the demand for government debt is sufficiently high: without debt, there is no steady-state deficit or surplus, and with large levels of debt, R rises above G, requiring a surplus. With intermediate levels of debt, $R < G$ and the government can run an indefinite deficit. The maximum of the deficit-debt locus is the largest permanently stable deficit a government

[1] This idea can be traced back to Abba Lerner's "Functional Finance" view (Lerner 1943), but has recently been articulated by Blanchard and Tashiro [2019] in their discussion of policy in Japan: "under current forecasts about the rest of the Japanese economy, primary deficits may be needed for a long time ... they are probably the best tool for maintaining demand and output at potential."

[2] Among others, the recent work by Jiang, Lustig, Van Nieuwerburgh, and Xiaolan [2019, 2020a] has raised questions on how the pricing of government debt can be consistent with the evolution and riskiness of revenues and expenditures going forward.

can run. To the left of the maximum, we show that the government has access to a "free lunch": it can permanently raise the deficit, without ever having to raise taxes.

Whether a free lunch is possible or not is captured by a simple condition: either $R = 0$, the economy is at the ZLB, or $R < G - \varphi$, where φ is the (semi-)elasticity of R to the level of debt. The reason φ plays a prominent role here is similar to why price and marginal revenue differ for a monopolist: a marginal increase in debt increases interest rates on all infra-marginal units of debt already issued. $R < G$ is therefore not the right criterion to gauge to what extent a free lunch is feasible. It is neither sufficient nor necessary.[3]

The notion of a free lunch formalizes an intuition that is often associated with "Modern Monetary Theory" (MMT). However, unlike common renditions of MMT (see Bisin 2020 for a critical review), our model spells out the exact conditions under which a free lunch policy works or does not work. According to our model, it always works if any economy faces a persistent demand shortage at the ZLB (as already intuited by Lerner 1943), and otherwise if the debt level sufficiently low that R lies below $G - \varphi$.

To gauge whether the U.S. economy is in its free lunch region or not, we calibrate a version of our model to pre-Covid economic data. A crucial number in our calibration is φ, for which we survey the relevant literature, finding an average of $\varphi \approx 1.7\%$. The calibrated deficit-debt diagram suggests that the U.S. economy's maximum permanent primary deficit is about 2%, reached at a debt level of about 110%; and that the interest rate R would rise above the growth rate G at around 220%. While a permanent deficit of 2% is significant, we note that it is about half the structural deficit that is forecast by the Congressional Budget Office (CBO) for the next three decades. Moreover, as of the fourth quarter of 2020, U.S. federal debt stands at 126%, above 110%. This suggests that, if the economy recovers to its pre-Covid state, fiscal consolidation is necessary to prevent unsustainable debt accumulation; a free lunch is no longer feasible; and any further increase in outlays has to be matched by future tax increases despite $R < G$.

We use our model to investigate the role of several important factors for fiscal space and the deficit-debt locus. We begin by exploring the role of weak aggregate demand. captured by a reduced discount rate of households. By reducing natural interest rates, this increases the locus, thus allowing for greater fiscal space, in line with Furman and Summers [2020]; it also tightens the ZLB constraint, therefore requiring that part of the increase fiscal space be used to avoid the ZLB. A slowdown in growth rates G—e.g. due to slowing technological progress or population aging—only has the second effect in our model: it tightens the ZLB constraint, requiring more expansionary fiscal policy without increased fiscal space.

[3]The condition is not necessary in that $R = 0$ and $G < 0$ are possible at the ZLB.

Among the most relevant and interesting factors to study is rising inequality. Inequality matters for government debt since as much as 69% of U.S. government debt is directly or indirectly held by households in the top 10% of the U.S. wealth distribution (Mian, Straub, and Sufi 2020). We model inequality by assuming that only a fraction of households can save in government bonds ("savers"), while the rest are hand-to-mouth. We find that greater inequality unambiguously expands fiscal space and the deficit-debt locus. This is because savers need resources in order to be able to purchase government bonds. The more resources they have, the greater is their demand for government bonds.

The connection with inequality has two important implications. First, it implies that redistribution narrows fiscal space, by shifting resources away from savers. This is especially relevant when debt levels are high and governments contemplate raising taxes to finance their debts. Greater progressive taxation requires greater increases in tax revenue than greater regressive taxation. In other words, there is a trade-off between the sustainability of high levels of debt and a more egalitarian distribution of resources.

Second, an important caveat to this result emerges when an economy not just has high debt levels but is also close to the ZLB. In that case, greater regressive taxation could lead to a ZLB-induced recession, and, as we show, paradoxically imply increased, rather than reduced, debt levels. Greater progressive taxation can still be used to finance the debt, however, as its impact on aggregate demand is weaker and can be offset by redistribution.

Other extensions to our framework flesh out implications for governments financed by long-term debt, quantitative easing, and governments that are part of monetary unions. We conclude our paper by providing calibrations for several other advanced economies, showing how, by borrowing in its own currency, Japan has vastly more fiscal space than Italy, despite similar fundamentals.

Related Literature. This study is part of a growing body of theoretical research that has emerged around two important facts on government debt. The first fact is that the nominal interest rate on government debt is lower than the nominal growth rate on average, $R < G$ (Feldstein 1976, Bohn 1991, Ball, Elmendorf, and Mankiw 1998, Blanchard 2019, Mehrotra and Sergeyev 2020). The second, and more recent fact, is that the demand curve for government debt slopes down empirically, that is, the interest rate on government debt rises in the volume of government debt (Engen and Hubbard 2004, Laubach 2009, Krishnamurthy and Vissing-Jorgensen 2012, Greenwood, Hanson, and Stein 2015, Presbitero and Wiriadinata 2020). This fact is typically either attributed to effects of government debt on the marginal product of capital (due to crowding out of capital), to effects of debt on default premia, or to effects of debt on its "convenience benefits", capturing regulatory requirements, liquidity premia, and safety premia.

The literature has explored several ways to explain one or both of these facts. Bohn [1995] suggests that $R < G$ can naturally occur in complete markets economies with aggregate risk. Due to Barro [1974] - Ricardian equivalence, the model suggests that government debt neither affects R, nor can the government run a permanent deficit in each state of the world. Jiang et al. [2019] demonstrate that this approach cannot explain the valuation of US government debt. A potential way out is the recent work by Barro [2020] relying on the risk of rare disasters (Barro and Ursua 2008) that may not have realized yet for the US economy.[4]

The perhaps largest literature on $R < G$ is based on OLG models, going back to Samuelson [1958] and Diamond [1965]. An early motivation of this literature has been to understand when $R < G$ is a sign of dynamic inefficiency (Abel, Mankiw, Summers, and Zeckhauser 1989, Blanchard and Weil 2001, Ball and Mankiw 2021), as well as when a possibility for a "free lunch" policy exists (Blanchard and Weil 2001, Blanchard 2019), whereby deficits can be increased today without future tax increases, in any state of the world. Such a policy was found to be more likely to exist when the economy is inefficient, and when capital is not crowded out. When a free lunch policy does not exist, deficits resemble a "gamble" (Ball et al. 1998).[5] Relative to this literature, our model emphasizes that $R < G$ is generally not a threshold that is relevant for fiscal policy; instead, it is $R < G - \varphi$. We demonstrate that this holds even in a standard Diamond [1965] OLG model in Appendix B.4.

The above facts have also been approached using liquidity premia. Woodford [1990] illustrates how liquidity demand by producers or consumers can lead to $R < G$. Angeletos, Collard, and Dellas [2020] microfound a convenience yield function based on liquidity needs to revisit the optimality of the Barro [1979] tax smoothing results.[6] Bayer, Born, and Luetticke [2021] estimate the response of the liquidity premium to fiscal policy shocks empirically and model it with an estimated two-asset HANK model. The closest paper to ours among this class of models is Reis [2021]. The paper microfounds liquidity and safety

[4]The Barro [2020] model can be used to generate an endogenous "safety premium" in the presence of default risk. We demonstrate in Section 7.1 that this can microfound the concave convenience utility we work with in Section 2.

[5]There is also a long literature on the private production of assets when the return on non-government assets is also below the growth rate, see, e.g., Tirole [1985], Kocherlakota [2009], Farhi and Tirole [2012], Hirano and Yanagawa [2016], and Martin and Ventura [2018].

[6]See also Canzoneri, Cumby, and Diba [2016], Bhandari, Evans, Golosov, and Sargent [2017], Azzimonti and Yared [2019].

premia of government debt and shows that the economy leads to a "bubble premium" on public debt, which can be used to sustain permanent primary deficits. Among the model predictions are that more government spending requires lower R, while greater inequality leads to greater R. Our analysis complements that of Reis [2021] by exploring a different microfoundation for why $R < G$, which is shown to lead to different mechanisms and results, such as our characterization of the free lunch region and the effects of inequality. Moreover, instead of modeling capital and investment, we focus on the interaction of $R < G$ with the ZLB.[7]

Mehrotra and Sergeyev [2020] share with our paper the assumption of a convenience utility function $v(b)$ over government debt, which they employ in a model with aggregate risk, and allowing for default risk. Different from Mehrotra and Sergeyev [2020], our focus is on a deterministic model, which we show can be tractably analyzed using phase diagrams for arbitrary $v(b)$. We also allow for a ZLB constraint, which we show meaningfully interacts with important comparative statics (e.g. that of rising inequality or falling growth rates).

Our model is based on the assumption that monetary policy remains active in stabilizing inflation and economic activity whenever it can. A recent branch of the literature explores deviations from this assumption.[8] Brunnermeier, Merkel, and Sannikov [2020a,b] derive a Laffer curve for the rate of inflation in a model with liquidity needs among producers. Sims [2019] argues that fiscal policy should, in general, use this "inflation tax" to generate seignorage-like revenue and reduce distortionary taxes (different from Chari and Kehoe 1999). The deficit-debt schedule that we derive, and on which our phase diagram is based, may seem similar to the inflation Laffer curve, but is quite distinct. In our schedule, debt is on the horizontal axis instead of inflation. With a binding ZLB, we find a positive relationship between inflation and debt levels, while this branch of the literature generally finds a negative one. With interest rates above the ZLB, inflation is independent of debt in our economy. We discuss the relationship in more detail in Appendix B.2.

Finally, this study is closely related to the burgeoning literature on the sources and implications of safe asset demand (e.g., Caballero, Farhi, and Gourinchas 2008, Caballero and Farhi 2018b, and Farhi and Maggiori 2018). In their model of the international monetary system, Farhi and Maggiori [2018] explore an equilibrium in which there is large demand for debt issued by a hegemon government. When this is met by too much issuance, default risk emerges. Farhi and Maggiori [2018] explain how a zero lower bound constraint can

[7] In Appendix B, we derive the deficit-debt diagram for the Reis [2021] economy.

[8] See also the recent work by Bassetto and Cui [2018] and Bianchi and Melosi [2019].

make this a more likely outcome.

2 Model

We begin with a stylized model that we extend in later sections. The model runs in continuous time and is deterministic.[9] It involves two actors, a government and a representative household. The government issues government debt, spends, and raises lump-sum taxes. The representative agent consumes and draws convenience benefits from holding government debt.

Throughout, we denote by R_t the nominal interest rate on government debt and by $G_t \equiv \gamma + \pi_t$ the nominal growth rate, which is equal to real trend growth γ plus inflation π_t. $G^* \equiv \gamma + \pi^*$ corresponds to nominal trend growth, when inflation is at its target π^*.

To save on notation, we will conduct our analysis entirely in the context of a model that was de-trended with the nominal growth rate. Potential output y^* in the de-trended model is constant and we normalize it to one, $y^* \equiv 1$. Any quantities, such as the level of government debt b_t are to be understood as government debt relative to potential GDP. Moreover, we refer to $R_t - G_t$ as the "de-trended rate of return" on government debt, as it is the return R_t net of the re-investment that is necessary to keep a constant ratio of government debt to potential GDP.

Households. The economy is populated by a representative household choosing paths of consumption c_t and government debt holdings b_t in order to maximize

$$\max_{\{c_t, b_t\}} \int_0^\infty e^{-\rho t} \left\{ \log c_t + v\left(b_t\right) \right\} dt \tag{1}$$

subject to the budget constraint

$$c_t + \dot{b}_t \leq \left(R_t - G_t\right) b_t + w_t n_t - \tau_t. \tag{2}$$

The objective (1) involves flow utility from consumption $\log c_t$ and a utility $v(b_t)$ from holding government debt (relative to potential GDP). The latter captures safety and liquidity

[9] As Jiang et al. [2019] show, aggregate risk alone cannot jointly explain why in the data, R lies persistently below G and governments run persistent deficits at the same time. In standard models with aggregate risk, either R rises above G after adverse shocks, or the government runs a surplus. One potential way out is the assumption of disaster risk, which has not materialized in the Jiang et al. [2019] sample. In Section 7.1, we provide an extension of the Barro [2020] model along those lines, which microfounds the reduced-form convenience utility assumed in this section.

benefits that have been used extensively and are well documented in the literature (e.g. Sidrauski 1967, Krishnamurthy and Vissing-Jorgensen 2012). In line with this literature, we assume that the utility over government debt is twice differentiable, increasing and concave, $v' \geq 0, v'' \leq 0$.[10] Flow utility is discounted using a discount rate ρ. The discount rate pins down the steady state return on assets other than government debt, which, as we derive below, is given by $\rho + G^*$.

The budget constraint (2) involves labor income $w_t n_t$ and lump-sum taxes τ_t. Labor income derives from agents selling their labor endowment n_t. We assume that agents wish to sell an endowment of 1 but may be unable to do so due to a standard downward nominal wage rigidity (which happens at the ZLB in our model). In particular, the path of nominal wages W_t satisfies

$$\frac{\dot{W}_t}{W_t} \geq \pi^* - \kappa(1 - n_t). \tag{3}$$

This implies that whenever labor demand is falling short of the unit labor endowment, wage inflation will fall short of π^*. The lower labor demand is, the lower wage inflation will be, just like in a standard Phillips curve. $\kappa \geq 0$ parameterizes the slope of the Phillips curve.

Representative firm. We assume that labor is used by a representative firm with linear production technology $y_t = n_t$. The firm charges flexible prices, pinning down the real wage $w_t = 1$. Price inflation π_t in our de-trended model is equal to wage inflation and therefore determined by (3).

Government. The government sets fiscal and monetary policy. Fiscal policy consists of paths $\{x, b_t, \tau_t\}$ of government spending x, government debt b_t and taxes τ_t, subject to the flow budget constraint

$$x + (R_t - G_t)\, b_t \leq \dot{b}_t + \tau_t \tag{4}$$

The primary deficit is given by

$$z_t \equiv x - \tau_t \tag{5}$$

We assume taxes adjust to ensure that z_t follows a given fiscal rule $z_t = \mathcal{Z}(b_t)$. Typically, $\mathcal{Z}(b)$ is downward-sloping in debt b, corresponding to a lower deficits or greater surplus with higher debt levels.

Government debt b_t is short-term and real in our baseline model. We relax both of these

[10]We also assume that the range of v' is given by $[0, \infty)$ or $(0, \infty)$ and that $v'' < 0$ whenever $v' > 0$.

assumptions in Section 7.2. Government spending $x \geq 0$ is assumed to be constant for now. Our analysis below is parallel to one in which government spending is allowed to vary while taxes are kept fixed (see also Section 6).

Monetary policy is dominant in our model and successfully implements the natural allocation whenever feasible. In particular, we denote by $\{R_t^*\}$ the path of the nominal natural interest rate, which would materialize in the absence of nominal rigidities in our model, assuming inflation is constant at its target π^*. We assume that the actual nominal interest rate then follows

$$R_t = \max\{0, R_t^*\}. \tag{6}$$

In particular, whenever the natural interest rate is positive, R_t tracks the natural interest rate R_t^*, the economy is at potential, $y_t = n_t = 1$, and inflation is at its target $\pi_t = \pi^*$.[11] When the natural rate is negative, however, R_t is constrained to be equal to zero by the ZLB. In that case, we will find that the economy falls below potential, $y_t = n_t < 1$.[12]

Equilibrium. We define equilibrium in our model as follows.

Definition 1. Given an initial level of debt b_0 and a fiscal rule $\mathcal{Z}(\cdot)$, a (competitive) equilibrium consists of a tuple $\{c_t, y_t, n_t, b_t, R_t, G_t, \pi_t, \tau_t, z_t, w_t\}$, such that: (a) $\{c_t, b_t\}$ maximizes the household objective (1) subject to (2); (b) the deficit $\{z_t\}$ follows the fiscal rule $\mathcal{Z}$ and taxes are in line with (5); (c) debt evolves in line with the flow budget constraint (4) and remains bounded; (d) monetary policy sets the nominal rate R_t in line with the rule (6); (e) inflation π_t is determined by the Phillips curve (3); (f) output y_t is given by $y_t = n_t$ and the real wage is $w_t = 1$; (g) the goods market clears $c_t + x = y_t$. A *steady state equilibrium* is an equilibrium in which all quantities and prices are constant.

Stability at the ZLB. One concern with our analysis may be that, when the economy is at the ZLB, the nominal rate is fixed at zero, $R_t = 0$. In textbook New-Keynesian (NK) models, this leads to indeterminacy and thus multiplicity of bounded equilibria (Benhabib, Schmitt-Grohé, and Uribe 2001, Cochrane 2017). Our model differs from a textbook NK model in that it involves utility over government debt, which can lead to stability at the ZLB (Michaillat and Saez 2019). Following a similar logic, we show in Appendix A.1 that one can show that equilibria with a binding ZLB in our model are locally determinate

[11] As is well known, the inflation target can be implemented by an active Taylor rule $R_t = R_t^* + \phi\pi_t$ with $\phi > 1$.

[12] This is similar to the rationing equilibria in Barro and Grossman [1971], Malinvaud [1977], and Benassy [1986].

whenever the Phillips curve is not too steep,

$$\kappa < \frac{\rho + G^*}{1 - x}. \tag{7}$$

We assume that this condition is satisfied for the remainder of our analysis.

Features of government debt in the model. The model follows the extensive body of research arguing that government debt directly enters the utility of those who hold it, which explains why government debt has low yields relative to similar assets (e.g., Krishnamurthy and Vissing-Jorgensen 2012, Caballero, Farhi, and Gourinchas 2017). The underlying logic of this assumption is government debt has certain benefits that lead it to be valued above and beyond future cash flows. These benefits can be due to primitive factors such as household demand for liquidity and safety, institutional factors such as regulatory requirements facing financial institutions, or international factors such as the demand for dollar-denominated assets.[13] Such an assumption generates a convenience yield that helps considerably in explaining the pricing of government debt. For example, of all the potential solutions to the "U.S. public debt valuation puzzle" considered by Jiang et al. [2019], the assumption of a convenience yield is the only one that makes a serious dent in the puzzle. There is a large empirical literature arguing that the pricing advantage of government debt declines in the amount of government debt outstanding, which is a main feature of our model. This literature is discussed in detail below in Section 4.2.

