Macroeconomic Implications of COVID-19:
Can Negative Supply Shocks Cause Demand Shortages?*

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We present a theory of *Keynesian supply shocks*: supply shocks that trigger changes in aggregate demand larger than the shocks themselves. We argue that the economic shock caused by the COVID-19 epidemic may have this feature. In one-sector economies supply shocks are never Keynesian. We show that this is a general result that also holds in economies with incomplete markets and liquidity constrained consumers. In economies with multiple sectors, Keynesian supply shocks are possible, under some conditions. A 50% shock that hits all sectors is not the same as a 100% shock that hits half the economy. Incomplete markets make the conditions for Keynesian supply shocks more likely to be met. Firm exit and job destruction can amplify the initial effect, aggravating the recession. Standard fiscal stimulus, spending or non-targeted transfers, can be less effective than usual due to broken links in the Keynesian multiplier feedback. Monetary policy can have magnified effects by preventing firm exits and encouraging labor hoarding. Turning to optimal policy, closing down contact-intensive sectors and providing full insurance payments to affected workers can achieve the first-best allocation, despite the lower potency of fiscal policy.

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1 Introduction

Jean-Baptiste Say is famously misquoted for stating the Law "supply creates its own demand." In this paper, we introduce a concept that might be accurately portrayed as "supply creates its own *excess* demand". Namely, a negative supply shock can trigger a demand shortage that leads to a contraction in output and employment larger than the supply shock itself. We call supply shocks with these properties *Keynesian supply shocks*.

Temporary negative supply shocks, such as those caused by a pandemic, reduce output and employment.[1] As dire as they may be, supply shock recessions are partly an efficient response to a lower capacity of the economy to produce goods and services. However, can a supply shock induce too sharp a fall in output and employment, going beyond the efficient response? Can it lead to a drop in output and employment in sectors not directly affected? Which policy instruments can be used to restore efficiency?

These are the questions we seek to address in this paper. They are also the questions behind debates over monetary and fiscal policy responses to the COVID-19 epidemic and the ensuing economic fallout.

A simple perspective on the effects of the pandemic casts the issue as one of aggregate supply versus aggregate demand, whether the shock to one or the other is greater. Some have expressed skepticism that any demand stimulus is warranted in response to what is essentially a supply shock. Others claim that the pandemic shock can cause output losses larger than efficient. For example, Gourinchas (2020) argues for macro measures aimed at "flattening the recession curve." This unsettled debate illustrates that a discussion focused on demand versus supply opens up many possibilities, but leaves many questions unanswered. What forces would induce demand to contract more than supply?

The perspective we offer here is different and based on the notion that supply and demand forces are intertwined: demand is endogenous and affected by the supply shock and other features of the economy. Our analysis uncovers mechanisms by which supply shocks end up creating a demand deficiency. One basic intuition is that when workers lose their income, due to the shock, they reduce their spending, causing a contraction in demand. However, the question is whether this mechanism is strong enough to cause an

[1] Throughout the paper, to simplify, we make the semantic choice of referring to the macroeconomic effect of a pandemic as a "supply shock". More generally, our analysis applies to any shock that reduces the efficient level of output and shows under what condition they lead to an even larger drop in aggregate demand. This includes, for example, the effect of consumers choosing to stay at home to avoid infection. At the microeconomic level, it makes sense to describe such behavior as a negative demand shock. At the macroeconomic level, what matters is whether the gains from trade between producers and consumers are diminished. Section 3.3.3 spells out this equivalence in our model.

overall shortfall in demand. As we show, the answer is more subtle than one might guess at first glance.

First, we show that in one-sector economies the answer is negative: the drop in supply dominates. The result is well-known in a representative agent economy. Less obviously, we show that it holds in richer incomplete market models that allow for heterogeneous agents, uninsurable income risk and liquidity constraints, creating differences in marginal propensities to consume (MPC). In these models, a mechanism from income loss to lower demand is present, but although it makes the drop in aggregate demand larger than in the representative agent case, the drop is still smaller than the drop in output due to the supply shock. Intuitively, the MPC for someone losing income may be large, but is still bounded above by one. Thus, the drop in consumption is always a dampened version of their income losses.

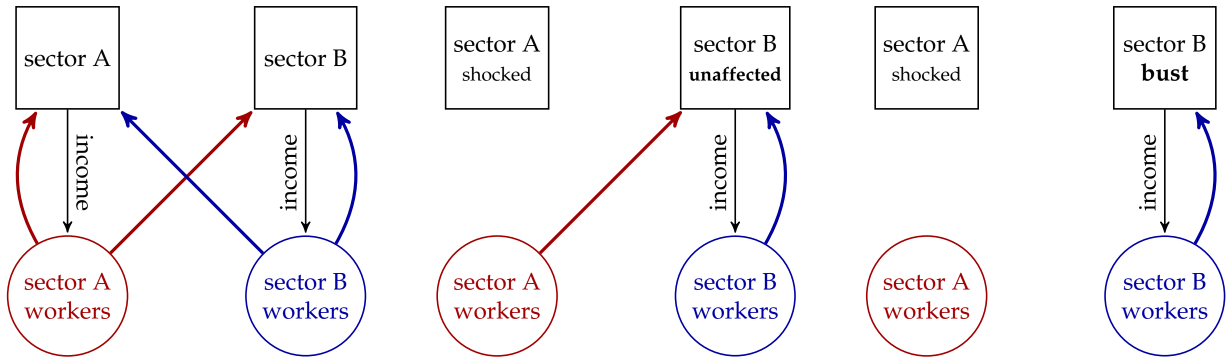
We then turn to economies with multiple sectors. When there is a shock that is concentrated in certain sectors, as, for example, a shutdown of contact-intensive businesses in response to an epidemic, there is greater scope for total spending to contract. The fact that some goods are no longer available makes it less attractive to spend overall. An interpretation is that the shutdown increases the shadow price of the goods in the affected sectors, making total current consumption more expensive, and thus inducing a shift away from current consumption and in favor of future consumption. On the other hand, the unavailability of some sectors' goods shifts spending towards the other sectors, through a substitution channel. Whether or not full employment is maintained in the sectors not shut down depends on the relative strength of these two effects. In particular, in a representative agent economy, a demand deficiency arises if the intertemporal elasticity of substitution is larger than the elasticity of substitution across sectors.[2]

We then turn to incomplete markets and show that the condition for a demand deficiency in the unaffected sectors becomes less stringent. Intuitively, if workers in the affected sectors lose their jobs and incomes, their consumption drops significantly if they are credit constrained and have high MPCs. To make up for this, workers in the unaffected sectors have to increase by more their demand for unaffected goods. If the degree of substitution across sectors is low enough or the intertemporal elasticity of substitution is high enough, the effect of the income losses dominates and employment falls in the unaffected sector.

Figure 1 illustrates this logic for two sectors, A and B. When sector A gets shocked, all output stops being produced in that sector. In a representative agent setting, agents

[2] Rowe (2020) provides a colorful and careful intuition for this possibility.

— Case with equal inter- and intra-temporal elasticities —



(a) Before the shock **(b)** Representative agent **(c)** Incomplete markets

Figure 1: How negative supply shocks can lead to demand shortages

working in both sectors pool their income and spend it across sectors identically. Here, the difference between inter- and intra-temporal elasticities is all that matters for whether sector B is affected by the shock in sector A. Figure 1(b) illustrates the knife-edge case where the two elasticities are equal and sector B is unaffected. Panel (c) then emphasizes that with incomplete markets, even this case causes sector B to go into a recession, as sector A workers cut back their spending on sector B.

The fact that aggregate demand causes a recession above and beyond the reduction in supply might lead one to expect that fiscal policy interventions are powerful in keeping aggregate demand up. We show that this is a false conclusion. First of all, the marginal propensity to consume (MPC) may be low, at least among consumers that are not borrowing constrained. More importantly, and more surprisingly, even if MPCs are high due to the loss of income and borrowing constraints, the standard Keynesian cross logic behind fiscal multipliers is not operational: there are no second-round effects, so the multiplier of government purchases is 1 and that of transfers is less than 1. To see this, note that the highest-MPC agents in the economy are the former employees of the shocked sector. They do not benefit from any government spending. They do benefit from direct transfers, but their spending does not return to them as income. Thus, the typical Keynesian-cross amplification is broken as the highest-MPC agents in the economy do not benefit from increased spending by households or by the government.

An important transmission channel for supply and demand shocks across sectors are supply chains. We highlight this issue in an extension of our basic model, in which sector A uses sector B goods as an input. In that case, a negative supply shock to sector A travels upstream to sector B as a demand shock. We show that this unambiguously increases the likelihood that the supply shock turns into a demand shortage in sector B.

We next extend our baseline model to a firm-level perspective, introducing individual firms that can decide whether to remain open for business and whether to pay their workers, even when inactive. This extension provides us with a basis to study two important phenomena.

First, we study the decision of sector A firms to keep paying their workers, i.e., to *hoard labor*. Firms may want to do so due to labor market frictions that make future rehiring costly. We show that labor hoarding unambiguously aids the economy, as it provides insurance to workers hit by the shock. We also show that easy monetary policy can be used to incentivize firms' labor hoarding. Similarly, if hoarding is stymied by liquidity problems, liquidity injections by the central bank can help.

Second, we study the decision of sector B firms to exit. We identify an amplification mechanism, which we dub the *business exit multiplier.* If the shock in sector A reduces demand in sector B, some businesses in that sector can become unwilling or unable to stay open. When these businesses close and lay off workers, that produces an additional, endogenous Keynesian supply shock, which amplifies the initial one. This force can lead to shutting down more of the economy than the original shock. We discuss policy proposals to keep businesses running during the pandemic.

Finally, we study the jointly optimal health and macroeconomic policy caused by a pandemic. We nest our previous model in a setting that explicitly captures health concerns, both private and social, and study the role of Pigouvian policies to correct health externalities, in combination with macro stabilization policies. We show that the first best policy in our model involves closing down contact-intensive sectors and making insurance payments to affected workers.

Related Literature

Our paper is related both to papers motivated by the COVID-19 pandemic and to earlier work about demand spillovers across sectors.

Among early COVID-motivated papers is Fornaro and Wolf (2020) which considers a standard New Keynesian representative-agent economy and studies a pandemic as a negative shock to the growth rate in productivity. They also consider endogenous technological change and stagnation traps. Faria e Castro (2020) studies different forms of fiscal policy in a calibrated DSGE New Keynesian model. The model builds on Faria e Castro (2018) and features incomplete markets in the form of borrowers and savers with financial frictions. The pandemic is modeled as a large negative shock to the utility of consumption. In contrast to both papers, we focus on temporary shocks to supply and and study cross-sectoral demand spillovers. There is also a set of papers that have been building on epidemiological models of contagion, embedding them in an economic environment, including Alvarez et al. (2020), Atkeson (2020), Berger, Herkenhoff and Mongey (2020), Eichenbaum, Rebelo and Trabandt (2020), Kaplan et al. (2020).

Jorda et al. (2020) provide time-series evidence from historical pandemics on the impact on rates of return. The pandemics they study are persistent, with large numbers casualties. They find evidence that pandemics reduce the real rate of interest. It is not clear if this is comparable to the events we focus on, since we do not focus on the longer-term effects of death, but instead on the shorter-term effects of shutdowns that respond to the pandemic.