An alternative reason for low yields on government debt is that they provide a hedge against aggregate risk (Bohn 1995), and specifically disaster risk (Barro 2020). These papers demonstrate that permanent deficits cannot be sustained in every state of the world, including after disaster shocks. Building on these insights, we provide a microfoundation of $v(b)$ as "safety premium" in a model with disaster risk and default in Section 7.1 below. In this microfoundation, disasters occur in different sizes. Debt b is "safe" if it is repaid after disasters of (almost) all sizes, carrying a large premium. Greater b increases the range

[13] The demonstration of such a convenience yield on U.S. government debt is the subject of a large body of research including studies by Krishnamurthy and Vissing-Jorgensen [2012], Vandeweyer [2019], Van Binsbergen, Diamond, and Grotteria [2019], Koijen and Yogo [2020], Mota [2020]. There is a considerable range of estimates given in the literature for the size of the convenience yield based on the fact that comparable assets such as AAA rated bonds may also have a convenience yield. Mota [2020] estimates an average convenience yield of U.S. Treasuries of 130 basis points. Studies focused on demand for dollar-denominated debt estimate a convenience yield of U.S. Treasuries of 200 to 215 basis points (e.g., Jiang, Krishnamurthy, and Lustig [2021], Koijen and Yogo [2020]). Del Negro, Giannone, Giannoni, and Tambalotti [2017] estimated that rising convenience yields may have been an important factor behind the recent decline of the riskless rate in the US.

of disasters for which debt is defaulted on, reducing the premium. We show that the dynamics of this economy *before* the disaster occurs are isomorphic to those implied by a model with an exogenous convenience utility $v(b)$.

More broadly, as we show next, our model generates a positive relationship between government debt and the interest rate R. Our calibration in Section 4 will target moments of that relationship directly. We demonstrate in Appendix B that other environments generate a similar relationship, including those based on precautionary saving as in Aiyagari and McGrattan [1998], or those based on OLG models with crowding out of capital, as in Blanchard [2019]. Those models can be calibrated to match the same moments, and we speculate that our further analysis would be similar in those models, albeit less tractable.

3 The deficit debt diagram

We begin our equilibrium analysis by studying steady state equilibria.

3.1 Steady state equilibria

Our model admits a set of steady state equilibria, indexed by the level of steady state debt $b \geq 0$. For each b, one can find a primary deficit z such that $\dot{b} = 0$ and the economy remains steady at that level of debt b. We distinguish two cases, according to whether the economy is above or at the zero lower bound (ZLB).

Above the ZLB. When the economy is above the ZLB, monetary policy implements the natural allocation. This implies that the interest rate is equal to the natural rate, $R_t = R_t^*$, output and employment are at potential, $y_t = n_t = 1$, inflation is at its target, $\pi_t = \pi^*$, and the nominal growth rate is equal to nominal trend growth, $G_t = G^*$.

To see how the natural rate is determined, consider the household's Euler equation

$$\frac{\dot{c}_t}{c_t} = R_t^* - G^* - \rho + v'(b_t)c_t \tag{8}$$

Here, $v'(b_t)$ enters as it is the marginal convenience utility from saving one more unit in government bonds. It enters with the opposite sign as the discount rate ρ and therefore effectively makes the household more patient when saving in government bonds.

In a steady state, consumption is constant and equal to $1 - x$ by goods market clearing.

This lets us solve (8) for the natural interest rate,

$$R^*(b) = \rho + G^* - \underbrace{v'(b) \cdot (1 - x)}_{\text{convenience yield}}. \tag{9}$$

This expression for the natural interest rate on government debt is intuitive. The natural rate is equal to $\rho + G^*$, which would be the steady state return on any non-convenience-bearing assets, minus the steady state convenience yield $v'(b) \cdot (1 - x)$. The expression already suggests how R^* moves with debt. As v is a concave utility function, R^* weakly increases in government debt b.

At ZLB. For low levels of government debt b, the natural rate $R^*(b)$ may be negative and therefore unattainable for monetary policy due to the ZLB. In this region of the state space, output falls short of potential and labor is rationed. Using the goods market clearing condition, output is given by

$$y_t = c_t + x \tag{10}$$

where consumption follows an Euler equation again,

$$\frac{\dot{c}_t}{c_t} = \underbrace{0 - (G^* - \kappa (1 - y_t))}_{R_t - G_t} - \rho + v'(b_t) c_t \tag{11}$$

but this time involving a nominal interest rate of zero, $R_t = 0$, and an endogenous nominal growth rate $G_t = \gamma + \pi_t$ with inflation determined by the Phillips curve (3). Solving the system of (10) and (11) at a steady state with $\dot{c}_t = 0$, we find

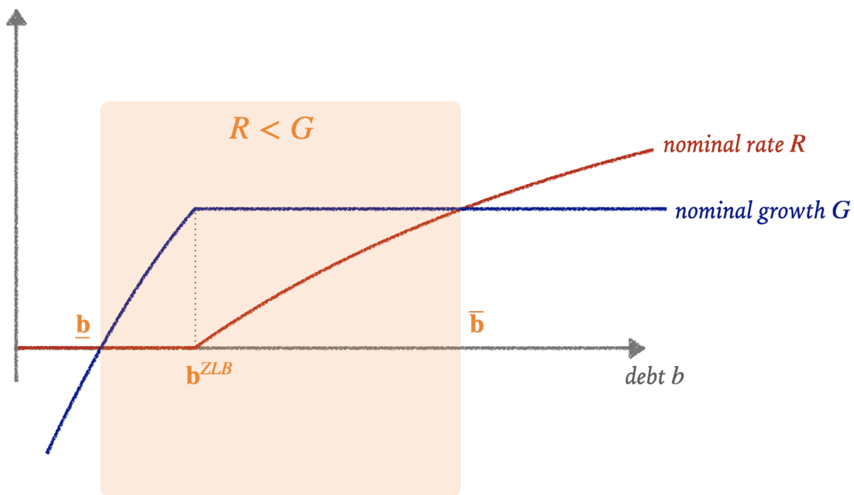
$$G(b) = G^* - \frac{\kappa}{v'(b) - \kappa} \left(-R^*(b) \right). \tag{12}$$

At the ZLB, the interest rate is exogenous, $R = 0$, but the nominal growth rate G is endogenous to the debt level. It lies below nominal trend growth G^*, and by more so the more negative $R^*(b)$ is. As we show in the appendix, the fraction $\frac{\kappa}{v'(b)-\kappa}$ is well defined with $v'(b) > \kappa$ in expression (12) due to our assumption (7).

3.2 Steady state deficits

Taken together, the previous two sections can be summarized by Figure 1. To explain it, we go from high to low levels of debt. For high levels of debt, the economy is above the ZLB

Figure 1: Nominal interest rates and nominal growth rates across steady state debt levels



and the interest rate R on government debt lies above the growth rate G. As we reduce debt levels, R falls below G at some $b = \overline{\mathbf{b}}$ and hits the ZLB at some $b = \mathbf{b}^{ZLB}$. At the ZLB, G starts falling with lower b, as inflation undershoots its target. Eventually, nominal growth falls below zero due to deflation at $b = \underline{\mathbf{b}}$. Left of $\underline{\mathbf{b}}$, R lies above G again.

We can characterize the thresholds in closed form.

Proposition 1. *Define*

1. $\underline{\mathbf{b}}$ *by*
$$v'(\underline{\mathbf{b}})(1-x) = \frac{1-x}{1-x-G^*/\kappa} \cdot \rho$$
if $1 - x > G^*/\kappa$. *Else set* $\underline{\mathbf{b}}$ *equal to infimum of the domain of* v'.

2. $\mathbf{b}^{ZLB}$ *by*
$$v'(\mathbf{b}^{ZLB})(1-x) = \rho + G^*$$

3. $\overline{\mathbf{b}}$ *by*
$$v'(\overline{\mathbf{b}})(1-x) = \rho \tag{13}$$

Then, $\underline{\mathbf{b}} < \mathbf{b}^{ZLB} < \overline{\mathbf{b}}$. *Moreover,* $R(b) > G(b)$ *iff* $b < \underline{\mathbf{b}}$ *or* $b > \overline{\mathbf{b}}$, *and* $R(b) = 0$ *iff* $b < \mathbf{b}^{ZLB}$.

Proposition 1 characterizes the three thresholds that split up the state space into the four regions shown in Figure 1 according to the relative positions of $R(b)$ and $G(b)$. One noteworthy implication of the equation for the upper bound (13) is that the size of $\overline{\mathbf{b}}$ can be large, and is in no meaningful way constrained by existing household or private wealth of agents. In fact, with $\rho \to 0$, $\overline{\mathbf{b}}$ would diverge to infinity, allowing the government to run permanent deficits even for very large debt levels. In this limit, private wealth relative to potential GDP would become unboundedly large. This distinguishes our analysis from that in Reis [2021], who finds that the level of government debt relative to GDP is bounded above by the level of private wealth to GDP. There is no such bound in our economy.

An immediate corollary to the proposition is that the gap between the two, $G(b) - R(b)$, is hump-shaped and positive over the interval $(\underline{\mathbf{b}}, \overline{\mathbf{b}})$. It peaks right at the point at which the economy hits the ZLB, $\mathbf{b}^{ZLB}$. This has an important implication for the level of the primary deficit $z(b)$ that the government is required to choose for the economy to be in a steady state equilibrium at b,

$$z(b) = \left(G(b) - R(b)\right) b.$$

Indeed, the primary deficit is also positive over the interval $(\underline{\mathbf{b}}, \overline{\mathbf{b}})$. Since $G(b) - R(b)$ increases for b left of $\mathbf{b}^{ZLB}$, the peak of $z(b)$ must lie weakly to the right of $\mathbf{b}^{ZLB}$. To characterize the shape of $z(b)$ more formally, we introduce notation to characterize the semi-elasticity of the convenience yield,

$$\varphi\,(b) \equiv -(1-x)\frac{\partial v'(b)}{\partial \log b} = -(1-x)v''(b)b$$

We then have the following result.

Proposition 2. *The steady state primary deficit is positive over the interval* $(\underline{\mathbf{b}}, \overline{\mathbf{b}})$ *and given by*

$$z(b) = \big(v'(b)\cdot(1-x) - \rho\big)\, b - \frac{v'(b)}{v'(b)-\kappa}\left(R^*(b)\right)^{-} b \tag{14}$$

- *When the interest rate is at the ZLB,* $b < \mathbf{b}^{ZLB}$*, and growth is positive,* $b > \underline{\mathbf{b}}$*, the deficit* $z(b)$ *strictly increases in debt.*
- *If* φ *declines monopolistically and* $\varphi\left(\mathbf{b}^{ZLB}\right) > G^*$*, then it peaks at* $\mathbf{b}^z = \mathbf{b}^{ZLB}$*.*
- *If it peaks at some* $\mathbf{b}^z > \mathbf{b}^{ZLB}$*,* $\mathbf{b}^z$ *satisfies*

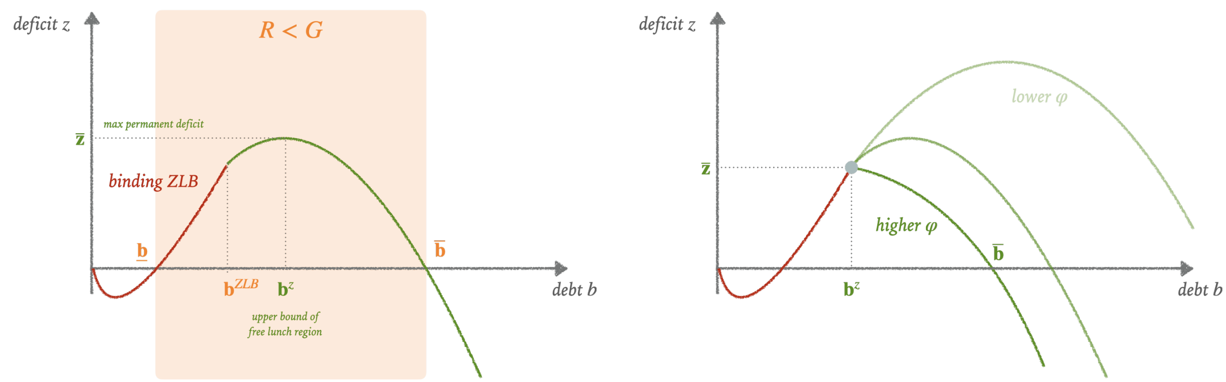
$$R^*(\mathbf{b}^z) = G^* - \varphi(\mathbf{b}^z) \qquad \Leftrightarrow \qquad v'(\mathbf{b}^z)(1-x) = \rho + \varphi(\mathbf{b}^z) \tag{15}$$

At the peak, the maximum deficit is given by

$$\overline{\mathbf{z}} = \varphi(\mathbf{b}^z)\mathbf{b}^z \tag{16}$$

Figure 2: The deficit-debt diagram

(a) Baseline **(b)** Varying φ



Proposition 2 characterizes the shape of steady state primary deficits $z(b)$ as a function of the debt level b. We sketch the shape in Figure 2. The function $z(b)$ increases when the ZLB is binding. It stops increasing as soon as the ZLB stops binding if the elasticity of the convenience yield $\varphi\left(\mathbf{b}^{ZLB}\right)$ is higher than G^*. This ratio corresponds to the ratio of nominal trend growth to the nominal return on assets other than government debt. With an elasticity below this ratio, primary deficits keep increasing further, above the ZLB, until they peak at some debt level $\mathbf{b}^z$. As shown in (15) this debt level is greater the smaller the elasticity of the convenience yield $\varphi\left(\mathbf{b}^z\right)$ is. The maximum deficit is given by $\overline{\mathbf{z}}$. Equations (15) and (16) are implicit equations in general. We solve them below explicitly for special functional forms of the convenience utility $v(b)$.

However, we already note here that in our model, there is no force that sets a "hard limit" as to how large the maximum permanent deficit $\overline{\mathbf{z}}$ may be. In fact, as we let the convenience utility $v(b)$ approximate a linear function of government debt, not only does $\overline{\mathbf{b}}$ diverge to infinity, so do $\mathbf{b}^z$ and $\overline{\mathbf{z}}$ with it. In this limit, therefore, there exist steady states with debt levels and deficits that are many multiples of potential GDP. In practice, realistic values for the elasticity φ may prevent such large deficits and debt levels. This will be one motivation behind our measurement exercise in Section 4.

3.3 The deficit debt diagram

Figure 2 plots the steady state deficits $z(b)$ as a function of the debt. This plot provides us a meaningful notion of "fiscal space" which we will use extensively in the remainder of the paper. For lack of a better term, we call it the *deficit-debt diagram*.

The diagram neatly illustrates the "Goldilocks dilemma" we mentioned in the introduction: If the debt level is too low, the economy faces the consequences of a binding ZLB in the form of output losses and under-employment. If the debt level is too high, a potentially large primary surplus has to be run to finance it.

3.4 When is rising debt a "free lunch"?

One idea that has garnered considerable attention in the literature surrounding $R < G$ (see, e.g., Blanchard 2019) is that the condition seemingly allows economies to run larger deficits temporarily, and then simply "grow out" of the resulting increased debt levels without a need to raise taxes. We refer to this idea as the "free lunch" property of higher deficits. A stronger version of the "free lunch" idea is that *permanent* increases in deficits do not require tax increases going forward, even if they lead to permanently greater (non-explosive) debt levels.

Both versions of the free lunch idea can easily be derived from the government budget constraint (4), under the assumption of a constant interest rate R and a constant growth rate $G > R$. Then,

$$\dot{b}_t = -(G - R)\, b_t + z \tag{17}$$

describes a stable differential equation for debt b. This implies that temporary increases in deficits of arbitrary magnitude, leading to greater debt levels, can always be grown out of over time. Also, a permanent increase in deficits by some Δz simply raises steady state debt levels by $\Delta z/(G - R)$, with no need for a reduction in deficits, i.e. an increase in taxes, at any point. Both versions of the free lunch property are satisfied with exogenous R and G in (17). This is clearly a stylized example but it captures one, if not *the* most, important reason why fiscal policy in a world with $R < G$ is thought to be so different from fiscal policy with $R > G$.

We next investigate the extent to which the free lunch idea holds true in our model. What distinguishes our model from the stylized analysis in (17), is that both G and R are endogenous to the debt level. To understand the dynamics of b_t, it is crucial to incorporate this endogeneity. To do so, we first describe the behavior of the debt level for a general exogenous path z_t of primary deficits. Then, we feed in the specific paths for deficits that correspond to the two versions of the free lunch property. We separate again the case at the ZLB from the case above the ZLB.

Transitions above the ZLB. Above the ZLB, even along transitions, consumption remains constant at $1-x$. Thus, the natural rate $R^*(b_t)$ is still given by (9),

$$R^*(b_t) = \rho + G^* - v'(b_t) \cdot (1-x).$$

Therefore, the dynamics of the debt level simplify follow

$$\dot{b}_t = -\left(G^* - R^*(b_t)\right) b_t + z_t \tag{18}$$

for an exogenous path of deficits z_t.[14] Notably, the dynamics of debt are perfectly backward looking, despite households being forward looking with rational expectations. This stems from the fact that consumption is constant even along transitions due to the goods market clearing condition, pinning down the natural interest rate in each instant.

Transitions at the ZLB. At the ZLB, the situation is more complex as consumption is no longer constant. In this case, the economy is described by a system of two differential equations, the Euler equation

$$\frac{\dot{c}_t}{c_t} = -\left(G^* - \kappa\left(1-x-c_t\right)\right) - \rho + v'(b_t)c_t \tag{19}$$

in addition to the government budget constraint

$$\dot{b}_t = -\left(G^* - \kappa\left(1-x-c_t\right)\right) b_t + z_t. \tag{20}$$

Representing transitions in the deficit-debt diagram. A useful diagram to study the effects of temporary or permanent changes in deficits is the deficit-debt diagram.[15] In Figure 3 we indicate with arrows the direction the economy travels in when deficits are moved above or below the steady state locus.

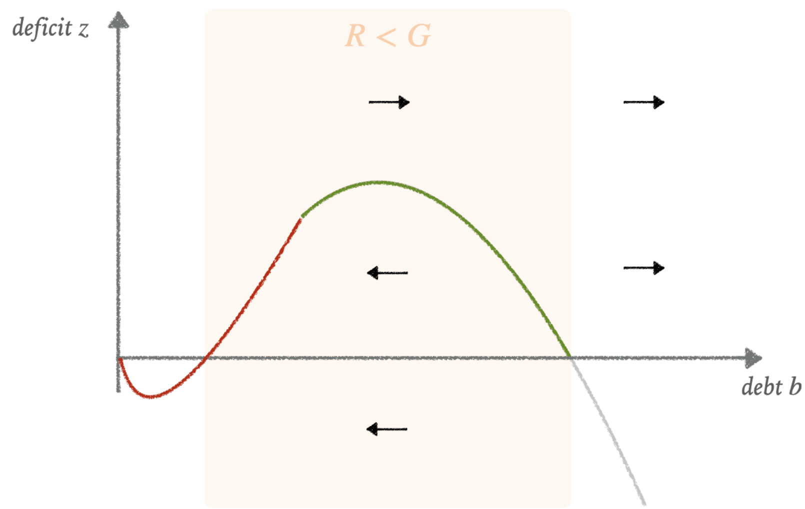
The behavior is intuitive. When deficits are raised above the steady state locus, debt grows, until either the steady state locus is hit, or until, at some point in the future, the deficit is reduced again down to the steady state locus. When deficits are reduced below the steady state locus, debt falls over time.

Mathematically, this behavior follows immediately from (18) in the case where the economy is above the ZLB and the evolution of debt is purely backward looking. In the

[14] If deficits followed a fiscal rule $z_t = \mathcal{Z}(b_t)$ instead, one would simply have to replace z_t in (18).

[15] An alternative is a (c, b) phase diagram, which we develop in Appendix A.4.

Figure 3: Transitions when changing the deficit



case where the economy is at the ZLB, the behavior follows directly from a phase diagram analysis of (19) and (20) (see Appendix A.4).

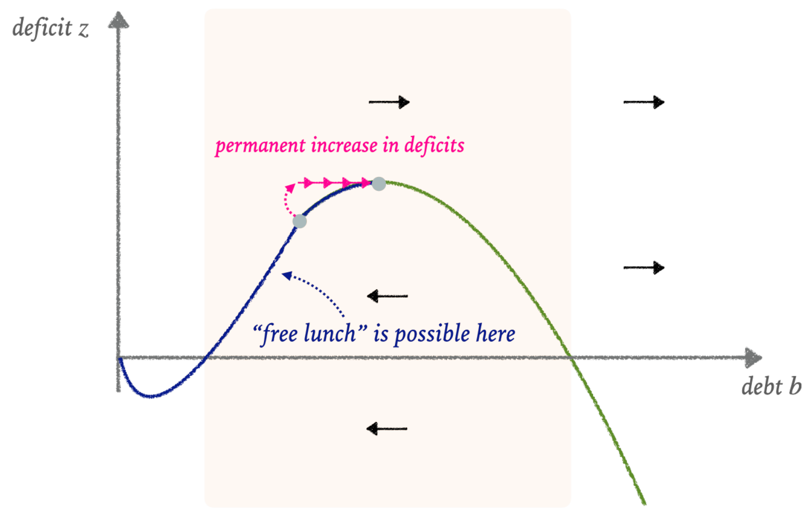
The free lunch region in our model. The directions in Figure 3 allows us to see the region of the state space in which the government can obtain a "free lunch". Indeed, any steady state on the increasing part to the left of the peak at $\mathbf{b}^z$ allows for some form of a free lunch. For example, starting at any of these steady states, a permanent increase in the deficit to any value below or equal to $\bar{\mathbf{z}}$ can be sustained indefinitely. If the deficit increase is temporary, it can exceed $\bar{\mathbf{z}}$, as long as it is reduced back to $\bar{\mathbf{z}}$ or below in time. We show an example transition along these lines in Figure 4.

An important implication of this behavior of our model is that a free lunch policy is always available when the interest rate is at the ZLB and $b < \mathbf{b}^{ZLB}$, since the deficit-debt locus always increases in the ZLB region. This is a result that is often informally made by advocates of "Modern Monetary Theory" (e.g. Kelton 2020). Deficit-financed fiscal stimulus can always be used to ensure that an economy at the ZLB returns to full employment, and the resulting increased level of debt need not be repaid by greater tax levels.

However, while the diagram in Figure 4 illustrates how a "free lunch" policy is indeed possible, it also makes the limits of such a policy very clear. For example, if deficits are increased by too much and / or for too long, a free lunch cannot obtained.

More fundamentally, a free lunch policy cannot work if the initial debt level already exceeds $\mathbf{b}^z$, that is, the initial steady state lies on the downward-sloping branch of the

Figure 4: Free lunch (or not)



deficit-debt locus in Figure 4.[16] In this case, any deficit increases, however temporary, must ultimately be met by reduced deficits, or even, surpluses. In other words, taxes must increase. Crucially, this logic applies despite the fact that the economy displays $R < G$ throughout.

How is this possible? The aspect of our theory that is responsible for this result is the endogeneity of interest rates $R^*(b)$ to the debt level. As the debt level increases, the convenience yield of government debt falls, raising the interest cost on all (infra-marginal) outstanding debt positions. This can undo the positive effect of a greater debt position on the government budget constraint when $R < G$ that we highlighted at the beginning of this section. In fact, as Figure 4 illustrates, this precisely happens for debt levels greater than $\mathbf{b}^z$. This analysis shows that, if the level of debt is larger than $\mathbf{b}^z$, the economics behind the financing of fiscal deficits are entirely conventional: greater debt must be repaid by raising taxes. Whether $R < G$ or $R > G$ is irrelevant for this question when debt is above $\mathbf{b}^z$. As the following corollary shows, based on our results in Proposition 2, the correct threshold is not G, but $G - \varphi$.

Corollary 1. *Assume the deficit-debt diagram $z(b)$ is single-peaked. Fix a level of debt b_0 with corresponding natural rate $R = R^*(b_0)$ and growth rate $G = G(b_0)$. It admits a free lunch if and only if it either*

- *has a binding ZLB, or*

[16]Strictly speaking, there could be multiple local maxima of $z(b)$ in our model. The condition for the absence of a free lunch policy is that there can be no steady state with a greater debt level and a greater or equal deficit z.

- *satisfies* $R < G - \varphi(b_0)$.

4 Measuring the deficit-debt diagram

We have shown in the previous section that the shape of the deficit-debt locus pins down several key quantities that help us understand the abilities and limits of fiscal policy. In this section, we set out to calibrate the model to illustrate the key factors determining the deficit-debt locus, and to offer an attempt to measure the locus as accurately as possible. Along the way, we focus on three specific quantities that are determined by the deficit-debt locus: the maximum permanent deficit $\overline{\mathbf{z}}$, the associated debt level $\mathbf{b}^z$, beyond which a free lunch policy ceases to be feasible, and the level of debt $\overline{\mathbf{b}}$ at which the interest rate R rises above the growth rate G.

The crucial object to calibrate is the shape of the convenience yield $v'(b)(1-x)$. We view the fact that $v'(b)(1-x)$ is a crucial object in our model as a promising feature because there is already a large body of research that seeks to estimate this derivative. We discuss this literature in detail below.

Our calibration strategy for $v'(b)$ proceeds in two steps. We first assume a parametric family of functional forms for $v'(b)$ and then determine the parameters that match a given steady state with debt b_0 (the US economy in 2019) as well as estimates of the local (semi-)elasticity of the convenience yield $\varphi(b_0)$. We henceforth abbreviate $\varphi(b_0)$ by φ. Since matching the elasticity only provides accuracy in a neighborhood of b_0, we provide analyses of robustness with respect to the functional forms below.

4.1 Functional forms for the convenience yield

We consider two functional forms for $v'(b)$ that the empirical literature has documented a good empirical fit of (e.g., Krishnamurthy and Vissing-Jorgensen 2012, Presbitero and Wiriadinata 2020). The first is a linear specification, which will be our baseline,

$$\text{linear:} \quad v'(b)\,(1-x) = v'(b_0)\,(1-x) - \varphi\frac{b-b_0}{b_0}. \tag{21}$$

The second is a log-linear specification,

$$\text{log-linear:} \quad v'(b)\,(1-x) = v'(b_0)\,(1-x) - \varphi\log\frac{b}{b_0}. \tag{22}$$

In both cases, we set $v'(b) = 0$ for any b sufficiently large to cause the right hand side to move below zero. The intercept $v'(b_0)(1-x)$ is determined by the initial steady state (assumed to be above the ZLB for now), for which the Euler equation pins down the convenience yield $v'(b_0)(1-x)$ as

$$v'(b_0)(1-x) = \rho + G^* - R_0. \tag{23}$$

For both cases, we can explicitly solve the three main quantities of interest.

Proposition 3. *For the linear specification* (21)*, we have*

$$\frac{\overline{\mathbf{b}}}{b_0} = 1 + \frac{1}{\varphi}(G^* - R_0)$$

$$\frac{\mathbf{b}^z}{b_0} = \begin{cases} \frac{1}{2}\frac{\overline{\mathbf{b}}}{b_0} & \text{if } \varphi < G^* + R_0 \\ 1 - \frac{1}{\varphi}R_0 & \text{if } \varphi \geq G^* + R_0 \end{cases}$$

$$\overline{\mathbf{z}} = \begin{cases} b_0 \frac{\varphi}{4}\left(1 + \frac{1}{\varphi}(G^* - R_0)\right)^2 & \text{if } \varphi < G^* + R_0 \\ b_0\left(1 - \frac{1}{\varphi}R_0\right)G^* & \text{if } \varphi \geq G^* + R_0 \end{cases}$$

For the log-linear specification, we have

$$\log \frac{\overline{\mathbf{b}}}{b_0} = \frac{1}{\varphi}(G^* - R_0)$$

$$\log \frac{\mathbf{b}^z}{b_0} = \begin{cases} \log \frac{\overline{\mathbf{b}}}{b_0} - 1 & \text{if } \varphi < G^* \\ -\frac{1}{\varphi}R_0 & \text{if } \varphi \geq G^* \end{cases}$$

$$\overline{\mathbf{z}} = \begin{cases} \varphi \cdot \mathbf{b}^z & \text{if } \varphi < G^* \\ G^* \cdot \mathbf{b}^z & \text{if } \varphi \geq G^* \end{cases}.$$

These are simple expressions that allow us to translate empirical estimates of φ directly into the three objects of interest. Interestingly, the objects are pinned down by only four statistics: the elasticity φ, the initial debt level b_0, nominal trend growth G^*, and the initial interest rate R_0.[17] Neither government spending x nor the discount rate ρ are relevant. The elasticity φ takes a crucial role in the formulas we introduced in this section, which is why we measure it next.

[17] The ZLB threshold can also be computed in closed form based on these statistics. We find $\mathbf{b}^{ZLB}/b_0 = 1 - \varphi^{-1}R_0$ for the linear specification and $\log\left(\mathbf{b}^{ZLB}/b_0\right) = -\varphi^{-1}R_0$ for the log-linear one.

4.2 Measuring the elasticity φ

Given the importance of the elasticity φ to the determination of the shape of the deficit-debt locus, we discuss the measurement of this parameter at length in this section. There are different ways to estimate the elasticity φ that are equivalent within the context of the model. Expanding (23), we can write

$$\varphi = -\frac{\partial\left(\rho+G-R\right)}{\partial \log b} = -b_0 \frac{\partial\left(\rho+G-R\right)}{\partial b} \tag{24}$$

Alternative ways to obtain φ are given by

$$\varphi = -\frac{\partial\left(G-R\right)}{\partial \log b} = -b_0 \frac{\partial\left(G-R\right)}{\partial b} \tag{25}$$

because, in the model, ρ is independent of b. As both (24) and (25) are valid ways to obtain φ, we will compare estimates across these specifications. Note, however, that specifications estimating (25) are slightly more robust, as they do not hinge on a convenience-yield interpretation of $R^*(b)$ and apply equally well to the alternative models in Appendix B.

The derivatives in equations (24) and (25) have been estimated in the literature, and we summarize these estimates in Table 1.[18] For equation (24), Krishnamurthy and Vissing-Jorgensen [2012] focus on estimates of $\frac{\partial(\rho+G-R)}{\partial \log b}$. This derivative measures how the difference between the rate of return on government debt R and the return on other assets $\rho+G$ varies with a change in the log government debt to GDP ratio. Krishnamurthy and Vissing-Jorgensen [2012] use the yield spread difference between corporate bonds rated Baa and 10-year Treasuries as the measure of $\rho+G-R$, and they show a semi-elasticity of -1.3% to -1.7%, depending on the sample. This implies that a 10 percent increase in debt to GDP leads to a 13 to 17 basis point decline in the convenience yield. Alternatively, one can use the Krishnamurthy and Vissing-Jorgensen [2012] estimates to measure $b_0\frac{\partial(\rho+G-R)}{\partial b}$, which gives estimates between -1.1% and -1.8% when using the average debt to GDP ratio over the relevant sample period for b_0.[19] Finally, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan [2020b] provide estimates of the effect of government debt to GDP ratios on

[18] A detailed explanation of the exact specifications used from the existing literature to construct Table 1 is in Appendix C. We thank Sam Hanson, Andrea Presbitero, Quentin Vandeweyer, and Ursula Wiriadinata for helpful discussions.

[19] Two other studies in the literature use short-term T-bills and more high frequency data. Greenwood et al. [2015] find estimates for $b_0\frac{\partial(\rho+G-R)}{\partial b}$ in this range, around -1.4%, whereas Vandeweyer [2019] finds an estimate of -0.4%. These two studies provide estimates that are not as close to the object of interest in the model given that they are based on a local estimate of the demand for T-bills as opposed to all government debt.

convenience yields for Eurozone countries from 2002 to 2020. The implied estimate of $b_0 \frac{\partial(\rho+G-R)}{\partial b}$ from their main specification is -0.8%.

There is also a literature estimating the derivative in equation (25), which is $b_0 \frac{\partial(G-R)}{\partial b}$. In particular, the recent study by Presbitero and Wiriadinata [2020] estimate this derivative in a sample of 56 countries from 1950 to 2019. They provide estimates of $\frac{\partial(G-R)}{\partial b}$ for 17 advanced economies and for the full sample. After multiplying these estimates by b_0, which is the average debt to GDP ratio in each of the respective samples, the implied estimates of $b_0 \frac{\partial(G-R)}{\partial b}$ are -1.4%. For this study, we replicated the Presbitero and Wiriadinata [2020] results for the 17 advanced economies and also for the Group of 7 (G7) countries, and the coefficient estimate ranges are also reported in Table 1. The appendix shows the full results from the regressions. The estimates of interest are robust to the inclusion of both time and country fixed effects. Overall, most of the estimates across the different samples and the two different objects fit between -1.0% and -2.5%.

An alternative technique to estimate $\frac{\partial(G-R)}{\partial \log b}$ is an analysis of the 2021 Georgia Senate run-off elections that took place on January 5th in the United States. Ex-ante, there was about an even probability of the two Democrat candidates winning their elections as there was that at least one of the two winning candidates was Republican. In the event of a Democrat win, Democrats would obtain the Senate majority, and would likely pass the $1.9 billion deficit-financed stimulus package already proposed by President-elect Biden. This was unlikely to happen otherwise. As shown in Figure 21 in Appendix C, the wins by both Democrats in Georgia led to a significant persistent increase in real 10 year Treasury yields of about 8 basis points. The effect is concentrated right after the election. It is unlikely that the election was associated with a change in long-term growth prospects; as a result, we interpret the evidence as suggesting that the prospect of the $1.9 trillion stimulus package, approximately corresponding to 7.4% of outstanding debt, led to a persistent 8 basis point reduction in $G - R$. As this the Democrat win was anticipated with 50% likelihood, this gives

$$\frac{\partial (G-R)}{\partial \log b} = -2.2\%.$$

The natural experiment yields an effect of government debt on $G - R$ that is in the same range as the estimates from the existing literature. Please see the Appendix C for details on this calculation.

Finally, Laubach [2009] and Engen and Hubbard [2004] estimate the effect of government debt to GDP on real interest rates, finding effects in the range 3% to 4.4%. The average level of government debt (total federal debt) to GDP over their sample period was about 50%.

Table 1: How does government debt to GDP affect convenience yield and $G-R$?

Study	Countries	Sample	Object	Estimated φ
Convenience yield: $\rho+G-R$				
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1926-2008	$b_0\frac{\partial(\rho+G-R)}{\partial b}$	-0.011
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1969-2008	$b_0\frac{\partial(\rho+G-R)}{\partial b}$	-0.018
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1926-2008	$\frac{\partial(\rho+G-R)}{\partial \log b}$	-0.013
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1969-2008	$\frac{\partial(\rho+G-R)}{\partial \log b}$	-0.017
Greenwood et al. [2015]	USA	1983-2007	$b_0\frac{\partial(\rho+G-R)}{\partial b}$	-0.014
Vandeweyer [2019] (natural experiment)	USA	2014-2016	$\frac{\partial(\rho+G-R)}{\partial \log b}$	-0.009
Jiang et al. [2020b]	Eurozone	2002-2020	$b_0\frac{\partial(\rho+G-R)}{\partial b}$	-0.008
Growth minus Interest Rate: $G-R$				
Presbitero and Wiriadinata [2020]	17 AEs	1950-2019	$b_0\frac{\partial(G-R)}{\partial b}$	-0.014
Presbitero and Wiriadinata [2020]	31 AEs & 25 EMs	1950-2019	$b_0\frac{\partial(G-R)}{\partial b}$	-0.013
This paper	17 AEs	1950-2019	$\frac{\partial(G-R)}{\partial \log b}$	-0.015 to -0.031
This paper	G7	1950-2019	$\frac{\partial(G-R)}{\partial \log b}$	-0.020 to -0.028
This paper	USA, Senate	Jan 2021	$\frac{\partial(G-R)}{\partial \log b}$	-0.022
Negative Real Interest Rate: $-R$				
Laubach [2009]	USA	1976-2006	$b_0\frac{\partial(\pi-R)}{\partial b}$	-0.015 to -0.022*
Engen and Hubbard [2004]	USA	1976-2003	$b_0\frac{\partial(\pi-R)}{\partial b}$	-0.015*

Notes. This table summarizes estimates from the existing literature of the effect of government debt to GDP ratios on convenience yields (upper panel) and $G-R$ (lower panel). All of the details on the exact specifications used are in the appendix. Further details on the country-year panel regressions done in this study and the evaluation of the Georgia Senate election results of January 2021 are also in the appendix.

* Estimates in Laubach [2009] and Engen and Hubbard [2004] are stated in terms of $\frac{\partial(-R)}{\partial b}$. To obtain $b_0\frac{\partial(-R)}{\partial b}$, estimates were multiplied by $b_0=0.5$, the average level of total federal debt to GDP over the sample period.

Table 2: Baseline calibration

Parameter	Description	Chosen to match target	Value
b_0	initial debt to GDP	2019 US federal debt to GDP	100%
x	gov. spending to GDP	2019 gov outlays to GDP	20%
R_0	initial nominal rate	fall 2019 5-year Treasury yield	1.5%
G^*	nominal trend growth	CBO long-term growth projection	3.5%
ρ	discount rate	return on private wealth other than gov. debt	0.03
κ	slope of the wage Phillips curve	condition (7)	0.075

Together, this gives an estimate of φ of $b_0 \frac{\partial(G-R)}{\partial b} \approx -1.5\%$ to -2.2% under the assumption that the real growth rate is unaffected by government debt.

Overall, while there is some variation, most of the implied elasticity estimates φ lie in the range 1.1% – 2.5%. We pick the average estimate $\varphi = 1.7\%$ as our baseline parameter and explore robustness to $\varphi = 1.2\%$ and $\varphi = 2.2\%$ below.

4.3 Calibrating other parameters to the US

We calibrate the remaining model parameters to match the US economy in the fall of 2019, before the pandemic recession of 2020/21. We set the initial debt level to $b_0 = 100\%$ of GDP, assume government spending of $x = 20\%$. We set the initial nominal rate to $R_0 = 1.5\%$ in line with nominal interest rates in the fall of 2019.[20] We set the nominal trend growth rate to $G^* = 3.5\%$. In line with $G^* - R_0 = 2\%$, the US was indeed running a primary deficit of about 2% before the pandemic. We set the discount rate to $\rho = 3\%$, in line with about a $\rho + G^* = 6.5\%$ rate of return on private wealth other than government debt. Finally, condition (7) requires that the slope of the Phillips curve κ lies below 0.081. Since this is already a relatively flat Phillips curve, we choose a value close to the bound of 0.075.[21] We collect all our baseline parameters in Table 2.