Since our paper was first circulated, there has been an unprecedented growth in macroeconomic research on the effects of COVID-19. Woodford (2020) studies a multi-sector model with incomplete markets like ours, but allows workers in difference sectors to have different preferences. This kind of heterogeneity creates a “circular flow of payments” that has the potential to amplify the effect of the shock and increase the transfer multiplier. As in our paper, targeted transfers are key to an optimal policy response. Caballero and Simsek (2020) emphasize the role of risk-taking, finding that redistribution through asset prices can induce drops in demand. Baqaee and Farhi (2020) explore the implications of a rich input-output network structure, more general than the simple one we explore in Section 3.3.1. In the more empirical area, several papers look at the responses of consumer spending in real time, including Andersen et al. (2020), Carvalho et al. (2020), Chetty et al. (2020), and Cox et al. (2020). Two papers, Brinca et al. (2020) and Balleer et al. (2020), explicitly try to separate demand from supply shocks. There is also a growing set of quantitative/calibrated papers focusing on the effects of fiscal policy, including Mitman et al. (2020) and Bayer et al. (2020), and on the sectoral structure, including Danieli and Olmstead-Rumsey (2020).

There have also been a number of policy proposals related to COVID-19. Among the ones closest to the results in this paper are those in Baldwin and Weder di Mauro (2020), Gopinath (2020), Gourinchas (2020), Hamilton and Veuger (2020), Hanson et al. (2020), Saez and Zucman (2020), and Sahm (2020).

The role of demand spillovers across sectors plays an important role in non-pandemic-related areas too, in particular in the study of economic growth and sectoral change (Murphy et al. (1989), Matsuyama (2000)) and in international macro, which focuses on the transmission of productivity changes from the domestic tradable sector to non-tradables and foreign produced goods (Corsetti et al. (2008)). A recent paper that uses these cross-sectoral spillovers to provide an interpretation of structural forces at work in the Great Recession is Gatti et al. (2012), and a paper that estimates regional demand spillovers is Huber (2018). Cross-sectoral effects also play a central role in understanding comovement and labor supply responses following demand shocks, as investigated in Beaudry and Portier (2014).

2 Single Sector: Standard Supply Shocks Even with Incomplete Markets

We being by studying the effects of a supply shock in a one-sector model. Supply shocks have standard features here: they never cause demand effects strong enough to dominate the effects on the supply side. This applies even in economies with heterogenous agents and incomplete markets.

The framework we use for this section, and that we expand on in later sections, is a standard infinite horizon model with a single good. The model is populated by a unit mass of agents whose preferences are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t\right), \tag{1}$$

where c_t is consumption and $U\left(c\right) = c^{1-1/\sigma} / \left(1 - 1/\sigma\right)$, so agents have constant intertemporal elasticity of substitution σ. Each agent is endowed with $\overline{n} > 0$ units of labor which are supplied inelastically. Competitive firms produce the final good from labor using the linear technology

$$Y_t = N_t.$$

The supply shock we study is inspired by the recent COVID-19 epidemic but relevant more generally: a fraction $\phi > 0$ of agents is unable to produce in period $t = 0$. This is meant to capture that the epidemic makes it unsafe for some goods to be produced and consumed, because it would increase the chance of infection for consumers and workers.[3] Our shock is a stand-in for the effects of individual decisions or government regulations seeking protection from contagion.[4] The important point is that a subset of agents can no longer supply their labor endowments. We assume the shock was not anticipated and

[3] At a microeconomic level, one might call cases where working is unsafe a supply shock, and cases where consuming is unsafe a demand shock. At a macroeconomic level, both have the same implication, as explained in Section 3.3.3.

[4] In Section 7 we model explicitly health objectives.

that starting at $t = 1$ all agents can again supply their full labor endowments of $\overline{n}$, so the economy recovers in one period.

We analyze the effects of this supply shock separately for two versions of the model; first, with complete markets, or equivalently, a representative agent; then, with incomplete markets. In both cases, we look at two indicators—the response of the natural interest rate and the response of output if the real interest rate does not (or cannot) adjust in line with the natural rate. These indicators reveal whether the supply shock has standard effects or Keynesian effects. In the first case, the natural interest rate increases and aggregate demand falls less than aggregate supply at a constant interest rate. In the second, the natural interest rate falls and aggregate demand falls more than aggregate supply at a constant rate.

2.1 Complete Markets

Consider first the case of complete markets, which is equivalent to that of a representative agent. The argument here is well known, but it is useful to review it to set the stage for the rest of the analysis. Since markets are complete, the supply shock reduces the representative agent's labor supply from $\overline{n}$ to $(1-\phi)\overline{n}$ in period $t = 0$.

What happens to the natural interest rate? Consider the flexible price version of this economy in which labor is always fully employed. The effect of the labor supply shock is mechanical: consumption falls at $t = 0$ before returning to its previous level. Thus, at date $t = 0$, the natural interest rate is

$$1 + r_0^* = \frac{1}{\beta} \frac{U'\left((1-\phi)\overline{n}\right)}{U'\left(\overline{n}\right)} > \frac{1}{\beta}, \tag{2}$$

and is above its steady state level of $1/\beta$.

The fact that the natural interest rate increases is a sign that there is no shortage of demand, in fact the opposite. To corroborate this logic, we introduce nominal rigidities. A convenient and tractable way to do so is to assume that nominal wages W_t are downwardly rigid. If labor demand falls below labor supply, nominal wages remain unchanged at W_{t-1} and there is unemployment. We continue to assume that firms are competitive, so nominal prices are equal to nominal wages, $P_t = W_t$, and the real wage is $w_t = 1$.

After the supply shock, however, demand falls less than supply so there is no unemployment. To see why, consider the following experiment. Assume that the central bank ensures full employment at all future dates so $c_t = \overline{n}$ for $t = 1, 2, ...$ Assume also that at $t = 0$ the central bank tries to keep the real interest rate r_t at its steady state level $1/\beta - 1$.

Consumption is then purely determined by the forward looking condition

$$U'(c_0) = \beta \frac{1}{\beta} U'(\overline{n}).$$

Aggregate demand is then $c_0 = \overline{n}$, and exceeds aggregate supply, which is $(1-\phi)\overline{n}$.[5] We summarize the results with complete markets.