Figure 5: Calibrated deficit-debt diagram

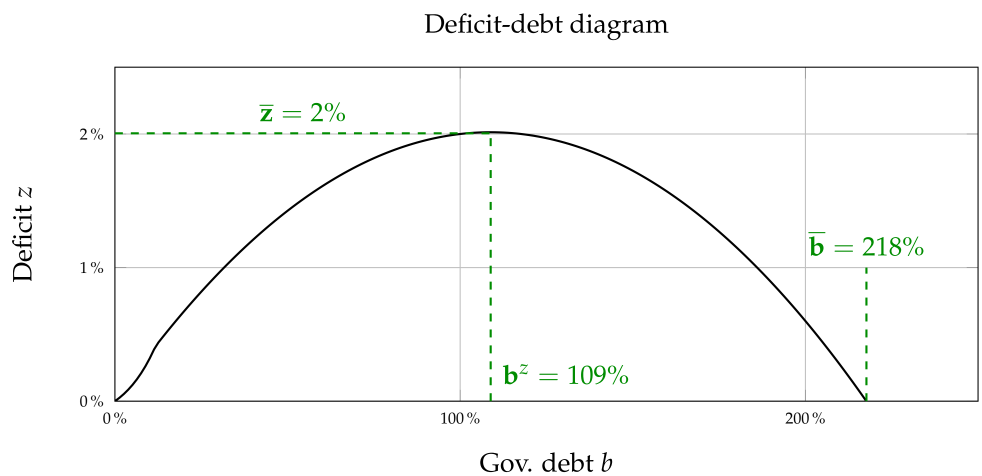
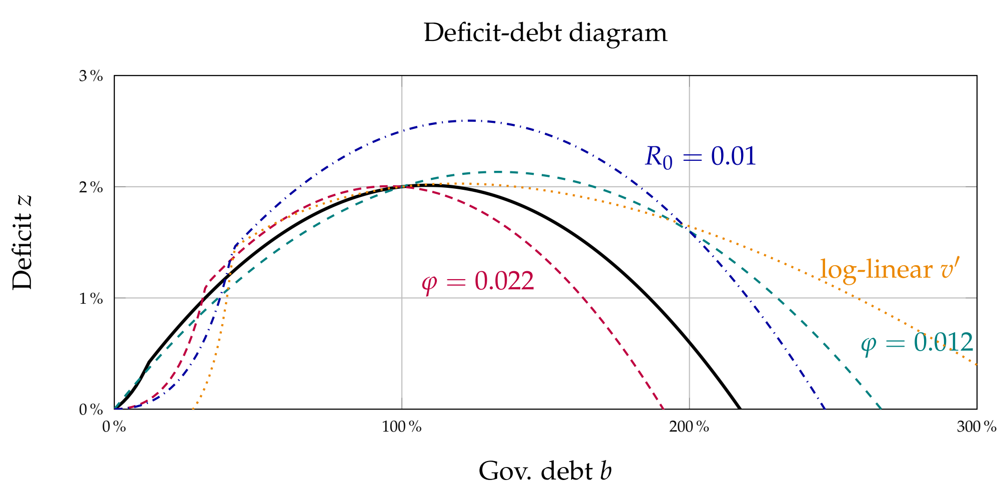


Figure 6: Robustness in deficit-debt diagram



4.4 Results for the US

Figure 5 shows the deficit-debt diagram that is implied by our calibration, as well as the three main quantities we set out to measure. The calibration suggests that the permanent primary deficit that the United States can run is $\overline{\mathbf{z}} = 2.0\%$ of GDP; it can run that at a debt level of $\mathbf{b}^z = 109\%$ of GDP; and the debt level at which R rises above G is $\overline{\mathbf{b}} = 218\%$ of GDP. These numbers are interesting against the backdrop that US government debt currently stands at around 126% as of the fourth quarter of 2020.[22] According to this calibration, any increase in debt above 109% of GDP would have to be met by increases in taxes going forward (even if R remains below G). Moreover, note that even if debt remained stable at 109%, this would only allow the US government to run a 2% permanent deficit. As of March 2021, the average projected annual primary fiscal deficit to GDP ratio by the CBO for 2021 to 2030 is 3.3%. For 2021 to 2050, it is 3.9%. If the large projected deficit of 2021 is excluded, then the averages are 2.7% and 3.7%, respectively.

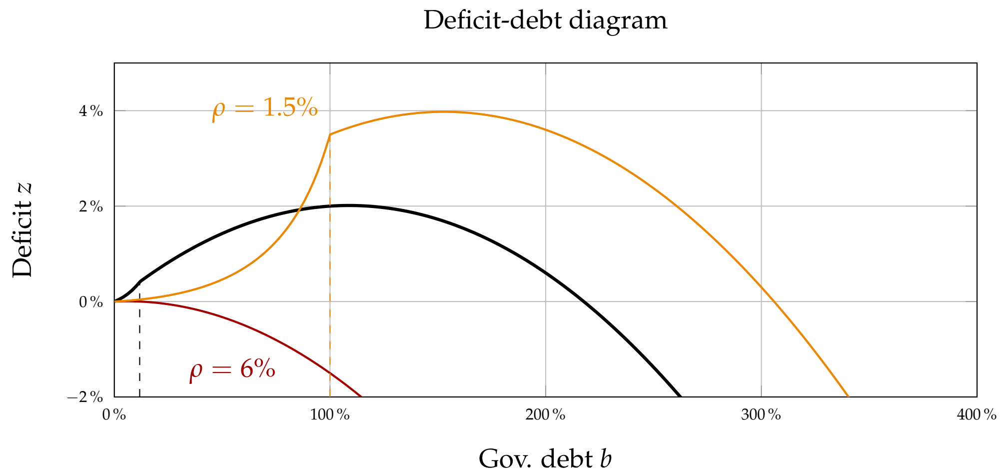
Clearly the implied numbers by our calibration should not be taken as gospel. They instead illustrate how our stylized model can be put to work to parse through recent US data. We show robustness across alternative assumptions about the convenience yield in Figure 6. In particular, we show deficit debt diagrams for smaller or greater elasticities φ; for the log-linear functional form (22); and for a reduced pre-Covid natural interest rate $R_0 = 1\%$. Shifting G^* and R_0 in parallel (e.g. $G^* = 4\%$, $R_0 = 2\%$) does not affect the deficit-debt schedule above the ZLB. Also, as we discussed above, neither ρ nor the level of government spending x affects the deficit-debt schedule conditional on calibrating R_0. Across the alternatives shown in Figure 6, the maximum deficit $\overline{\mathbf{z}}$ remains relatively robust around 2-2.5%. Among alternatives with the linear functional form (21), the debt level $\mathbf{b}^z$ at which $\overline{\mathbf{z}}$ is attained varies in the range 100% to 130%. The debt level at which R crosses G is most uncertain, with estimates varying between just below 200% to just above 300%.

[20] The effective federal funds rate was 1.55%, the 5-year Treasury yield was about 1.65%, the 10-year yield just above that.

[21] Note that κ only matters in the ZLB region. We adjust κ below when we shift parameters in order to ensure that (7) continues to hold.

[22] The current government debt to GDP ratio represents all government debt outstanding as measured by the Financial Accounts of the Federal Reserve scaled by nominal GDP as measured in the National Income and Product Accounts. As of the end of 2020, the U.S. Treasury had a large cash balance at the Federal Reserve, which should be netted out. Using an unveiling process as in Mian et al. [2020], the U.S. Government (other than the Federal Reserve) held 14% of GDP in claims on government bonds, primarily through this large cash balance at the Federal Reserve. If this position is netted out, then the outstanding government debt to GDP ratio held by entities other than the U.S. Government as of the end of 2020 was 112%. It is not appropriate to net out other holdings of the Federal Reserve, as these other holdings are financed by the private banking system through reserves.

Figure 7: Shifts in discount rates



Note. We set $\kappa = 0.055$ for the plot with reduced ρ to ensure (7) holds.

5 What determines fiscal space?

Across most advanced economies, there appears to be ample fiscal space; government debt to GDP ratios are high, and yet government deficits continue to be substantial. An examination of the underlying parameters of the model sheds light on how this situation continues to persist. Furthermore, the model highlights challenges to governments running large deficits while experiencing a decline in trend growth due to population aging or lower productivity growth. We focus in particular on three factors that are relevant for amount of fiscal space: aggregate demand (as captured by discount rates), trend growth rates, and inequality.

5.1 Aggregate demand

We capture movements in aggregate demand by shifts in the discount rate ρ. A greater discount rate means agents are more impatient and would like to spend more and save less, which raises R^*. Vice versa, a smaller discount rate means agents are more patient and would like to spend less and save more, which lowers R^*.[23]

Figure 7 plots the deficit-debt diagram for ρ's equal to 1.5%, 3% (our baseline), and 6.5%. For the case $\rho = 1.5\%$ (orange line), we see that fiscal space generally expands, as R^* is reduced and $G^* - R^*$ grows. However, the lower ρ also leads there to be a tight

[23]The discount factor ρ does affect fiscal space in this experiment, as we do not calibrate R_0 and G^* here, different from our calibration in the previous section.

ZLB constraint, with $\mathbf{b}^{ZLB}$ at 100% (dashed orange line). We confirm this in the following proposition.

Proposition 4. *A reduction in the discount rate ρ*

- *increases fiscal space above the ZLB.*
- *reduces fiscal space at the ZLB.*
- *raises the ZLB threshold $\mathbf{b}^{ZLB}$ according to $v'(\mathbf{b}^{ZLB}) = \frac{\rho+G^*}{1-x}$.*

The opposite holds for an increase in ρ.

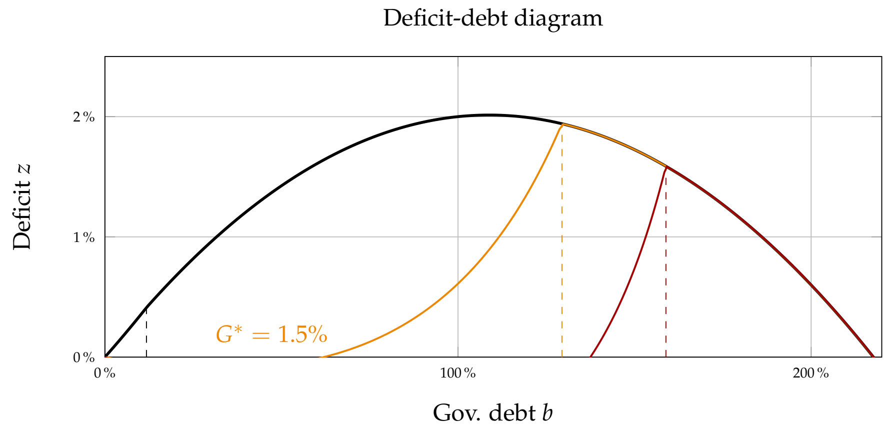
One way to interpret a reduction in ρ is that during recessions such as the Great Recession or the Covid recession, agents have a greater desire to save. This raises $\mathbf{b}^{ZLB}$, possibly above b_0, pushing the economy into the ZLB region. Deficit-financed fiscal stimulus can mitigate the recession and move the economy to debt levels above $\mathbf{b}^{ZLB}$. Just like before, we see that governments can always run permanent deficits at the ZLB (as long as $G^* > 0$). This also illustrates that negative shocks to aggregate demand are unlikely to cause fiscal sustainability concerns.

The case $\rho = 6.5\%$ corresponds to a situation with less desire to save and a greater R^*. In fact, in this example, R^* is sufficiently high for any level of debt b that it consistently lies above the trend growth rate G^*. The economy needs to run a primary surplus. This can be thought of as the state of the US economy in the 1980s or early 1990s. One can say that the amount of fiscal space that the United States can currently afford is a "symptom" of low secular demand / low secular interest rates R^*.

5.2 Trend growth

A reduction in nominal trend growth G^*—whether caused by a productivity growth slowdown, falling inflation expectations, or declining population growth—seems like it may tighten fiscal space by moving G^* closer to R. But this is not obvious as slower growth rates lead to a greater desire for saving by households, pushing R^* down alongside G^*. With log preferences over consumption as in (1), R^* falls one for one with G^*, as in (9), leaving $G^* - R^*$ unchanged. This is why, in our model, growth does not affect steady state deficits above the ZLB. This implies that in that region, the deficit-debt schedule is unchanged. However, slowing growth does affect the ZLB threshold and deficits at the ZLB, as the following proposition demonstrates.

Figure 8: Shifts in trend growth



Note. We set $\kappa = 0.04$ to ensure that (7) holds with reduced G^*.

Proposition 5. *A slowdown in nominal trend growth G^**

- *leaves fiscal space above the ZLB unchanged.*
- *shrinks fiscal space at the ZLB.*
- *raises the ZLB threshold $\mathbf{b}^{ZLB}$ according to $v'(\mathbf{b}^{ZLB}) = \frac{\rho+G^*}{1-x}$.*

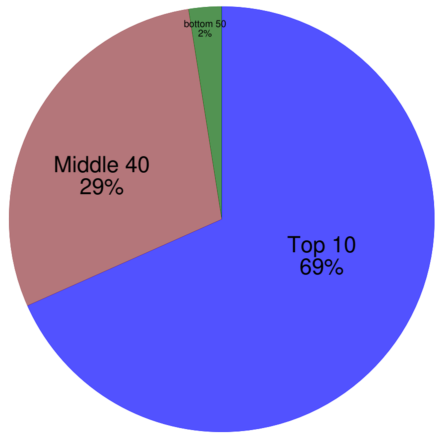
The proposition follows directly from the results in Section 3. We illustrate it in Figure 8. Above the ZLB, the economy can continue running the same primary deficits as before, even as growth slows. However, as soon as R^* falls sufficiently to cross zero, the economy is at the ZLB, where output is no longer at potential, and the permanent primary deficit actually falls in G^*.

In Figure 8, this can be seen as the orange and red lines showing tighter and tighter ZLB thresholds $\mathbf{b}^{ZLB}$ (vertical dashed lines) that force policymakers to run greater deficits temporarily so as to move the economy out of the ZLB region. Ultimately, the economy ends up with much greater levels of government debt and possibly smaller steady state deficits. At a stylized level, this seems to capture Japan's experience over the past three decades.

5.3 The role of inequality

Inequality is relevant for fiscal sustainability, as it is mainly richer households that, directly or indirectly, own government debt. Figure 9 shows that the top 10% of the wealth distribution in the United States holds almost 70% of the government debt outstanding

Figure 9: Holdings of U.S. Government Debt across the Household Wealth Distribution, as of 2020



Notes. This figure plots the fraction of U.S. government debt held by households across the wealth distribution. These are fractions held after unveiling the financial sector. Please see Mian et al. [2020] for more details on the unveiling process. The unveiling process that produces this figure uses the Distributional Financial Accounts of the Federal Reserve to measure the share of assets owned by each wealth group. U.S. households hold almost 60% of U.S. government debt, with the rest of the world holding most of the remaining U.S. government debt.

held by the U.S. household sector. Furthermore, the bottom 50% of the wealth distribution holds almost no government debt at all. This implies that the willingness or ability of richer households to save is a primary factor in the determination of R^*. To speak to these issues, we extend our model to allow for two types of agents.

Savers (or *bondholders*) earn a share $\omega^s \in (0, 1)$ of labor income and behave just like the representative agent did above. In particular, savers maximize utility $\int_0^\infty e^{-\rho t} \{\log c_t^s + v(b_t)\}\, dt$ subject to the budget constraint

$$c_t^s + \dot{b}_t \leq (R_t - G_t)\, b_t + \omega^s w_t n_t - \tau_t^s. \tag{26}$$

Hand-to-mouth agents earn a share $\omega^h = 1 - \omega^s$ of labor income and have no access to financial markets, that is,

$$c_t^h = \omega^h w_t n_t - \tau_t^h.$$

Here, τ_t^h may be negative in order to capture transfers to hand-to-mouth agents. Observe that, above the ZLB, this model can be thought of as a reinterpretation of our representative-agent model, in which the government spends on behalf of hand-to-mouth agents.

We calibrate this model by identifying savers with the top 10% and setting $\omega^s = 50\% = \omega^h$. For now, we set $\tau_t^h = 0$, which we relax in Section 6 below. We keep the other parameters unchanged.

How is fiscal space affected when inequality increases? It is now the Euler equation of savers that determines the steady state natural interest rate,

$$\frac{\dot{c}_t^s}{c_t^s} = R_t^* - G^* - \rho + v'(b_t)c_t^s$$

where, above the ZLB, $c_t^s = 1 - \omega^h - x$. We thus obtain the natural interest rate R^* as

$$R^*(b_t) = \rho + G^* - v'(b_t)\left(1 - \omega^h - x\right)$$

Different from before, the income distribution now directly influences the convenience yield. Following the same steps as in Section 3, we obtain:

Proposition 6. *Greater inequality ($\omega^s \uparrow$, $\omega^h \downarrow$ with $\omega^s + \omega^h$ unchanged)*

- *expands fiscal space above the ZLB, where*

$$z(b) = \left(v'(b_t)\left(1 - \omega^h - x\right) - \rho\right) b.$$

- *shrinks fiscal space at the ZLB, where*

$$z(b) = \frac{v'(b)}{v'(b) - \kappa} G^* b + \frac{\kappa}{v'(b) - \kappa}\left(\rho - v'(b)\left(1 - \omega^h - x\right)\right) b.$$

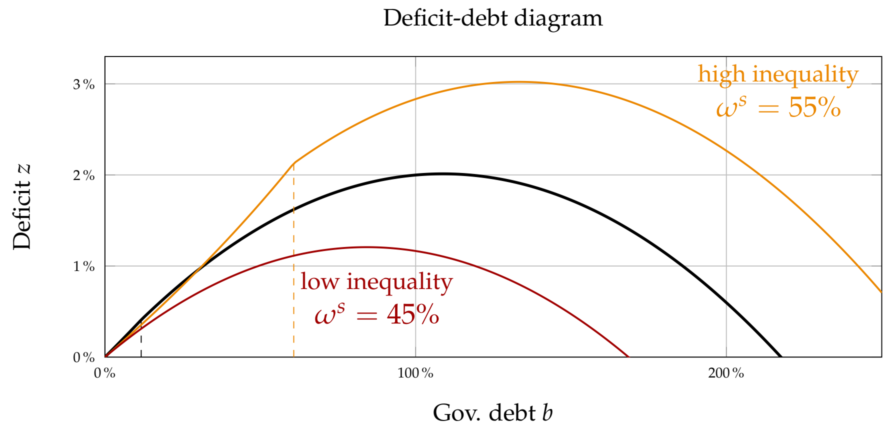
- *raises the ZLB threshold $\mathbf{b}^{ZLB}$ according to $v'(\mathbf{b}^{ZLB}) = \frac{\rho + G^*}{1 - \omega^h - x}$.*

The results are reversed with lower inequality.

Proposition 6 shows that inequality is tightly connected to the amount of fiscal space. On the one hand, it increases fiscal space above the ZLB, allowing the government to run greater permanent deficits. This is because inequality puts more resources into the hands of savers, increasing their ability and willingness to hold larger quantities of government debt. But this very fact also implies that, at the ZLB, greater inequality tightens fiscal space, as it reduces demand, inflation, and ultimately nominal growth G. Figure 10 illustrates these findings.

The model provides intuition behind the observation that rising income inequality has been accompanied by rising fiscal deficits and government debt levels in many advanced economies. Rising income inequality allows governments to borrow more cheaply from

Figure 10: Rising income inequality



savers. However, accumulating high government debt in response to a rise in income inequality raises challenges going forward. For example, if the government implements policies that reduce inequality, it will need to raise taxes to cover the higher debt costs associated with the high debt levels accumulated when inequality was high. It also presents challenges to tax policy, which we discuss in the next section.

6 The role of tax policy

The fact that inequality, and in particular the economic position of savers, matters for the amount of fiscal space has important implications for tax policy, which we explore next. We continue to use the model with two types of agents, as introduced in section 5.3.