Proposition 1. *In the single-sector model with complete markets, a negative supply shock increases the natural interest rate. If the real rate does not adjust, there is excess demand in the labor market.*

An alternative interpretation of Proposition 1 is that the negative supply shock is equivalent to a positive news shock, as at $t = 0$ the representative agent learns that labor endowments will increase over time, from $(1-\phi)\overline{n}$ to $\overline{n}$. This explains why labor demand exceeds supply at $t = 0$ unless the real interest rate adjusts.

2.2 Incomplete Markets

Next we move to an economy with incomplete markets. The effects of the supply shock are less obvious here. After all, those hit by the shock lose their earnings and, in the presence of market incompleteness, will need to cut back their spending. We focus on a simple incomplete markets model, but our results hold more generally.[6]

We label agents by $i \in [0, 1]$. Each agent i maximizes utility (1) subject to the budget constraint

$$c_{it} + a_{it} \leq w_t n_{it} + (1 + r_{t-1}) a_{it-1}.$$

[5]This cannot be an equilibrium, so the real rate has to increase up to the natural rate. For our purposes here, we do not need to specify the mechanism by which equilibrium is restored. A simple possibility is to assume that the central bank follows the rule $i_t = i^* + b\pi_t$, where $i^* = 1/\beta - 1$ is the steady state natural rate and π_t is current inflation. At $t = 0$ excess demand in the labor market causes nominal wages to increase. This produces price inflation up to the point where $i^* + b\pi_0$ equals the value r_0 in (2). From $t = 1$ onwards the equilibrium has $i_t = i^*$ and zero inflation.

Note that we could also relax the fixed labor endowment assumption to consider elastic labor supply decisions, such as with an additive disutility of labor, and introduce some upward wage rigidity or price rigidity with monopolistic competition. In such extensions, one can allow the real interest rate to stay below the natural rate and a boom in employment relative to its natural and efficient level obtains. We work with the fixed endowments as it is simpler and sufficient for our purposes.

[6]By focusing on a single shock at time $t = 0$, we intentionally abstract from well-studied precautionary effects (e.g. Ravn and Sterk, 2017; Challe, 2020). One reason for doing so is simply to focus and isolate less studied forces. Another is related to the application to the COVID-19 epidemic: the shock is relatively front loaded and the idiosyncratic uncertainty as to who it hits is also resolved early on. Arguably the main source of uncertainty during an epidemic is its duration and this form of uncertainty has no effects on our results.

Agents have access to real, one-period bonds in zero net supply. A fraction $\mu \in [0, 1)$ of agents face the borrowing constraint

$$a_{it} \geq 0. \tag{3}$$

We refer to these agents as "potentially constrained".

The economy starts in a symmetric steady state with $a_{it} = 0$ and $c_{it} = \bar{n}$ for all agents. At $t = 0$, a fraction ϕ of agents are unexpectedly hit by the shock and their labor supply goes to $n_{it} = 0$. The remaining fraction $1 - \phi$ has $n_{it} = \bar{n}$. The labor supply shock is independent of whether the agent is a potentially constrained agent or not.

Once again, we begin by characterizing the flexible price equilibrium. Consider first the fraction $\mu\phi$ of agents that are potentially constrained and hit by the shock. Since their income drops to zero and they cannot borrow, their consumption falls to zero, $c_{it} = 0$. We call those agents "constrained".

Consider next the remaining group of $1 - \mu\phi$ agents. These agents are all on their Euler equation

$$U'(c_{i0}) = \beta (1 + r_0) U'(c_{i1}),$$

those not hit by the shock because they turn out to be net savers in equilibrium, those hit but not subject to (3) because they do not face a borrowing constraint. We call this group of $1 - \mu\phi$ agents "unconstrained". Due to homothetic preferences (implied by the power function $U(c)$) the average consumption of unconstrained agents, denoted in bold $\mathbf{c}_t$, also satisfies the Euler equation,

$$U'(\mathbf{c}_0) = \beta (1 + r_0) U'(\mathbf{c}_1). \tag{4}$$

Since the shock is gone after $t = 0$ and, on average, unconstrained agents have zero assets, their average consumption will be equal to their income, so

$$\mathbf{c}_1 = \overline{n}. \tag{5}$$

Note that some unconstrained agents are saving while others are borrowing so that their consumption paths do not coincide.

Aggregate demand at date $t = 0$ equals $(1 - \mu\phi)\, \mathbf{c}_0$, so goods market clearing requires

$$(1 - \mu\phi)\, \mathbf{c}_0 = (1 - \phi)\overline{n}. \tag{6}$$

Substituting (5) and (6) into the Euler equation (4), we arrive at the natural interest rate

$$1 + r_0^* = \frac{1}{\beta}\frac{U'\left(\frac{1-\phi}{1-\mu\phi}\overline{n}\right)}{U'\left(\overline{n}\right)} > \frac{1}{\beta}. \tag{7}$$

Once again, the natural interest rate cannot decrease in response to the shock, as $1 - \phi < 1 - \mu\phi$. This is a surprising result. Even in the case where the supply shock loads almost entirely on constrained agents, $\mu \to 1$, and thus causes the largest possible demand response, the real interest does not decline, it merely stops increasing.[7] In the opposite extreme, $\mu \to 0$, we obtain the same aggregate outcome as the complete markets case.

Now turn to the economy with rigid nominal wages, with the central bank holding the interest rate at $1 + r_0 = 1/\beta$. To simplify, we assume that if any rationing of labor takes place, then all workers are equally rationed.[8] From (4) and (5), we have average demand of the unconstrained equal to

$$\mathbf{c}_0 = \overline{n},$$

implying that aggregate demand is $(1 - \mu\phi)\bar{n}$ which is larger than aggregate supply $(1 - \phi)\overline{n}$. Again, the supply shock leads to an excess of labor demand, except in the extreme case $\mu \to 1$.[9]

To get an intuition for this result it is useful to think in terms of individual consumption functions. When the supply shock hits, it reduces agents' incomes. Assuming full employment, some agents' incomes remain unchanged at $\bar{n}$, and some agents' income fall to 0. At constant interest rates, the consumption of the first group of agents remains the same. The consumption of the second group of agents falls in proportion to their marginal propensity to consume (MPC). But MPCs are never bigger than 1, so these agents' demands can never fall by more than $(1 - \phi)\,\bar{n}$. In particular, in our model $1 - \mu$ agents have MPCs equal to $1 - \beta$, while μ agents have MPC equal to 1. The largest drop in demand we can obtain happens if $\mu = 1$ when all agents have MPC equal 1. In this case, the shock removes $\phi\overline{n}$ units of aggregate supply and removes $\phi\overline{n}$ units of aggregate demand. In all other cases, the drop in demand is weaker than the drop in supply.

[7] By contrast, if the supply shock were to load disproportionately on unconstrained, rather than constrained, agents, the complete markets conclusion would be reinforced, with an amplified increase in the natural interest rate. Bilbiie (2008) provides an example of such an economy, as negative technology shocks in his economy reduce profits going to unconstrained agents.

[8] Indeed, our results would only be strengthened if rationing were discrete and took place along an extensive margin, with some workers being fully rationed and others not at all.

[9] As discussed earlier, in our model with fixed labor endowments, an equilibrium will thus require a raise in interest rates.

As is clear from this intuition, the result transcends the specific simple model used here and applies to much richer incomplete-markets models with uninsurable idiosyncratic shocks, as long as there is only a single consumption good. In those models the distribution of MPCs is richer than in our case. But the fact that MPCs are bounded above at 1 is a robust feature of this class of models, and it implies the result that aggregate demand can never fall more than aggregate income, following a shock that shifts down all current incomes.[10]

We summarize the results with incomplete markets.

Proposition 2. *In the single-sector model with incomplete markets and $\mu < 1$, a negative supply shock increases the natural interest rate. If the real rate does not adjust, there is excess demand in the labor market. In the special case in which all agents are constrained, $\mu \to 1$, the natural interest rate remains constant.*

3 Multiple Sectors: Keynesian Supply Shocks

We now extend the model to include more than one sector. This is a natural extension to capture a shock like the COVID-19 pandemic with asymmetric impacts across sectors.

We consider two sectors, A and B. Sector A captures activities that require personal contact, such as restaurants, hotels, and entertainment venues. This is the sector that is directly impacted due to the health concerns of consumers and workers, or due to public health policies. Sector B captures the rest of the economy that is not directly impacted. Workers in sector B can work from home (e.g. accounting services) or their work can be re-organized to limit contacts (e.g. manufacturing). Assuming all sectors fit in these two polar extremes (one closed, the other fully open) is simply a useful modeling device.

In the model, a fraction ϕ of agents works in sector A and the remaining fraction $1 - \phi$ in sector B. All agents inelastically supply labor $\overline{n}$ to their respective sector. For now, we assume that workers are perfectly specialized and cannot move between sectors. In 3.3.2, we consider labor mobility.

The technology to produce both goods is linear

$$Y_{jt} = N_{jt}, \tag{8}$$

[10] The conclusion can be overturned by departing from purely negative shocks, to assume that the fundamental shock is positive for some agents. For example, suppose we reduce labor endowment for some fraction ϕ of agents but increase it for the remaining fraction, leaving the average endowment less than or equal to before. Such a "redistributive" shock can obviously lead to a fall in aggregate consumption beyond the drop in the aggregate endowment.

for $j = A, B$. Competitive firms in sector j hire workers at the sector-specific wage W_{jt} and sell good j at price P_{jt}. Prices P_{jt} are flexible and, given the technology above, the price of good j is $P_{jt} = W_{jt}$ in equilibrium.

Consumer preferences are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t U\left(c_{At}, c_{Bt}\right), \tag{9}$$

where

$$U\left(c_{At}, c_{Bt}\right) = \frac{\sigma}{\sigma - 1}\left(\phi^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}\frac{\sigma-1}{\sigma}}, \tag{10}$$

so the utility function features constant elasticity of substitution ϵ between the two goods and constant intertemporal elasticity of substitution σ.

For the derivations in the text we use the restriction $\epsilon > 1$, so utility is well defined even when $c_A = 0$. However, all results apply in the general case with ϵ greater or smaller than 1, by considering a straightforward infinitesimal perturbation of the CES utility function (10) that allows us to set $c_A = 0$ also with $\epsilon \leq 1$. Appendix A.1 contains the details.

Next, we characterize the response of this multi-sector economy to a supply shock that completely shuts down sector A. As we did for the single sector model, we begin with complete markets.

3.1 Complete Markets

At the steady state with full employment before the shock we have a unitary relative price $p^* = 1$ of good A in terms of good B, equalized wages $w_A = w_B = 1$, and quantities

$$c_A^* = Y_A^* = \phi\overline{n}, \quad c_B^* = Y_B^* = (1-\phi)\overline{n}.$$

For reasons we make clear shortly, it is useful to focus on the real interest rate in terms of good B, defined as

$$1 + r_t \equiv (1 + i_t)\frac{P_{Bt}}{P_{Bt+1}},$$

where i_t denotes the nominal interest rate and P_{Bt} is the nominal price of the good produced in sector B. The real interest rate $1 + r_t$ enters the Euler equation in terms of good B:

$$U_{c_B}\left(c_{At}, c_{Bt}\right) = \beta(1 + r_t) U_{c_B}\left(c_{At+1}, c_{Bt+1}\right), \tag{11}$$

where U_{c_B} denotes the partial derivative of U with respect to c_{Bt}. In steady state, this real interest rate is $1 + r_t = 1/\beta$ as in the one good economy, since all consumption levels are constant.

At date $t = 0$, the economy is hit by an asymmetric supply shock: all production in sector A is shut down, so

$$c_{A0} = Y_{A0} = N_{A0} = 0.$$

Of course, employment falls in sector A. That is the inevitable effect of the shock. So, we ask what happens in sector B. As before, the shock is temporary and the economy goes back to steady state at $t = 1$. And, as before, we look first at the natural interest rate and then we ask what happens if the central bank keeps the real rate unchanged.