Tax instruments. We begin by introducing the tax instruments. In addition to separate taxes on savers and hand-to-mouth agents, we also allow for consumption taxes τ_t^c (which are equivalent to value added taxes in our model) and capital income taxes τ_t^b. Savers thus maximize their utility function subject to the modified budget constraint

$$(1+\tau_t^c)\, c_t^s + \dot{b}_t \leq \left(\left(1-\tau_t^b\right) R_t - G_t\right) b_t + \omega^s w_t n_t - \tau_t^s \tag{27}$$

while the budget constraint of hand-to-mouth agents is given by

$$(1+\tau_t^c)\, c_t^h = \omega^h w_t n_t - \tau_t^h. \tag{28}$$

Together, this gives us four tax instruments: regressive income taxes τ_t^h on the hand-to-mouth, progressive income taxes τ_t^s on savers, consumption taxes τ_t^c, and capital income taxes τ_t^b. In the presence of capital income taxes, before-tax and after-tax returns on government bonds differ. Throughout this section we follow the convention that R_t^* denotes the after-tax return on government bonds. We do so because (a) it is the after-tax return that matters for the ZLB and (b) the government effectively pays the after-tax return on its debt once tax revenue from capital income taxes is netted out.

Tax policy and fiscal space. With this convention, and the four tax instruments, savers' Euler equation now reads

$$\frac{\dot{c}_t^s}{c_t^s} = R_t^* - G^* - \rho + v'(b_t)\left(1+\tau_t^c\right)c_t^s \tag{29}$$

Goods market clearing requires that $c_t^s + c_t^h + x = 1$, where (28) implies that $c_t^h = \frac{\omega^h - \tau_t^h}{1+\tau_t^c}$ in the natural allocation. With constant tax rates, the natural interest rate can be obtained by rearranging (29),

$$R^*(b_t) = \rho + G^* - v'(b_t)\left(\left(1+\tau^c\right)(1-x) - \omega^h + \tau^h\right). \tag{30}$$

The steady state primary deficit is then given by

$$z(b) = \begin{cases} \frac{v'(b)}{v'(b)-\kappa}G^*b + \frac{\kappa}{v'(b)-\kappa}\left(\rho - v'(b)\left(\left(1+\tau^c\right)(1-x) - \omega^h + \tau^h\right)\right)b & \text{at the ZLB} \\ \left(v'(b)\left(\left(1+\tau^c\right)(1-x) - \omega^h + \tau^h\right) - \rho\right)b & \text{above the ZLB} \end{cases} \tag{31}$$

Our main result in this section uses (31) to characterize the effect of the four taxes on the shape of the deficit-debt locus.

Proposition 7. *The tax instruments affect the deficit-debt locus as follows.*

- *Increased regressive income taxes τ_t^h and consumption taxes τ_t^c expand fiscal space the most, by $v'(b)b$ per tax dollar. They reduce steady state deficits at the ZLB, and raise the ZLB threshold $\mathbf{b}^{ZLB}$ in line with $v'(\mathbf{b}^{ZLB}) = \frac{\rho+G^*}{(1+\tau^c)(1-x)-\omega^h+\tau^h}$.*
- *Increased capital income taxes leave fiscal space zone unchanged.*

Proposition 7 studies the effects of raising taxes on the deficit-debt schedule. To interpret the results, we bear in mind that a change in permanent deficits is, by construction, accommodated by a change in the tax on savers τ^s, which is why τ^s itself mechanically has no effect on fiscal space and is excluded from Proposition 7.

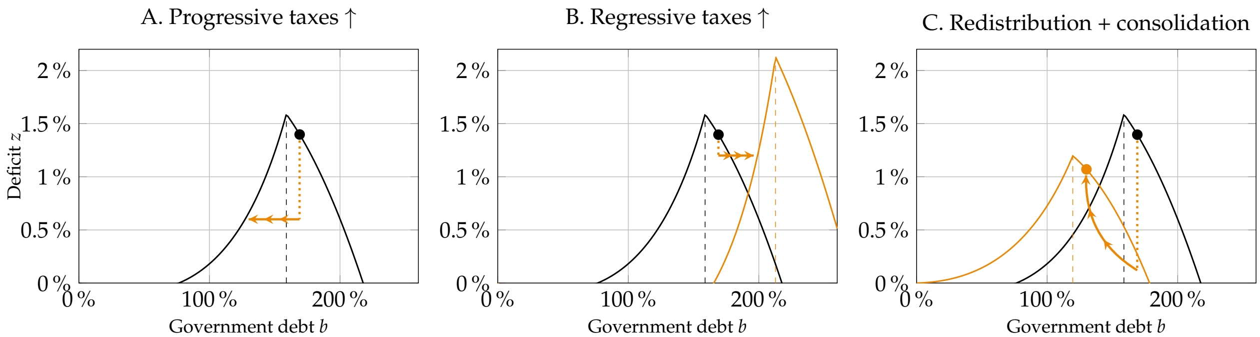
The first part of Proposition 7 studies the effects of raising income taxes on the hand-to-mouth and consumption taxes (using the proceeds to reduce τ^s). Both kinds of taxes are regressive, reducing aggregate demand and natural interest rates R^* in (30). Above the ZLB, this relaxes the government budget constraint and allows for greater permanent primary deficits. However, it also tightens the ZLB, and reduces primary deficits at the ZLB. The second part of Proposition 7 shows that increased capital income taxes are irrelevant for fiscal policy. In fact, they do not even raise any revenue in our model, as the before-tax return on government debt immediately adjusts upwards to keep the after-tax return constant. Two caveats are in place here. First, with longer-duration debt or large surprise taxes at date $t = 0$, some initial expropriation occurs, which can be used for a one-time reduction in government debt. Second, the only source of capital income in our model is interest income from government debt. If other types of capital income, such as dividends, were present, the capital income tax would adopt some of the properties of a tax on savers' income.

There are three immediate implications of Proposition 7, which we now go through in detail.

Implication 1: Redistribution narrows fiscal space. The first implication is that redistributive policies narrow "fiscal space" in the sense of reduced permanent deficits. Taxing savers and giving it to the hand-to-mouth means taxing the buyer of government debt. This reduces the demand for government debt and pushes up R^*, requiring a greater reduction in the deficit. Vice versa, a more regressive tax response limits the required correction in the deficit. This highlights an important dilemma. Retaining enough fiscal space, especially when a "free lunch" policy is to be successful, necessitates that savers have enough resources to purchase bonds at high prices, i.e. at low R^*. One cannot simultaneously eliminate inequality—by redistributing from rich bondholders to poorer hand-to-mouth agents—and sustain a large debt level at low interest rates. The model suggests that a government that chooses to use aggressive fiscal policy in response to a rise in income inequality should recognize that large debt levels make any subsequent effort at redistribution more costly from a fiscal perspective.

Taking this logic one step further establishes that regressive taxation (e.g. consumption taxes) is able to finance a greater level of government debt than progressive taxation, holding fixed the overall tax burden. Governments with sufficiently large debt levels and

Figure 11: Fiscal consolidation during a growth slowdown



Note. This plot assumes $\kappa = 0.03$ to satisfy (7).

interest rates R near or above G may thus be forced to resort to such regressive taxation.[24]

Implication 2: Handling a growth slowdown. As we saw in Section 5.2, a slowdown in growth, causing high levels of government debt in a very deficit-debt schedule, poses a conundrum to policymakers. On the one hand, debt levels are high and R is close to G, raising concerns about fiscal sustainability. On the other, R is close to the ZLB, making it imperative to avoid reductions in the natural interest rate.

This conundrum leads to counterintuitive results. For example, it may seem that regressive tax policies, such as consumption taxes, relax fiscal space by reducing R^* and allow for fiscal consolidation. However, regressive tax policies also tighten the ZLB constraint, requiring increased deficits and debt levels as remedy. This is best seen in the equation for $\mathbf{b}^{ZLB}$ in Proposition 7, $v'(\mathbf{b}^{ZLB}) = \frac{\rho+G^*}{(1+\tau^c)(1-x)-\omega^h+\tau^h}$. Increased τ^c requires increased debt $\mathbf{b}^{ZLB}$ to stay out of the ZLB. Other simple alternative policies do not work either. For example, lowering deficits by reducing taxes on the rich is contractionary as well (albeit by less), and hence pushes the economy into the ZLB as well. We illustrate these policies as paths (A) and (B) in Figure 11.

How can the conundrum be resolved? What the above discussion shows is that strategies that reduce the natural interest rate R^* are doomed to fail, as they further tighten the ZLB. This is despite the fact that such strategies allow for greater fiscal space above the ZLB. Instead, strategies that raise R^* are more promising, when coupled with deficit reductions. An example of such a strategy is increased taxation of savers that is partly used to redistribute to the hand-to-mouth, and partly used to reduce deficits. Path (C) of

[24] This argument can, in principle, be taken even further. Any policy instrument that discourages demand reduces natural interest rates R^* and thus has beneficial effects on the government's interest expenses.

Figure 11 shows the effect of such a strategy. Increased R^* implies a thinner deficit-debt locus, yet one with a ZLB threshold further to the left. This gives the economy room to move to the left with deficit reductions, without hitting the ZLB.[25]

Implication 3: Financial repression. Our third implication concerns financial repression. Financial repression can be modeled as the government imposing a lower bound on the required bond holdings of the saver, $b_t \geq \underline{b}$, thereby allowing it to reduce the interest rate it pays on government debt, from the market rate R_t to some $R_t - \xi_t$, where ξ_t measures the extent of financial repression. Modeled this way, financial repression corresponds to nothing other than a tax on bondholders. When large debt positions are financed this way, a significant amount of repression is necessary. Since it acts like a tax, it reduces the resources of savers and hence reduces their demand for bonds, requiring even more stringent financial repression.

7 Extensions

We next discuss several important extensions of our baseline model.

7.1 Microfoundation of the convenience yield as safety premium

Throughout, we have assumed that $v(b)$ captures exogenous convenience benefits. These benefits are typically thought of as stemming from either liquidity or safety. Many microfoundations exist for liquidity (e.g. Lagos and Wright 2005), and some have been shown to reduce to a $v(b)$ function (Angeletos et al. 2020). In this section, instead, we propose a model of safety premia, interpreting bonds as being safe if they are very likely pay out even in a big disaster.[26]

Consider an economy like the one in Section 2, with two changes. First, there is no ad-hoc convenience utility function v. Second, there is a flow probability $\lambda > 0$ with which a disaster occurs. Conditional on the disaster occurring, it reduces potential output y^* from 1, our normalized pre-disaster value, to $\delta \in (0, 1)$, with probability $f(\delta)$, where $\int_0^1 f(\delta) d\delta = 1$. The only friction that we assume in this model is that the government can

[25] In practice, when debt levels are very high, even such a strategy may not be without dangers. For example, when R^* increases too far, say beyond G, fiscal sustainability may be at risk as the government may have to shift towards running primary surpluses.

[26] We describe in Appendix B a number of alternative models and show that they numerically have similar implications to our reduced-form convenience-yield model of Section 2.

only raise tax revenue τ_t up to some fraction $\overline{\tau} + x$ of output.[27] If debt service requires greater taxes, we assume that the government defaults. For simplicity, we assume that default entails default costs (in the form of transfers to households, not resource costs) that are sufficiently large so that all bond wealth is lost.

We analyze this model in two steps. First, we analyze the economy after a disaster of size δ happened. Then, we study the economy before the disaster shock, and argue that it is largely isomorphic to our model in Section 2.

When a disaster of size δ materializes, the interest rate rises to $R = G^* + \rho$, as bonds lose their specialness. This requires the economy to run a primary surplus of $\rho b/\delta$ relative to GDP. Given the upper bound on taxes of $\overline{\tau} + x$, default occurs when output after the shock δ falls below $\underline{\delta} \equiv \rho b/\overline{\tau}$. We denote by $\tilde{V}_t(b;\delta)$ the utility of an individual agent with bond position b after shock δ realizes.

Before the disaster occurs, households now maximize utility

$$\rho V_t(b) \equiv \max_c \log c + \lambda \int_0^1 f(\delta) \left(\tilde{V}_t(b;\delta) - V_t(b) \right) d\delta + \dot{V}_t(b) + V_t'(b)\dot{b}_t \tag{32}$$

where $\dot{b}_t$ is given by the budget constraint (2). Combining the first order condition for c and the Envelope theorem for $V_t(b)$, this formulation can be shown to imply a natural rate before the disaster that depends on b and is given by

$$R^*(b) = \rho + G^* + \lambda - \lambda \int_{\rho b/\overline{\tau}}^1 f(\delta)\delta^{-1} d\delta$$

This expression has exactly the same shape as (9), just with $\lambda - \lambda \int_{\rho b/\overline{\tau}}^1 f(\delta)\delta^{-1} d\delta$ instead of $v'(b)(1-x)$. As before, the convenience yield falls in b. In the special case where the density is equal to $f(\delta) = 2\delta$, we find that the convenience yield is given by

$$\lambda \int_{\rho b/\overline{\tau}}^1 f(\delta)\delta^{-1} d\delta - \lambda = \lambda - 2\frac{\rho\lambda}{\overline{\tau}} b$$

microfounding our affine-linear specification (21).

This illustrates that our analysis in Section 3 applies to a model with a microfounded convenience yield based on safety premia.

[27] We include the share of government spending x here so that the government can always finance its spending. This is equivalent to a cap on the primary surplus of $\overline{\tau}$.

7.2 Long-term bonds and QE

We next relax our assumption of short-term debt. We do so in two steps. First we study an economy with only nominal long-term debt. Then, we assume short and long-term debt co-exist and are imperfectly substitutable.

Only long-term debt: revaluation effects, but same deficit-debt diagram. We assume that debt is both nominal and long-term. We model this with coupons that are exponentially declining at some rate $\psi > 0$ as in Hatchondo and Martinez [2009]. We denote by $\tilde{b}_t$ the nominal principal outstanding, normalized by (nominal) potential GDP as before, and assume each pays a fixed coupon of 1. The price of a bond is

then given by

$$q_t = \int_t^\infty e^{-\int_t^s (R_u+\psi)du} ds$$

We continue to denote by $b_t \equiv q_t \tilde{b}_t$ the present value of government debt, and assume it is b_t that enters agents' utility. One can show that the government's and representative agent's budget constraints, (4) and (2), are unchanged. The only difference in this model, relative to our baseline in Section 2, is that there is a date-0 revaluation effect, changing the value of government debt relative to the previous steady state value b,

$$\frac{b_0}{b} = (R+\psi) \int_0^\infty e^{-\int_0^s (R_u+\psi)du} ds \tag{33}$$

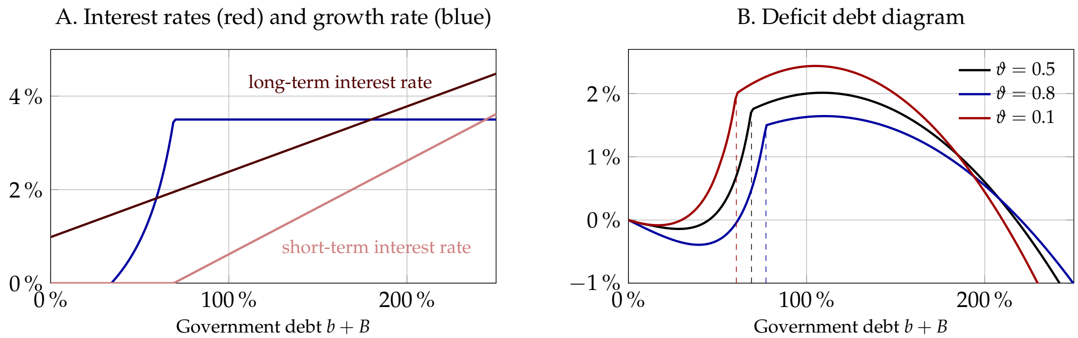
The sole factor determining the strength of the revaluation is the nominal interest rate. In particular, revaluation only depends on inflation through the nominal interest rate.

Equation (33) implies that long-term nominal bonds neither affect the deficit-debt schedule nor the "free lunch" region. While a temporary increase in deficits z_t necessarily leads to a reduction in the real value of outstanding government debt—due to the expectation of rising debt levels and interest rates—this reduction can never outweigh the increased deficits. Debt will eventually always rise above the previous steady state level.[28] However, long-term nominal bonds do help slow down the increase in debt after a given deficit policy.[29]

[28]This follows from a simple proof by contradiction. Imagine debt b_t was to remain below the initial steady state level b. Then, R_t must remain below R and (33) prescribes an increase in b_0, not a reduction, which contradicts our hypothesis.

[29]One question not answered here, is whether expansionary fiscal policy without simultaneous monetary tightening can be self financing due to increasing inflation. Long-term nominal debt is not special to this question as the nominal rate remains unchanged. We answer this question in appendix A.5, showing that this can happen, but only if κ is above the bound in (7).

Figure 12: Fiscal space with various shares ϑ of long-term debt



Note. Plot uses $\alpha = 0.7$. Left panel: $\vartheta = 0.5$. Right panel: $\vartheta \in \{0.1, 0.5, 0.8\}$.

Imperfect substitution: QE helps at ZLB, but reduces fiscal space. Next we combine short term and long-term debt into a single model. In particular, denote the stock of long-term debt relative to potential GDP by B_t. We assume that long-term debt also carriers convenience benefits for investors, albeit less than short-term debt. Thus, we assume a convenience utility of

$$v\left(b_t + \alpha B_t\right)$$

where $\alpha \in (0, 1)$. This specification implies that the natural interest rate on short-term debt, which we continue to denote by R_t^*, is given by

$$R_t^* = \rho + G^* - v'(b_t + \alpha B_t)\,(1 - x)$$

with $R_t = \max\{R_t^*, 0\}$ as before. The interest rate on long-term debt, which we denote by R_t^{LT}, is then

$$R_t^{LT} = R_t + (1 - \alpha)\, v'(b_t + \alpha B_t)\,(1 - x)\,.$$

In particular, R_t^{LT} is strictly greater than R_t, and the spread between the two shrinks in $b_t + \alpha B_t$.

To see how this affects the deficit-debt diagram, we denote the share of LT debt issued by the government by ϑ. The government budget constraint is then

$$\frac{d}{dt}\left(b_t + B_t\right) = \left(\overline{R}_t - G_t\right)\left(b_t + B_t\right) + z_t$$

where $\overline{R}_t = (1 - \vartheta)\, R_t + \vartheta R_t^{LT}$ and G_t is determined by (12) as before.

Figure 12(A) plots R_t, R_t^{LT}, and G_t as function of total debt $b + B$, illustrating the positive spread between R_t and R_t^{LT}, which shrinks at higher debt levels. Figure 12(B) plots the deficit debt locus $z(b+B)$, as function of total debt $b+B$, for various shares of long-term debt ϑ. We choose the parametrization of our model with weak demand (see Figure 7). Two observations are noteworthy. First, with greater shares of long-term debt ϑ, there is less fiscal space at small debt levels; the ZLB region is greater; and the boundary of the free lunch region $\mathbf{b}^z$ generally shifts to the left, possibly even into the interior of the ZLB region. Second, with greater ϑ, there is generally *more* fiscal space at higher debt levels. This is a direct consequence of the fact that long-term debt has smaller convenience benefits, so both interest rates R_t and R_t^{LT} increase less rapidly in long-term debt.

A stylized way to think of large scale purchases of long-term debt (one type of quantitative easing, QE) is that it changes the maturity composition of government liabilities towards short-term debt, effectively lowering ϑ. As Figure 12(B) shows, this can help an economy escape the ZLB (as in Caballero and Farhi 2018a and Cui and Sterk forthcoming), and gives it greater fiscal space at low debt levels. However, it also highlights that QE may reduce fiscal space at higher debt levels.

7.3 Fiscal space of an open economy: foreign vs local currency debt

One important question in international macroeconomics is why many countries borrow in foreign currency, and when might they be better of borrowing in domestic currency. Here, we revisit this question in the context of fiscal space.

To do so, we sketch a small open economy version of our model. The world consists of a continuum of countries. Country i is populated by a representative household with preferences

$$\max \int_0^\infty e^{-\rho t}\left\{\alpha \log c_{iTt} + (1-\alpha)\log c_{iNt} + \int_0^1 v(a_{ijt})dj\right\} dt$$

subject to the budget constraint

$$\mathcal{E}_{it}c_{iTt} + P_{iNt}c_{iNt} + \mathcal{E}_{it}\int_0^1 \dot{a}_{ijt}dj \leq \mathcal{E}_{it}y_{iTt} + W_{it}n_{it} + \mathcal{E}_{it}\int_0^1 \left(R_{jt} + \frac{\dot{\mathcal{E}}_{it}}{\mathcal{E}_{it}} - \frac{\dot{\mathcal{E}}_{jt}}{\mathcal{E}_{jt}} - G_{it}\right) a_{ijt}dj - \tau_{it}.$$

Here, $\mathcal{E}_{it}$ is the price of tradable goods in local currency, and a measure of the nominal exchange rate between the rest of the world and the domestic economy; P_{iNt} is the nominal price of non-traded goods; y_{iTt} is an endowment of traded goods; a_{ijt} is country i's position in the bond of country j, in units of tradable goods. Non-traded goods are produced using the linear production function $y_{iNt} = n_{it}$. W_{it} is the nominal wage. We ignore the ZLB in this analysis and set $n_{it} = 1 - \alpha$ at all times. In the steady state, $y_{iT} = \alpha$, so that real GDP of any economy is equal to 1.

The Euler equation for country i and bond j is then

$$\frac{\dot{c}_{iTt}}{c_{iTt}} = R_{jt} + \frac{\dot{\mathcal{E}}_{it}}{\mathcal{E}_{it}} - \frac{\dot{\mathcal{E}}_{jt}}{\mathcal{E}_{jt}} - G_{it} - \rho + v'(a_{ijt})\frac{c_{iTt}}{\alpha}. \tag{34}$$

When all countries are in a steady state, $\frac{\dot{\mathcal{E}}_{it}}{\mathcal{E}_{it}} - \frac{\dot{\mathcal{E}}_{jt}}{\mathcal{E}_{jt}} = G_i - G_j$, and so

$$R_j = G_j + \rho - v'(a_{ij})\frac{c_{iT}}{\alpha}.$$

Focusing on country i and assuming that the rest of the world is symmetric, we have $c_{Tj} = \alpha(1-x)$, $a_{ji} = b_i$, and $x_j = x$ for all countries, where b_i is country i's steady state debt to potential GDP ratio. The interest rate is then given by

$$R_i = G_i + \rho - v'(b_i)(1-x).$$

In other words, *conditional* on the convenience utility function $v(b_i)$, the small open economy i faces the exact same interest rate schedule as the closed economy in Section 3.

Where an interesting difference emerges is when $v(b_i)$ is microfounded as in Section 7.1. To do so, assume that there is a shock that reduces tradable output in all countries by factor $\delta \in (0,1)$, and in country i by a factor $\eta\delta$, where $\eta \in (0,1)$. This is a simple way to model a global recession that coincides with a depreciation of i's exchange rate by η.

If country i's debt is issued in foreign currency, the shock increases its debt liability relative to the size of the economy, raising the default threshold. The interest rate is then given by

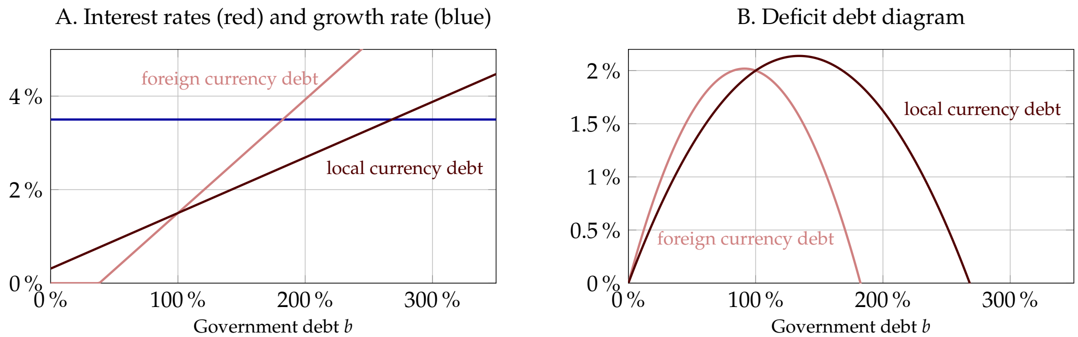
$$R_i^{FC} = G_i + \rho + \lambda - \lambda \int_{\eta^{-1}\rho b/\overline{\tau}}^{1} f(\delta)\delta^{-1}d\delta = G_i + \rho - \lambda\left(1 - 2\eta^{-1}\frac{\rho b}{\overline{\tau}}\right) \tag{35}$$

when we use $f(\delta) = 2\delta$ as above.

If country i's debt is issued in local currency, the shock does not increase its debt liability relative to the size of the economy. The flip-side is that the repayment to international investors is now reduced by a factor η. Thus,

$$R_i^{LC} = G_i + \rho + \lambda - \lambda \int_{\rho b/\overline{\tau}}^{1} \eta f(\delta)\delta^{-1}d\delta = G_i + \rho - \lambda\left(2\eta - 1 - 2\eta\frac{\rho}{\overline{\tau}}b\right) \tag{36}$$

Figure 13: Fiscal space with foreign and local currency debt



Note. We set $\eta = 0.7$. Both were calibrated to semi-elasticity $\varphi = 0.017$ when $\eta = 1$, and to include point with $b_0 = 100\%$, $R_0 = 1.5\%$, $G_0 = 3.5\%$.

We plot (35) and (36) and the implications for fiscal space in Figure 13. We see that for moderate debt levels, foreign currency debt is associated with lower interest rates and greater fiscal space. As long as default is unlikely, foreign investors value a riskless payoff. However, when default becomes more likely due to foreign currency borrowing, local currency debt enables greater fiscal space.

7.4 Fiscal space in a monetary union

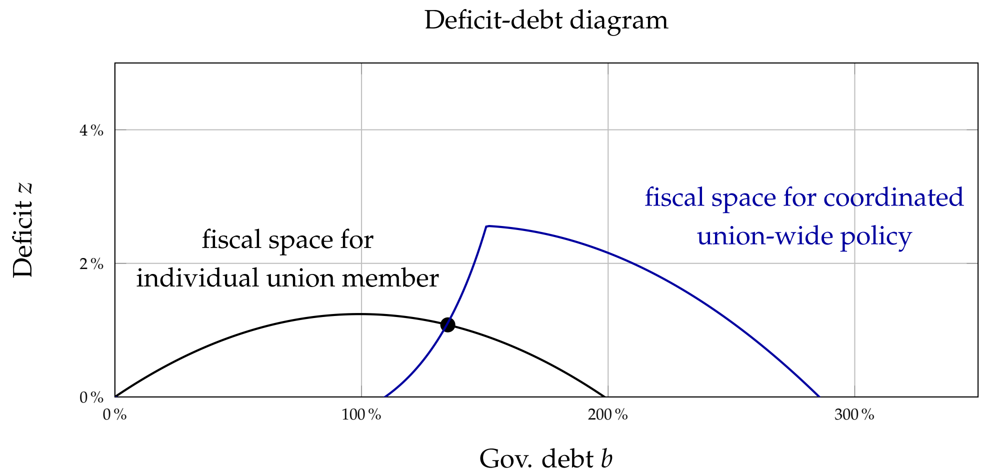
We continue with the model from the previous section, but now study the fiscal space of a country i in a monetary union, with $\mathcal{E}_{jt} = 1$ for all countries j. We focus on the case with a given convenience utility $v(b_i)$. Starting from (34), with a symmetric rest of the world (rest of the monetary union) in steady state, with inflation rate π, this implies for country i,

$$R_i = \gamma_i + \pi + \rho - v'(b_i)\,(1 - x)\,.$$

In other words, it is no longer a countries own nominal inflation rate π_i that enters R_j; instead, it is the union-average inflation rate of π. This difference is immaterial when the union achieves its inflation target; in that case, $\pi = \pi_i = \pi^*$, independent of b_i. However, when the union is at the ZLB, with $\pi < \pi^*$, expansionary fiscal policy of any individual country i cannot affect union inflation π. If fiscal policy was coordinated across members of the union, all members choosing the same level of government debt b, the inflation rate would be pinned down by the union-wide Phillips curve.

We illustrate this in Figure 14 for our calibration for Italy (see Section 8 below). According to the calibration, Italy has significantly less fiscal space going forward than if the entire union as a whole decided to stimulate. A union-wide policy would push up inflation, and

Figure 14: Fiscal space in a monetary union at the ZLB



Note. This plot assumes the calibration for Italy in Section 8, and that inflation $\pi_i = \pi^* - 0.9\%$, as CPI inflation in Italy 2009–2019 averaged 1.1%.

hence nominal growth, increasing each member country's fiscal space.

8 Comparison across countries

Our main calibration in Section 4 is meant to capture the pre-Covid US economy. In this subsection we explore simple calibrations to other countries, namely Japan, Italy and Germany.

Just as before, our calibration is mainly based on four objects—the elasticity φ, the initial debt level b_0, nominal trend growth G^*, and the initial interest rate R_0. For Japan, we further need to take a stand on the slope of the Phillips curve, as it currently is at the ZLB. We use a κ that is at the constraint (7).[30] Inflation in Japan was 1.7% below its target over the sample we consider, we add this to our measure of nominal trend growth G^*. We work with the same elasticity as before, $\varphi = 1.7\%$, but allow as robustness check $\varphi \in \{1.2\%, 2.2\%\}$. Our choices for the other parameters are listed in Table 3. We describe their choice in appendix A.6.

We begin by computing the three numbers that we focused on before, the debt level $\overline{\mathbf{b}}$ for which $R = G$, the highest permanent deficit $\overline{\mathbf{z}}$, and the upper bound $\mathbf{b}^z$ of the free lunch region (the level at which $\overline{\mathbf{z}}$ is attained). We do so for $\varphi = 1.7\%$ and the linear specification

[30]Italy and Germany, while at the ZLB, cannot influence inflation of the union, π, so that κ is not needed. See our monetary union extension in Section 7.4 for details.

Table 3: Calibration to other countries

Calibration parameters		Values by country			
Parameter	Description	USA	Japan	Italy	Germany
φ	semi-elasticity of conv. yield	$\varphi \in \{1.2\%, 1.7\%, 2.2\%\}$			
b_0	initial debt to GDP	100%	225%	135%	60%
R_0	initial nominal rate	1.5%	0%	0%	0%
G^*	nominal trend growth	3.5%	2.3%	0.8%	2.9%

Model statistics		Values by country			
Statistic	Description	USA	Japan	Italy	Germany
$\overline{\mathbf{b}}$	debt level for which $R = G$	218%	529%	199%	162%
$\overline{\mathbf{z}}$	highest permanent deficit	2.0%	5.3%	1.2%	1.9%
$\mathbf{b}^z$	upper bound of free lunch region	109%	265%	99%	81%

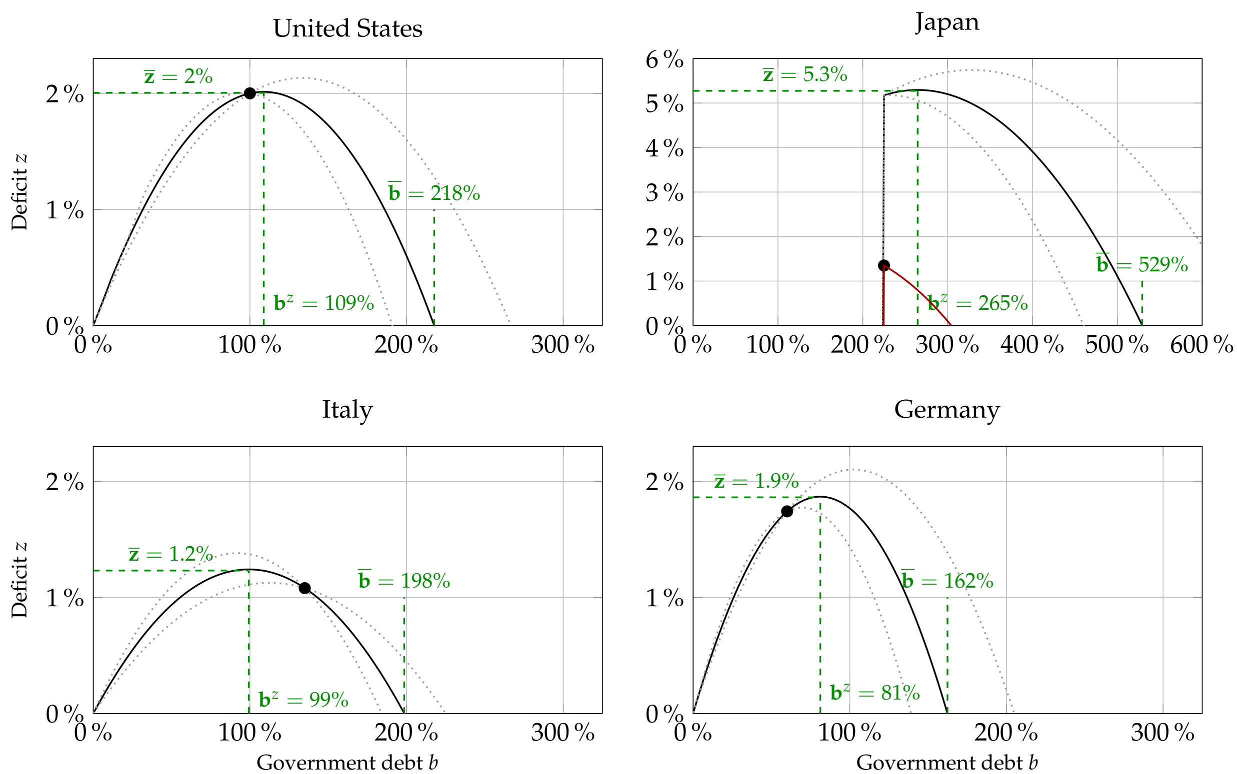
(21). The results are shown in Table 3 and illustrated in Figure 15. Several observations are noteworthy.

First, fiscal space across the countries is similar, except for Japan. Since inflation in Japan is so small (0.3% over 2009–2019), our calibration attests Japan a large amount of fiscal space. In fact, if inflation was equal to 2% in Japan, it's trend growth would be 2.3%. With an inflation targeting central bank that does not tighten interest rates before 2% inflation is reached, this implies a deficit of $2.3\% \times 225\% = 5.2\%$ of GDP. This already indicates that Japan has ample fiscal space.

The implicit assumptions in this calculation are that the inflation target can be reached with aggressive enough fiscal policy; and that, despite possibly significant overheating during the transition monetary policy stays put until the inflation target is reached. An alternative assumption is that inflation expectations and the implicit inflation target are indeed only 0.3%. In that case, the red line represents fiscal space for Japan.

Italy, which has comparable fundamentals to Japan (0.8% nominal growth over 2009–2019 compared to 0.6% for Japan), has much less fiscal space than Japan. The reason is that it does not borrow in its own currency. Aggressive Italian fiscal policy cannot sustainably increase Italian inflation, as it would if it had its own currency and borrowed in it. Despite being part of a monetary union, fiscal space of Germany, whose real growth rate exceeds that of Italy, looks more like that of the US.

Figure 15: Deficit-debt diagrams across countries



9 Conclusion

Government debt to GDP ratios in many advanced economies, including the United States, have entered into unchartered territory. At the same time, interest rates on government debt have been pinned against the zero lower bound. The situation has led to two types of concerns: first, there is a real threat of a long-run secular stagnation equilibrium with low demand and low output. Second, such high levels of debt threaten a future, potentially large, rise in taxes or cut in spending if interest rates rise. This study builds a model that helps characterize fiscal policy in the face of these two concerns.

The framework shows that there is a region in which fiscal policy is neither too cold, threatening secular stagnation, nor too hot, threatening a subsequent rise in taxes and cut in spending. Fiscal space can be quantified using empirical moments that are the subject of a large body of research. The framework shows that both nominal interest rates (R) and nominal growth rates (G) are endogenous to government debt levels. This endogeneity is crucial in understanding when exactly expansionary fiscal policy can be conducted without the need to raise taxes or cut spending going forward.

While we find that a government can sustain a large debt to GDP ratio without having to run primary surpluses, the maximum permanent deficit that can be sustained is in the range of 2-2.5%, which is lower than most projections of expected future deficits for the United States. Furthermore, fiscal space depends on underlying factors; a rise in aggregate demand, a decline in trend growth, or a decline in inequality can reduce fiscal space. The framework suggests that governments can try to live off permanent deficits, but this is potentially dangerous when deficits are used to address longer-run structural problems such as rising inequality and an aging population.

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Appendix

A Proofs and model details

A.1 Local determinacy at ZLB

When the ZLB is binding, the dynamics of the model are characterized by the following three equations:

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= 0 - \left(G^* - \kappa\left(1 - y_t\right)\right) - \rho + v'(b_t)c_t \\
y_t &= c_t + x \\
\dot{b}_t &= \mathcal{Z}(b_t) + \left(0 - \left(G^* - \kappa\left(1 - y_t\right)\right)\right) b_t
\end{aligned}$$

Simplifying the system into two equations for two unknowns, c_t, b_t, we find

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= \kappa\left(1 - x - c_t\right) - G^* - \rho + v'(b_t)c_t \\
\dot{b}_t &= \mathcal{Z}(b_t) - \left(G^* - \kappa\left(1 - x - c_t\right)\right) b_t
\end{aligned}$$

Fix a steady state (c, b). We denote small deviations in c_t and b_t from the steady state by $\hat{c}_t, \hat{b}_t$. In order to establish local determinacy we show that there is at most a single stable equilibrium in the neighborhood of (c, b). The linearized homogeneous ODEs for $\left(\hat{c}_t, \hat{b}_t\right)$ are given by

$$\begin{aligned}
\dot{\hat{c}}_t &= -\kappa\hat{c}_t + v'(b)\hat{c}_t + v''(b)c\hat{b}_t \\
\dot{\hat{b}}_t &= \left(\mathcal{Z}'(b) - G\right)\hat{b}_t - \kappa b\hat{c}
\end{aligned}$$

or stacked in vector form,

$$\begin{pmatrix} \dot{\hat{c}}_t \\ \dot{\hat{b}}_t \end{pmatrix} = \begin{pmatrix} -\kappa + v'(b) & v''(b)c \\ -\kappa b & \mathcal{Z}'(b) - G \end{pmatrix} \begin{pmatrix} \hat{c}_t \\ \hat{b}_t \end{pmatrix}$$

Define the matrix as $\mathbf{A}$. Its characteristic polynomial is given by

$$P(\lambda) \equiv \det\left(\mathbf{A} - \lambda\mathbf{I}\right) = \lambda^2 - \lambda\left(\mathcal{Z}'(b) - G - \kappa + v'(b)\right) - \left(\left(G - \mathcal{Z}'(b)\right)\left(v'(b) - \kappa\right) + \left(-v''(b)\right)c\kappa b\right)$$

We prove that the polynomial always admits a positive root if $v'(b) > \kappa$, and given that we work with a concave $v(b)$, that is, $v''(b) \geq 0$. We distinguish two cases, $G \geq \mathcal{Z}'(b)$ and $G < \mathcal{Z}'(b)$.