Using the representative agent's Euler equation (11), the natural rate after the shock is

$$1 + r_0^* = \frac{1}{\beta} \frac{U_{c_B}\left(0, c_B^*\right)}{U_{c_B}\left(c_A^*, c_B^*\right)}. \tag{12}$$

The natural interest rate falls due to the epidemic shock if the ratio of marginal utilities on the right-hand side is smaller than 1. Using the functional forms introduced above, this happens if

$$(1-\phi)^{\frac{1}{\sigma}\frac{\sigma-\epsilon}{\epsilon-1}} < 1. \tag{13}$$

An immediate implication is the following result.

Proposition 3. *In the multi-sector model with complete markets, a supply shock that shuts down sector A reduces the natural interest rate* (12) *if and only if*

$$\sigma > \epsilon. \tag{14}$$

To interpret this result, notice that consumer choice is driven by intratemporal and intertemporal considerations. When good A becomes unavailable, consumers substitute intratemporally in favor of good B. This increases demand for B. At the same time, when good A becomes unavailable, consumers respond by intertemporal substituting away from present consumption to future consumption, since in the future both goods will be available again. This reduces demand for B. When inequality (14) is satisfied the second effect is stronger than the first and there is an overall drop in demand for B. To induce consumers to consume enough to keep sector B at full employment, we need a drop in the interest rate. We graphically illustrate condition (14) in Figure 2. As mentioned above, the case $\epsilon \leq 1$ is covered by a limit argument developed in Appendix A.1.

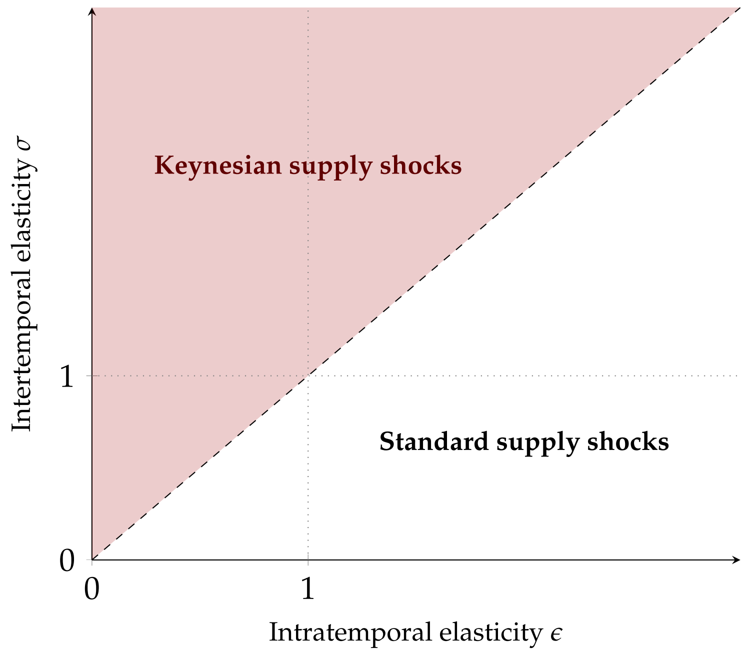


Figure 2: When are supply shocks Keynesian with a representative agent?

Consider now what happens if nominal wages are downwardly rigid and the central bank maintains the real interest rate at its steady state value. If (14) is satisfied then there is an inefficient recession in sector B, and the size of the output drop in sector B is given by

$$\frac{Y_{B0}}{Y_B^*} = (1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}. \tag{15}$$

Thus, when $\sigma > \epsilon$ and the central bank does not (or cannot) act, the economy features two types of job losses: the unavoidable job losses in sector A due to the direct effect of the shock and the inefficient job losses due to insufficient demand in sector B.

In interpreting the above condition that $\sigma > \epsilon$, it is important to bear in mind that ϵ represents an elasticity of substitution across broad sectors, not across different varieties of the same good. Thus, ϵ is plausibly quite low. Likewise, σ may be interpreted as capturing an average intertemporal substitutability, including durable goods, not just non-durables.[11] It seems difficult to interpret the large drop in consumer spending during the COVID epidemic without allowing for a relatively sizable degree of intertemporal substitution. Another perspective is that the borderline case with $\sigma = \epsilon$ corresponds to additive utility, so the condition $\sigma > \epsilon$ requires Hicksian complementarity, a plausible condition across

[11] We also should stress that our simple and stylized demand system abstracts from many other features, such as luxury versus necessities, and with only 2 sectors is aggregating broadly. At a lower level of aggregation many subcategories are likely more substitutable.

broad sectors. In sum, we think condition (14) might be satisfied for the case of the recent COVID-19 supply shock. In any case, we shall see that the conditions are weakened when we discuss incomplete markets, input-output linkages and labor mobility.

Measured, Ideal CPI and Real Interest Rates. Before turning to incomplete markets, we provide an alternative interpretation of the result above. Notice that the shutdown of sector A can be interpreted as making the shadow price of good A prohibitively high. The ideal consumer price index (CPI) in this economy is

$$P_t = \left(\phi P_{At}^{1-\epsilon} + (1-\phi) P_{Bt}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}.$$

If we set the price P_{At} to infinity in period 0, the price index is still well defined if $\epsilon > 1$. For a given nominal interest rate i_0, and assuming zero inflation in good 2, the real interest rate in terms of the aggregate consumption basket C_t is then

$$(1+i_0)\frac{P_0}{P_1} = (1+i_0)(1-\phi)^{\frac{1}{1-\epsilon}} > 1+i_0.$$

Proceeding in this way and using the Euler equation $U'(C_0) = \beta(1+i_0)\frac{P_0}{P_1}U'(C_1)$ it is then possible to re-derive equation (15). We can then reinterpret the shock causing unemployment in this economy as a shock that, for a given nominal rate, leads to a sharp temporary increase in the real interest rate, due to the fact that the shadow price of a number of goods goes up to infinity, as the goods cannot be bought. While this interpretation is useful, it is a bit harder to match to observables, because the shadow price of the goods not traded is not observed and their quantity goes to zero. So, in the following we remain focused on what happens to the real interest rate in terms of the goods that are still traded. In other words, we focus on the *measured* CPI, rather than the ideal CPI.

The discussion above also explains why we work with the real interest rate in terms of the goods that are traded in all periods, which in this section means good B, as this is the real interest rate that remains unchanged if the nominal interest is kept unchanged.

3.2 Incomplete Markets

We now generalize this economy to allow for market incompleteness, exactly as in Section 2. Again, we use a simple description of market incompleteness but all results in this section generalize to richer setups, e.g. those in Werning (2015). In particular, a random fraction μ

of households is potentially constrained, subject to the borrowing constraint (3), and all households have the same initial financial wealth $a_{i0} = 0$.

To derive the response of the natural rate, we focus again on the group of effectively unconstrained agents, those in sector B and those in sector A that are not constrained. Denote by $\mathbf{c}_{jt}$ the average consumption of good j for this group. The Euler equation holds for all these agents, and to homothetic preferences it then holds for their average consumption

$$1 + r_0 = \frac{1}{\beta} \frac{U_{c_B}\left(0, \mathbf{c}_{B0}\right)}{U_{c_B}\left(\mathbf{c}_{B1}, \mathbf{c}_{B1}\right)}. \tag{16}$$

To evaluate this expression at a full employment equilibrium we use the following observations.

At date $t = 0$ constrained agents consume zero. Therefore, unconstrained agents must absorb the whole output of sector B, which is $(1 - \phi)\overline{n}$. Since there is a mass $1 - \mu\phi$ of unconstrained consumers, their average consumption must then be

$$\mathbf{c}_{B0} = \frac{1 - \phi}{1 - \mu\phi}\overline{n}.$$

At date $t = 1$, the average asset holdings among unconstrained consumers is 0— by market clearing in the bonds market—and their income is $\bar{n}$. Moreover, the relative price of good A goes back to its original steady state value $P_A / P_B = 1$.[12] So their average consumption is

$$\mathbf{c}_{A1} = \phi\bar{n}, \quad \mathbf{c}_{B1} = (1 - \phi)\,\bar{n}.$$

Substituting $\mathbf{c}_{B0}$, $\mathbf{c}_{A1}$, and $\mathbf{c}_{B1}$ into (16), after a bit of algebra, yields

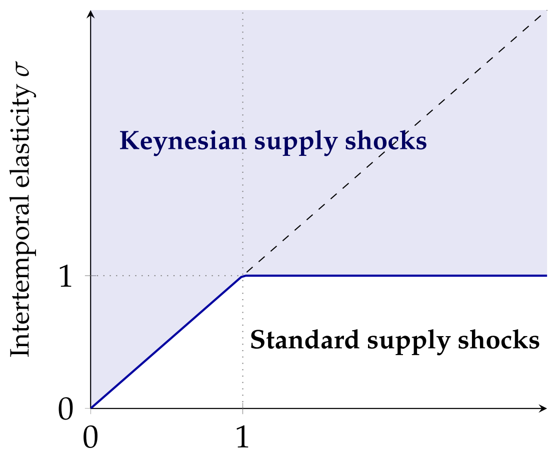
$$1 + r_0 = \frac{1}{\beta}\left(1 - \mu\phi\right)^{\frac{1}{\sigma}}\left(1 - \phi\right)^{\frac{1}{\sigma}\frac{\sigma - \epsilon}{\epsilon - 1}}. \tag{17}$$

Taking logs and rearranging gives us the following result.

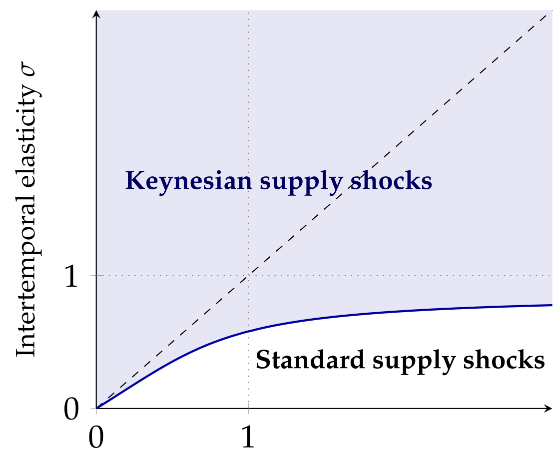
Proposition 4. *In the multi-sector model with incomplete markets, a supply shock that shuts down sector A reduces the natural interest rate in terms of good B if and only if $\sigma > \epsilon$ or*

$$\sigma > \left(1 - \frac{\log\left(1 - \mu\phi\right)}{\log(1 - \phi)}\right)\epsilon + \frac{\log\left(1 - \mu\phi\right)}{\log(1 - \phi)}. \tag{18}$$

[12] This price does not depend on the wealth distribution due to Gorman aggregation.



(a) Standard CES preferences



(b) CES locally but not globally

Figure 3: When are supply shocks Keynesian with incomplete markets?

This result is similar to Proposition 3, in that it provides a lower bound on the elasticity of intertemporal substitution. In Proposition 3, the lower bound was given by the intratemporal elasticity of substitution ϵ. Proposition 4 shows that market incompleteness relaxes this condition, possibly considerably so. In particular, condition (18) requires σ to lie above a convex combination of ϵ and 1. Moreover, the convex combination converges to 1 as $\mu \to 1$. Thus, in the extreme case where the borrowing constraint applies to all agents, the condition for Keynesian supply shocks is simply $\sigma > 1$. Interestingly, this condition no longer depends on the elasticity of substitution across goods (as long as $\epsilon > 1$). We illustrate condition (18) in Figure 3(a).