- Suppose first that $G \geq \mathcal{Z}'(b)$. If so, $P(0) \leq 0$ and $P'(0) < 0$. Given that $P(\lambda) > 0$ for large enough λ, this implies that there exists a positive root.
- Next suppose that $G < \mathcal{Z}'(b)$. Then, $P(\mathcal{Z}'(b) - G) = v''(b)c\kappa b \leq 0$, again implying that there exists a positive root.

Thus, the condition $v'(b) > \kappa$ establishes local determinacy around any ZLB steady state. As shown in Section 3, in any such steady state, $v'(b)(1-x) > \rho + G^*$. Thus, local determinacy necessarily holds if

$$\kappa < \frac{\rho + G^*}{1 - x}$$

which is (7). Note that (7) is not sufficient for global determinacy, as we explain in Appendix A.4.

A.2 Proof of Proposition 1

Given our assumptions on $v(b)$, there is a unique solution b to any implicit equation of the form $v'(b) = X$, where X is a number in $(0, \infty)$. This implies that $\underline{\mathbf{b}}$, $\mathbf{b}^{ZLB}$, $\overline{\mathbf{b}}$ are all well-defined. The ranking $\mathbf{b}^{ZLB} < \overline{\mathbf{b}}$ follows from $\rho + G^* > \rho$. The ranking $\mathbf{b}^{ZLB} > \underline{\mathbf{b}}$ if $1 - x > G^*/\kappa$ follows from

$$\frac{1-x}{1-x-G^*/\kappa} \cdot \rho > \rho + G^*$$

which after rearranging is equivalent to

$$\rho + G^* > \kappa\,(1-x)$$

and thus 7.

A.3 Proof of Proposition 2

To verify (14), first suppose the ZLB is not binding, $R^*(b) \geq 0$. Then, the formula is simply given by

$$z(b) = \big(v'(b) \cdot (1-x) - \rho\big)\, b \tag{37}$$

which is correct given that $v'(b) \cdot (1-x) - \rho = G^* - R^*(b)$ from (9). Next, suppose a binding ZLB, $R^*(b) < 0$. Then, the formula is

$$z(b) = \left(v'(b) \cdot (1-x) - \rho + \frac{v'(b)}{v'(b) - \kappa} R^*(b) \right) b$$

We rewrite this using (9),

$$z(b) = \left(G^* + \frac{\kappa}{v'(b) - \kappa} R^*(b) \right) b \tag{38}$$

which by (12) is precisely equivalent to $z(b) = G \cdot b$. This proves (14).

Next we prove the three bullets:

- At the ZLB, we have (38), whose derivative can be found to be necessarily positive

  $$z'(b) = G + \frac{\kappa \varphi(b)}{(v'(b) - \kappa)^2} \left(\frac{\rho + G^*}{1-x} - \kappa \right)$$

  under condition (7) and the assumption that $G > 0$.

- The derivative of $z(b)$ above the ZLB is given by

  $$z'(b) = G^* - R^*(b) - \varphi(b)$$

  At $b = \mathbf{b}^{ZLB}$, $R^*(b) = 0$ and so

  $$z'(\mathbf{b}^{ZLB}) = G^* - \varphi(\mathbf{b}^{ZLB})$$

  If $\varphi(\mathbf{b}^{ZLB}) > G^*$, the derivative is negative at $\mathbf{b}^{ZLB}$. When φ is monotone, this implies that the maximum is at $\mathbf{b}^{ZLB}$.

- If the peak is at some $\mathbf{b}^z > \mathbf{b}^{ZLB}$, it necessarily is associated with $z'(\mathbf{b}^{ZLB}) = 0$, that is,

  $$R^*(\mathbf{b}^z) = G^* - \varphi(\mathbf{b}^z)$$

  which is equivalent to (15) using (9). The maximum deficit in this case is

  $$z(\mathbf{b}^z) = \left(G^* - R^*\left(\mathbf{b}^z\right)\right) \mathbf{b}^z = \varphi(\mathbf{b}^z)\mathbf{b}^z.$$

A.4 Phase diagram analysis in (c, b) space

In this section, we develop an analysis of our model in a phase diagram in (c, b) space. This is complementary to the phase diagrams we developed in Section 3. The one advantage of considering (c, b) space is that it allows us to detect potential multiple equilibria (global indeterminacy). The disadvantage is that it is harder to see the dynamics of debt when changing the primary deficit z, or the associated policy function $\mathcal{Z}(b)$. This is why, in the present section, we work with a simple constant deficit policy.

We first state the ODEs for c_t and b_t that describe our economy. Above the ZLB, we have

$$c_t = 1 - x = const$$
$$\dot{b}_t = \left(R^*(b_t) - G^*\right) b_t + z$$

where consumption is constant even during transitions, due to the goods market clearing condition. In a steady state $\left(R^*(b) - G^*\right) b + z = 0$, which can have multiple solutions for b. We focus on the case where there is a unique solution for b above the ZLB, and possibly another steady state at the ZLB.

At the ZLB, dynamics of c_t and b_t are determined by

$$\frac{\dot{c}_t}{c_t} = -\left(G^* - \kappa\left(1 - x - c_t\right)\right) - \rho + v'(b_t)c_t$$
$$\dot{b}_t = -\left(G^* - \kappa\left(1 - x - c_t\right)\right) b_t + z$$

The locus for constant c is given by

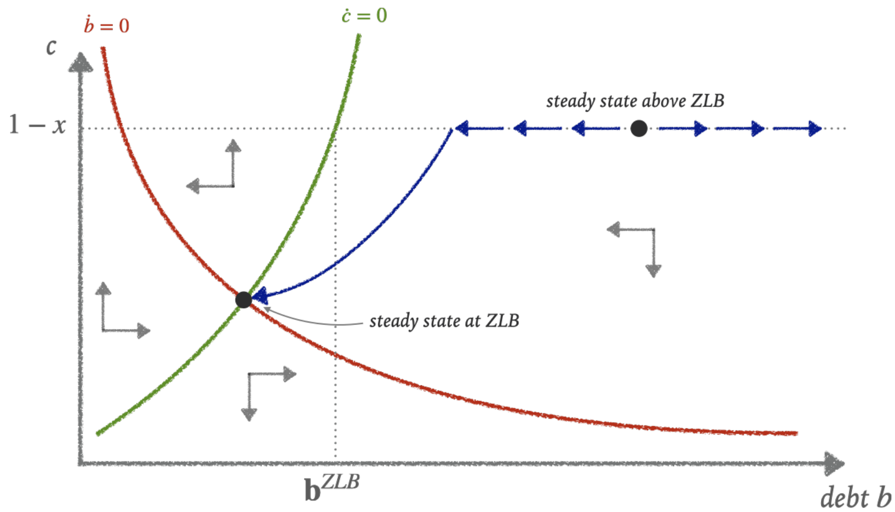
$$c = \frac{\rho + G^* - \kappa\left(1 - x\right)}{v'(b) - \kappa}.$$

The locus for constant b is given by

$$c = 1 - x - \frac{G^* - z/b}{\kappa}.$$

We illustrate the loci and transitions in Figure 16. For the z chosen here, there exist two steady states, one above the ZLB and one at the ZLB. In the diagram, there is a unique equilibrium for any debt level, since for debt levels in between the steady state levels, there

Figure 16: Phase diagram in (c, b) space



is a unique transition path towards the ZLB steady state. Along the transition path, c_t remains at $1 - x$ for a while, until it starts falling towards its level at the ZLB. Since agents are forward-looking, c_t falls already before $\mathbf{b}^{ZLB}$ is reached, at which point R_t falls to zero and the ZLB starts to bind.

There exist parametrizations (although not those considered in this paper) where the curved blue line only intersects with the $c = 1 - x$ line to the right of the above-ZLB steady state. In those cases, there exist multiple equilibria: starting in the above-ZLB steady state, it is a well-defined equilibrium for consumption to jump down below $1 - x$ immediately, pushing the economy against the ZLB and beginning a transition into the ZLB steady state. Those dynamics are similar to the ones in Benhabib et al. [2001], except that there is an additional state variable here (namely debt b_t).

A.5 What happens if $\kappa > \frac{\rho + G^*}{1-x}$?

We have assumed $\kappa < \frac{\rho + G^*}{1-x}$ in our analysis to ensure local determinacy for arbitrary deficit rules $\mathcal{Z}(b)$. Still, we can ask what the steady state deficit-debt locus looks like when the condition is not satisfied. Figure 17 plots deficit-debt loci when $\kappa < \frac{\rho + G^*}{1-x}$ and $\kappa > \frac{\rho + G^*}{1-x}$. We see that the loci with $\kappa > \frac{\rho + G^*}{1-x}$ are bending inwards. Notice that this causes local indeterminacy: with an infinitely steep $\mathcal{Z}(b)$, ensuring a constant debt level b, there are multiple equilibria. But if a different rule is followed, there may not be multiple equilibria. For example, imagine a constant z policy is followed, starting from the gray dot in the

Figure 17: Deficit-debt loci with flat and steep Phillips curves

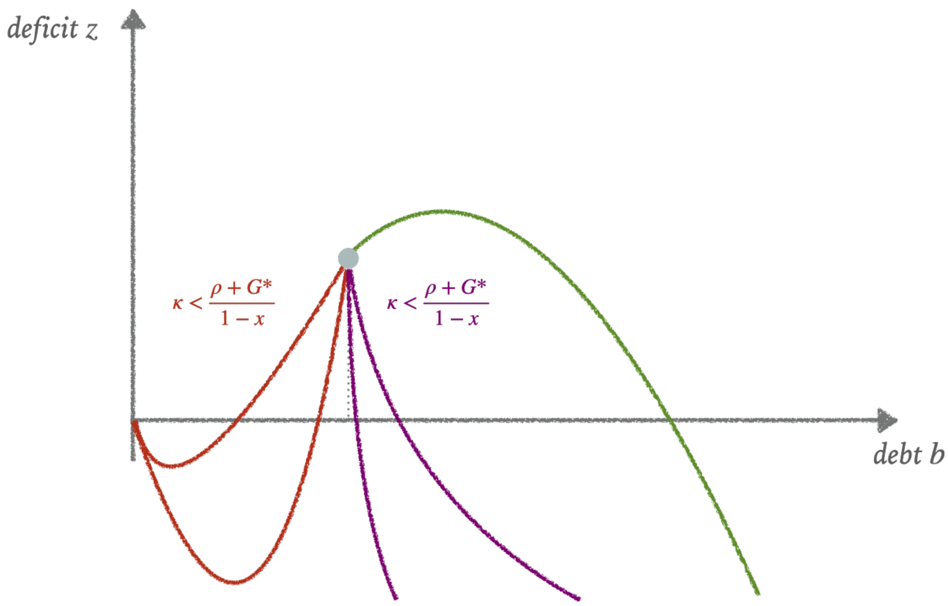


diagram.

What is interesting about the case $\kappa > \frac{\rho+G^*}{1-x}$ is that a reduction in the deficit can actually cause an *increase* in debt levels (relative to *potential* GDP). This happens as the reduction in the deficit reduces aggregate demand, and hence inflation. The reduction in inflation is then sufficiently strong to reduce nominal growth G by enough to cause an increase in debt. Vice versa, if an economy is at the ZLB and $\kappa > \frac{\rho+G^*}{1-x}$, an increase in the primary deficit can reduce debt levels.

A.6 Choices for the cross-country comparison

The values of b_0, R_0 and G^* in Table 3 are chosen as follows. The US parameters are those in Section 4.3. Japan, Italy, and Germany are clearly at the ZLB, so $R_0 = 0$. The pre-Covid debt to GDP levels of Japan, Italy, and Germany are given by 225%, 135%, and 60%, respectively. For growth rates, we use the average annual growth rate of nominal GDP from before the Great Recession to before the Covid pandemic, from 2008 through 2019. This yields 3.5% for the US, 0.6% for Japan, 0.8% for Italy and 2.9% for Germany. Japan's CPI inflation over this period was 0.3%, 1.7% below the 2% inflation target. This increases nominal trend growth G^* for Japan from 0.6% to 2.3%.

B The deficit-debt diagram in other models

In this section, we derive the interest rate and growth rate schedules $R(b)$ and $G(b)$ in a variety of models, and compute the deficit-debt locus $z(b) \equiv (G(b) - R(b))\, b$.

B.1 Model inspired by Reis [2021]

Here, we sketch a version of the two-agent model in Reis [2021] and use it to derive the corresponding functions for $R(b)$ and $G(b)$.

There are two types of agents, entrepreneurs E and financiers F. Each instant t, an agent i is randomly allocated to be either E or F, with probabilities α and $1-\alpha$ for E and F. Agents solve

$$\max \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^i dt \right]$$

subject to the budget constraint

$$da_t^i = \left(R_t b_t^i + r_t^l l_t^i + r_t^i k_t^i - \tau_t - c_t^i \right) dt \tag{39}$$

where $a_t^i = b_t^i + l_t^i + k_t^i$ is agent i's total wealth, and subject to the constraint

$$b_t^i \geq 0, k_t^i \geq 0.$$

Here, b_t^i is agent i's holdings of bonds, l_t^i agent i's lending (or if negative, borrowing), and k_t^i agent i's holding of capital. τ_t is a lump-sum tax. Thus, each agent can invest in three different assets each instant: government bonds b_t^i paying rate R_t, loans l_t^i paying rate r_t^l and capital paying rate r^i.

The return on capital r^i crucially differs by type. If i is type E, then r_t^i is constant, and equal to $r_t^i = A - \delta \equiv m > 0$. If i is type F, r_t^i is subject to idiosyncratic investment risk and given by

$$r_t^i = \eta(A - \delta) - \sigma dz_t^i$$

where $\eta \in (0,1)$ captures reduced capital quality in the hands of type F agents. We simplify the model here and set $\eta \to 0$. This essentially assumes that type F agents do not invest in capital.[31]

To avoid too much investment on the side of type E agents, we also impose a borrowing

[31] The case with $\eta > 0$ is similar, it just requires a case distinction.

constraint

$$-r_t^l l_t^i \leq \gamma r_t^i k_t^i$$

for some $\gamma > 0$. For type F agents, the borrowing constraint is simply assumed to be $l_t^i \geq 0$. In equilibrium, aggregate bonds outstanding B_t have to equal the sum of all individual positions,

$$B_t = \int b_t^i di$$

and the market for loans has to clear,

$$0 = \int l_t^i di.$$

Our goal is to use this description of the household side to solve for both the steady state interest rate R and the steady state growth rate G as a function of the overall supply of steady state bonds B.

Given the iid type switching, we can split total wealth a_t into wealth held by E's, $a_t^E = \alpha a_t$ and wealth held by F's, $a_t^F = (1-\alpha)\, a_t$. E's always borrow to their maximum. Further, we assume that γ is sufficiently high so that E's do not hold any government bonds. Then, from (39) and the fact that agents always consume $c_t^i = \rho a_t^i$, E's wealth evolves as

$$\dot{a}_t^E = \frac{(1-\gamma) m r_t^l}{r_t^l - \gamma m} a_t^E - \rho a_t^E$$

with positions in capital and lending markets given by

$$a_t^E = k_t^E - \gamma \frac{m k^E}{R}.$$

Given capital k_t^E, output is simply

$$y_t = A k_t^E. \tag{40}$$

F's hold all government bonds, and lend, so that $r_t^l = R_t$. Their wealth then evolves as

$$\dot{a}_t^F = (R_t - \rho)\, a_t^E$$

and is given by

$$a_t^F = \gamma \frac{m k_t^E}{R_t} + B_t. \tag{41}$$

In a steady state, total wealth evolves according to

$$\frac{\dot{a}_t}{a_t} = \alpha \frac{(1-\gamma) m R_t}{R_t - \gamma m} + (1-\alpha) R_t - \rho \tag{42}$$

and is given by

$$a_t = k_t^E + B_t. \tag{43}$$

We denote by $b_t \equiv B_t / y_t$ government debt relative to GDP.

This gives us all the equations we need. Assuming that b is constant, we combine (40), (42) and (43) to find a steady state growth rate G of the economy of

$$G = \alpha \frac{(1-\gamma) m R}{R - \gamma m} + (1-\alpha) R - \rho. \tag{44}$$

The interest rate R is itself determined by the amount of lending in equilibrium, using (41), (43) and the fact that $a_t^F = (1-\alpha) a_t$,

$$\gamma \frac{m k^E}{R} + B = (1-\alpha) \left(k^E + B \right).$$

Solving for R we find

$$R(b) = \frac{\gamma m}{1 - \alpha - \alpha A b}. \tag{45}$$

Together with (44), we can solve for G as function of b as well,

$$G(b) = \frac{(1-\gamma) m}{1 + Ab} + (1-\alpha) \frac{\gamma m}{1 - \alpha - \alpha A b} - \rho.$$

We sketch the two schedules in Figure 18 and the implied deficit-debt diagram.[32]

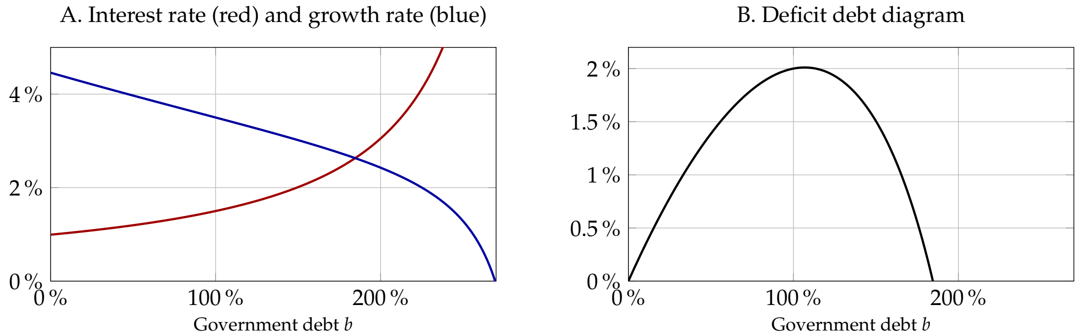
B.2 Model inspired by Brunnermeier et al. [2020a]

Next, we study a model that is inspired by recent models studying $R < G$ with the fiscal theory of the price level (Brunnermeier et al. 2020a,b, Bassetto and Cui [2018], Sims 2019).[33] We do so using a version of our model in Section 2. In particular, we assume away the zero lower bound, assume flexible prices with price level P_t, and nominal bonds B_t (now

[32] We calibrate the model exactly as above, matching $R_0 = 1.5\%$, $G_0 = 3.5\%$, $\varphi = 1.7\%$, $b_0 = 1$. This yields $\delta = 0.04$, $\rho = 0.03$, $\gamma = 0.033$, $A = 0.12$, $\alpha = 0.74$.

[33] For a recent book on the fiscal theory, see Cochrane [2019]. For a classic reference, see Leeper [1991].

Figure 18: $R(b), G(b)$ and deficits in the Reis [2021] model



measured relative to *real* GDP), so that the budget constraint of the government is given by

$$P_t z_t + (R_t - \gamma) B_t \leq \dot{B}_t \tag{46}$$

Since B_t is measured relative to real GDP, it is no longer G_t that is subtracted from R_t, but instead the real growth rate γ. Preferences continue to be those in (1) with the real value of government bonds still denoted by b_t, only that here, b_t is endogenous and given by $b_t = B_t/P_t$.

We follow the literature and assume that B_t grows at an exogenous rate $\mu_B > 0$ and pays an exogenous nominal interest rate R_t. We also assume a fixed level of the primary deficit $z \in \mathbb{R}$. In equilibrium, as we continue to normalize relative to potential, goods market clearing implies $c_t + x = 1$ at all dates. This means that along any transition, the Euler equation still implies a version of (9),

$$R_t = \rho + \gamma + \pi_t - v'(b_t) \cdot (1 - x). \tag{47}$$

Moreover, (46), together with $\dot{B}_t/B_t = \mu_B$, implies that

$$z + (R_t - \gamma - \mu_B) b_t = 0. \tag{48}$$

Finally, for any positive t, the real value of debt b_t changes according to

$$\dot{b}_t = (\mu_B - \pi_t) b_t. \tag{49}$$

We guess and verify that, irrespective of the initial level of nominal government debt B_0, this economy always exhibits a constant real value of debt b_t. Thus, guessing that $b_t = b = const$, we find that inflation is pinned down by growth in nominal debt due to

(49), $\pi_t = \mu_B > 0$. The level of debt is pinned down by (47),

$$v'(b) = \frac{\rho + \gamma + \mu_B - R}{1 - x}$$

and the primary surplus that can be financed follows from (48),

$$z = (\gamma + \mu_B - R)\, b. \tag{50}$$

This illustrates the key differences between our approach and an approach based on the fiscal theory of the price level. In the latter, the price level flexibly adjusts to achieve a given real value of debt, for an exogenously chosen nominal rate R and nominal growth rate $G = \gamma + \mu_B$. This happens because the monetary authority sets a (passive) fixed nominal interest rate R here while the fiscal authority sets an exogenous path for nominal debt. This leads to an expression for the primary deficit (50) that is to be read like the revenue from seignorage: setting $R = 0$ to capture money, we can rewrite this as

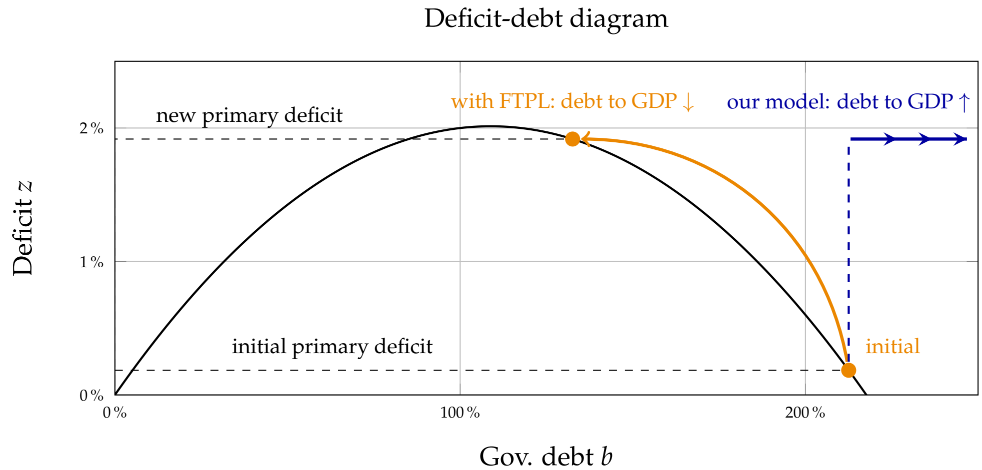
$$z = G \cdot (v')^{-1} \left(\frac{\rho + G}{1 - x} \right) \tag{51}$$

That is, by choosing inflation π, and thus nominal growth G, the fiscal authority can trace out a Laffer curve for seignorage revenue.

By contrast, the monetary authority in our model follows an active Taylor rule to implement the inflation target $\pi_t = \pi^*$, while the fiscal authority chooses primary deficits z_t. Thus, unless the economy is at the ZLB, nominal growth G is *entirely unaffected* by fiscal policy. In that sense, it cannot simply maximize the Laffer curve (51). Instead, debt b_t is a backward looking state variable that is controlled by primary deficits z_t.

The difference is not just semantics. To illustrate, imagine an economy that starts with a given level of debt b_0 that is to the right of the peak, $b_0 > \mathbf{b}^z$. What happens when policymakers would like to increase the deficit z? They can do so by raising $\mu_B - R$, that is, by either increasing the rate of nominal debt growth μ_B or reducing the nominal interest rate paid on debt. Crucially, and very differently from the dynamics in Section 3.4, the increase in the deficit z leads to a reduction in levels of debt to GDP. In our model, the same experiment would lead to an increase in debt to GDP. We view this as an important distinction between our model and models based on the fiscal theory of the price level (or models based on seignorage).

Figure 19: Deficit debt diagram and response to sudden increase in primary deficit with fiscal theory of the price level



B.3 Model inspired by Aiyagari and McGrattan [1998]

For this model, we move to discrete time. Compared to the model in Section 2, however, the main difference is that agents no longer enjoy any convenience utility from holding government debt; instead, they are hit by idiosyncratic income shocks and value government debts for their liquidity, which allows them to partially self insure against the income risk. Papers in this vein include Kocherlakota [2021] and Bayer et al. [2021].

Specifically, households solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} e^{-\rho t} u(c_{it}) \tag{52}$$

subject to the budget constraint

$$c_{it} + b_{it} \leq \frac{1+R_t}{1+G_t} b_{it-1} + (1-\tau_t)\, e_{it} w_t n_t \tag{53}$$

and a borrowing constraint $b_{it} \geq 0$. Here, e_{it} follows a Markov chain with a mean of the stationary distribution of 1. τ_t is a proportional labor income tax. We assume all agents have a labor endowment of 1, as before, and that, at the ZLB, all endowments are equally rationed, and equal to n_t. Inflation is downwardly rigid, just as before, with $1+\pi_t \geq (1+\pi^*)\,(1-\kappa\,(1-y_t))$. We continue to work with the same linear aggregate production function, so that the real wage w_t is still equal to one. Aggregating across households, we find the aggregate demand for bonds

$$b_t \equiv \int b_{it} di.$$

The government budget constraint in discrete time is given by

$$x + \frac{1+R_t}{1+G_t} b_{t-1} \leq b_t + \tau_t w_t n_t$$

where $z_t \equiv x - \tau_t w_t n_t$ continues to denote the primary deficit relative to GDP.

As is well known from Aiyagari [1994], and more recently studied in a two-asset context in Bayer et al. [2021], the household problem above implies a steady state schedule

$$1 + \mathcal{R}(b, y)$$

so that if $\frac{1+R_t}{1+G_t} = 1 + \mathcal{R}(b, y)$ and $(1 - \tau_t) w_t n_t = y$, for all t, in (53), the steady state demand for bonds is equal to b. Observe that $\mathcal{R}(b, y)$ is homogeneous of degree zero. This defines an increasing function for the natural interest rate $R^*(b)$,

$$1 + R^*(b) = (1 + G^*)(1 + \mathcal{R}(b, 1 - \tau)).$$

What happens when the natural rate is negative, $R^*(b) < 0$? In that case, the ZLB is binding. Output y and tax rate τ are pinned down jointly by

$$1 = (1 + G^*)(1 - \kappa(1 - y))(1 + \mathcal{R}(b, (1 - \tau) y))$$

and the government budget constraint

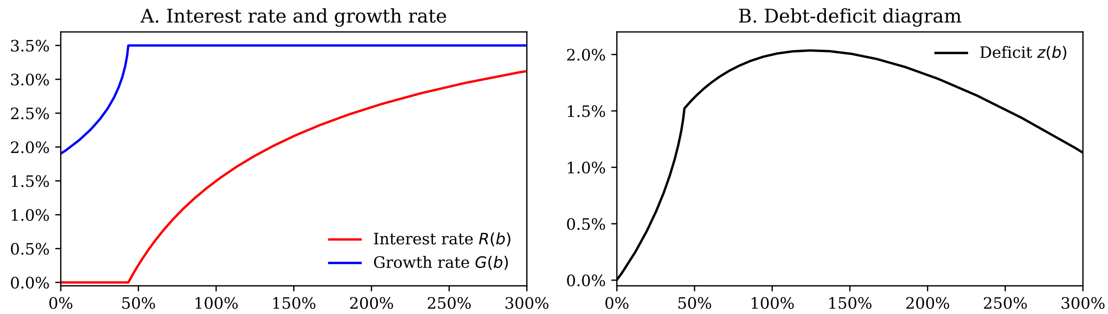
$$\tau y = \mathcal{R}(b, (1 - \tau) y) b + x.$$

This then determines $1 + G(b) = (1 + G^*)(1 - \kappa(1 - y))$.

To show that this procedure, while more involved, can predicts behavior similar to our more reduced form model in Section 2, we simulate this model with a standard AR(1) process for $\log e_{it}$ (annual persistence 0.90, standard deviation of $\log e_{it}$ of 0.70). We use the risk aversion in u to match the slope $\varphi = \frac{\partial \mathcal{R}(b,1)}{\partial \log b}$ at some initial debt level b_0. All other parameters are pinned down as in Section 4.3.[34]

[34] We choose ρ, G^* to match the same initial interest rate and growth rate at the same initial debt level b_0, we use the same steady state x and τ. The parameters we find are: $x = 0.20$, $\tau = 0.18$, EIS = 1.35,

Figure 20: $R(b)$, $G(b)$ and deficits in a model based on liquidity



The left panel of Figure 20 shows the interest rate $R(b)$ and growth rate $G(b)$ schedules as a function of the debt level. The right panel of Figure 20 shows the deficit-debt diagram $(G(b) - R(b))\, b$. The plots look very similar to those in Section 4.4, specifically those with the log-linear functional form for $v'(b)$.

B.4 Model inspired by Diamond [1965] and Blanchard [2019]

We sketch the well-known Cobb-Douglas version of the Diamond [1965] model and show that it implies a simple closed-form deficit schedule $z(b)$, and derive the conditions under which there is a free lunch (which in the Diamond [1965] model coincides with the region of dynamic inefficiency).

The model operates in discrete time and consists of two-period-lived overlapping generations. The generation born at date t has G^t members, where $G > 1$. Each maximizes preferences

$$(1-\beta)\log c_{yt} + \beta \log c_{ot+1}$$

over consumption when young c_{yt} and when old c_{ot+1}, subject to the budget constraints

$$c_{yt} + a_t \le w_t\,(1-\tau_t) \qquad c_{ot+1} = R_{t+1} a_t.$$

We have $\beta \in (0,1)$, τ_t is an income tax. The policy function is then

$$a_t = \beta w_t\,(1-\tau_t)\,. \tag{54}$$

The per capita saving a_t of generation t finances capital for $t+1$ and bonds maturing in $t+1$. Normalizing the latter two in terms of the population size at $t+1$, we have an asset

$\beta = 0.99$, $G^* = 3.5\%$. We choose $\kappa = 0.02$ to avoid indeterminacy.

market clearing condition

$$G^{-1}a_t = k_{t+1} + B_{t+1}. \tag{55}$$

Production in period t is neoclassical with aggregate output per capita

$$y_t = k_t^{\alpha} l_t^{1-\alpha}$$

where l_t is the labor endowment of each member of generation t, which we normalize to 1. Thus, the wage is $w_t = (1-\alpha)\, k_t^{\alpha}$ and, with a depreciation rate of 1, the return is $R_{t+1} = \alpha k_{t+1}^{\alpha-1} + 1 - \delta$. With (54) and (55), the law of motion for capital is then

$$k_{t+1} = G^{-1}\beta\,(1-\alpha)\,(1-\tau)\, k_t^{\alpha} - B_{t+1} \tag{56}$$

The government's budget constraint is simply given by

$$GB_{t+1} = R_t B_t - \tau_t w_t + X_t$$

where X_t denotes government spending per capita.

Next, we focus on steady states, at which all prices and per capita quantities are constant. Moreover, we normalize government debt and spending by output $y = k^{\alpha}$. We denote $b \equiv B/y$ as before and $x = X/y$. Then, (56) becomes

$$k^{1-\alpha} = G^{-1}\beta\,(1-\alpha)\,(1-\tau) - b$$

and we can rearrange it to obtain an expression for the interest rate

$$R(b) = \frac{\alpha G}{\beta\,(1-\alpha)\,(1-\tau) - Gb}.$$

The normalize government budget constraint can be written as usual

$$z(b) = (G - R(b))\, b$$

where we defined the primary deficit relative to GDP as $z(b) \equiv x - \tau\,(1-\alpha)$. Different from our model in Section 2, it turns out that for this analysis, it is somewhat more tractable to fix the tax rate τ and instead vary government spending x if $z(b)$ changes.

We can analyze the deficit schedule $z(b)$ just like before. In particular, we can ask when higher debt levels allow for a greater primary deficit $z(b)$, which in this model is equivalent to dynamic inefficiency. The condition for this is

$$R(b) < G - b \cdot R'(b) \tag{57}$$

where $\varphi = b \cdot R'(b)$. Observe that the standard condition for dynamic inefficiency that is usually taught in this model is $R < G$, or in terms of primitives, $\frac{\alpha}{1-\alpha} < \beta(1-\tau)$. Yet, as (57) highlights this condition is only accurate for levels of government debt around zero, where $\varphi = 0$. When $b > 0$, $\varphi > 0$, and the relevant condition becomes $R < G - \varphi$. In terms of primitives, this corresponds to

$$b < \frac{\beta(1-\alpha)(1-\tau)}{G} - \frac{1}{G}\sqrt{\alpha \cdot \beta(1-\alpha)(1-\tau)} \equiv \mathbf{b}^z$$

where $\mathbf{b}^z$ is, as before, the deficit-maximizing level of debt. The deficit associated with $\mathbf{b}^z$ is given by

$$\mathbf{z}^{max} = \left(\sqrt{\beta(1-\alpha)(1-\tau)} - \sqrt{\alpha}\right)^2.$$

We thus find that OLG models based on Diamond [1965] admit a similar interest rate schedule as the one we derived in Section 3, and the relevant condition for a free lunch (here equivalent to dynamic inefficiency) is given by $R < G - \varphi$, which only in the case without debt reduces to $R < G$.

C Details on estimation of φ

C.1 Further discussion of estimates from the literature

The estimates of $\frac{\partial(\rho+G-R)}{\partial \log b}$ from Krishnamurthy and Vissing-Jorgensen [2012] reported in Table 1above come from their Table 1, columns 4 and 5. The measure of the spread is the Baa corporate yield minus the Treasury bond yield, which they prefer because "Aaa bonds offer some convenience services of Treasuries and thus the Baa-Treasury spread is more appropriate for capturing the full effect of Treasury supply on the Treasury convenience yield." For the estimates of $b_0 \frac{\partial(\rho+G-R)}{\partial b}$, we collected the same data as in Krishnamurthy and Vissing-Jorgensen [2012] and regressed the Baa minus Treasury spread on the level of the debt to GDP ratio. We multiply this coefficient $\frac{\partial(\rho+G-R)}{\partial b}$ (which is -0.027 and -0.048 for the long and short time periods, respectively) by the average level of the debt to GDP ratio b_0 (which is 0.42 and 0.36 for the long and short timer periods, respectively) to get the estimate.

The Greenwood et al. [2015] estimate is from column 1 of Panel B of their Table 1. The measure of the spread is the difference between the actual yield on an 2-week Treasury bill and the 2-week fitted yield, based on the fitted Treasury yield curve in Gürkaynak, Sack, and Swanson [2007]. The derivative is with respect to the amount of Treasury bills outstanding scaled by GDP. The implied estimate of $\frac{\partial(\rho+G-R)}{\partial b}$ is -0.167, which we then multiply by the average Treasury bill to GDP ratio b_0 (which is 0.084) to get the estimate. We use the estimate from Panel B which goes only through 2007 because of the endogeneity issues discussed by Greenwood et al. [2015] surrounding the Great Recession and financial crisis (see the last full paragraph on page 1689 of their article). The Vandeweyer [2019] regression estimate comes from column 2 of Table 4 of his study. The measure of the spread is the 3-month T-bill rate minus the 3-month General Collateral Repo rate, and this is regressed on the ratio of outstanding T-bills to GDP. The implied estimate of $\frac{\partial(\rho+G-R)}{\partial b}$ is -0.040, which we then multiply by the average Treasury bill to GDP ratio b_0 (which is 0.010) to get the estimate. We use column 2 of Table 4, as this regression controls for the Federal Funds rate as suggested by Nagel [2016]. The Vandeweyer [2019] natural experiment involves the 2016 money market reform which led to a large rise in demand for T-bills by money market funds. Money market funds increased their holdings of T-bills by $400 billion, which was about 20% of the stock outstanding. Vandeweyer [2019] uses a model-based counter-factual to show that this shock led to an 18 basis point reduction in yields on government debt, which gives $\frac{\partial(\rho+G-R)}{\partial \log b} = 0.009$. The estimate from Takaoka [2018] comes from Table 4, and the estimate from Jiang et al. [2020b] comes from Table 5, panel A, column 2. For the Jiang et al. [2020b] estimate of -0.01, we multiply by the average government debt to GDP ratio in their sample to get the final estimate of -0.008.

The estimates of $b_0\frac{\partial(G-R)}{\partial b}$ come from Presbitero and Wiriadinata [2020], Table A3, column 1. The coefficients $\frac{\partial(G-R)}{\partial b}$come from that table (-0.027 for advanced economies, -0.024 for the full sample), and then these are multiplied by the average government debt to GDP ratiob_0 for the respective samples, which are 0.53 and 0.56 for the advanced economies and the full sample, respectively.

C.2 Regressions based on Presbitero and Wiriadinata [2020]

The other estimates from Table 1 come from our own data analysis using a data set constructed exactly as the one used by Presbitero and Wiriadinata [2020]. The associated regression table is Table 4.

Table 4: Results from regressions on Presbitero and Wiriadinata [2020] data

	Left hand side: G - R					
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Gov Debt/GDP)	-0.024***	-0.031***	-0.015**	-0.025**	-0.028**	-0.020**
	(0.006)	(0.005)	(0.004)	(0.007)	(0.006)	(0.003)
Observations	1184	1184	1184	490	490	490
R^2	0.103	0.179	0.553	0.162	0.209	0.698
FE		Country	Country and Year		Country	Country and Year
Sample						

* p < 0.1, ** p < 0.05, *** p < 0.01.
Standard errors are heteroskedasticity-robust, clustered by country.

Note. This table presents coefficient estimates of $G - R$ on government debt to GDP ratios. The sample for the first three columns are the 17 advanced economies covered by the JST Macrohistory data base. The sample for columns 4 through 6 is G7 countries (Canada, France, Germany, Italy, Japan, United Kingdom, United States). The time period covered is 1950 to 2019. Please see Presbitero and Wiriadinata [2020] for more details.

C.3 Georgia Senate election

The Georgia Senate election of January 5, 2021 offers a unique opportunity to assess how markets perceive a sudden rise in expected government debt. On the eve of the election, trading at Electionbettingodds.com implied a 50.8% probability of the Republicans controlling the Senate, and a 49.1% probability of the Democrats controlling the Senate. It was widely reported in the press that President-Elect Biden's administration would propose a \$1.9 trillion "American Rescue Plan" once the President-Elect took office. Our assumption in the calculation below is that the win by the two Democrats in the Georgia Senate election of January 5, 2021 increased the expected government debt by \$2 trillion, which at the time was about 7.4% of total debt outstanding.

Figure 21 shows the effect on the 10 year nominal interest rate, the 10 year TIPS interest rate, and expected inflation. As it shows the victory by the Democrats in the Georgia Senate election led to a 15 basis point immediate reaction which then declined to an 8 basis point reaction after a week. Taken together, these numbers imply that a 3.7% rise in total government debt outstanding relative to prior expectations led to an 8 basis point decline in $G - R$, which gives an estimate of $\frac{\partial(G-R)}{\partial \log b}$ of -0.022. The data for these calculations come from Bloomberg.

Figure 21: The change in real interest rates around the January 5th, 2021 Georgia run-off election.