As in the complete markets case, we extend the result to the region $\epsilon \leq 1$ using the limit arguments in Appendix A.1. As can be seen comparing Figures 2 and 3, in that region market incompleteness does not affect the contour. This, however, is an artifact of CES preferences. As supply in sector A shrinks, spending on sector A goods relative to sector B goods increases if $\epsilon < 1$. However, if sector A is entirely gone, intuition suggests that no spending should be done on sector A goods. To capture this idea, we repeat our comparison between incomplete and complete markets in Figure 9(b) in a case where CES preferences are modified to still have a constant elasticity locally, but no longer globally, allow for a spending share on good A that approaches zero as its price increases without bounds (see Appendix A.1 for details). With this correction, market incompleteness shifts down the contour line for any ϵ.

Consider now the case of downward rigid wages and a central bank that keeps the real interest rate unchanged, assuming condition (18) holds. The average consumption of unconstrained consumers is now

$$\mathbf{c}_{B0} = (1-\phi)^{\frac{\sigma-1}{\epsilon-1}}\,\bar{n}, \tag{19}$$

so there is a recession in sector B and the output contraction is

$$\frac{Y_{B0}}{Y_B^*} = (1-\mu\phi)\,(1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}. \tag{20}$$

An interpretation using consumption functions. We again provide some intuition for this result and the role of incomplete markets using individual consumption functions.

We plot the relation between current income y_0 and individual consumption of good B assuming the interest rate is fixed at $1/\beta - 1$ and future income and prices are back at steady state for $t = 1, 2, \ldots$ When the shock hits, two things happen. First, the consumption function shifts, due to the unavailability of good A. In addition, income falls for those in sector A. Income in sector B may or may not fall, but we examine what happens if it were to remain at full employment. Then, aggregating all consumers we compute total demand for good B. If it falls below Y_B^* we are in the Keynesian supply shock region.[13]

Figure 4 shows the exercise in the complete market case. In panel (a) we consider the case $\sigma = \epsilon$. In that case, the consumption function shifts up after the shut down of sector A. The shift of the consumption function is exactly compensated by the drop in income of the representative agent from $\bar{n}$ to $(1-\phi)\,\bar{n}$, so total spending on good B remains unchanged. In panel (b) we consider a case with $\sigma > \epsilon$, in which the consumption function shifts down. The drop in income from $\bar{n}$ to $(1-\phi)\,\bar{n}$ reinforces the effect, leading to an overall fall in demand for sector B. So we obtain a Keynesian supply shock, consistent with Proposition 3.

Turn now to incomplete markets. In Figure 5 we plot the consumption function after the shock, for an incomplete market economy with $\sigma = \epsilon$ in the limit case $\mu \to 1$. The dashed line is the consumption function of an unconstrained agent. The solid line is the consumption function of a constrained agent, which is piecewise linear and concave. We do not plot the pre-shock consumption functions to avoid clutter.

With complete markets the income of the representative agent goes from $\bar{n}$ to $(1-\phi)\,\bar{n}$ and consumption is equal to Y_B^* as in panel (a) of Figure 4. With incomplete markets consumers face heterogeneous income losses and are not insured. Namely, $1-\phi$ sector B workers get no income loss and stay at $\bar{n}$ (recall that we are asking what happens if we

[13] The formal details of the construction behind Figures 4 and 5 is in Appendix A.2.

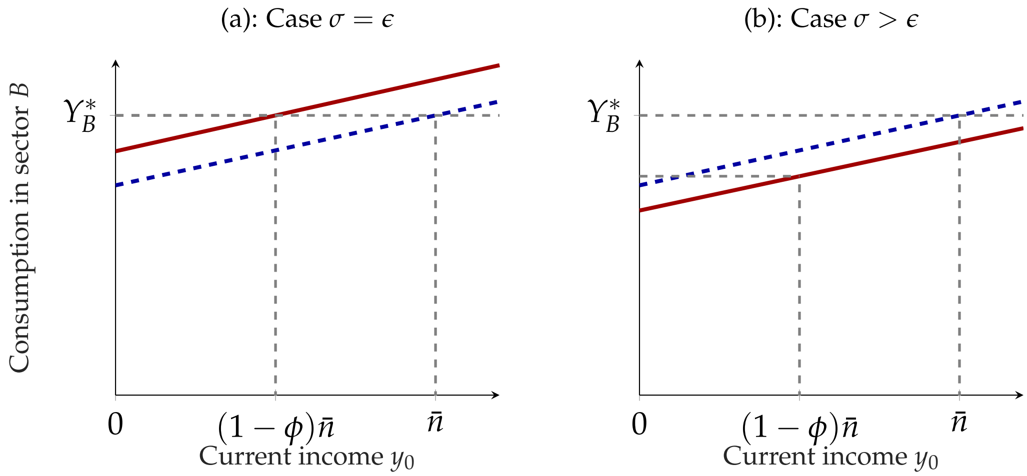


Figure 4: Consumption functions: complete markets. Dashed blue line: before the shock. Solid red line: after the shock.

were to stay at full employment), while ϕ sector A workers go from $\bar{n}$ to 0. We can read off their consumption choices from the constrained consumption function. Due to concavity, when we average their consumption levels we get an average consumption lower than Y_B^*, so we get insufficient aggregate demand.

The simple logic behind Figure 5 can be used to derive a simple condition for Keynesian supply shocks in terms of measurable statistics, instead of model parameters. In Appendix A.2, we show that the following condition is equivalent to condition (18) in Proposition (4):

$$\overline{MPC}^A > \frac{\Delta c_B}{\Delta c_A}. \tag{21}$$

The left hand side $\overline{MPC}^A$ is the average MPC of sector A workers. The right-hand side is the change in consumption of good B that is associated with a reduction in spending on good A—due to the shutdown of sector A—for a given level of income. It can be interpreted as a form of cross-good marginal propensity to consume as it answers the question: how would you change your consumption of good B if you were to save temporarily Δc_A by spending less on good A? This represents the shift in the consumption function in Figure 5. The left-hand side captures the effects of the income drop in the shocked sector, that is, the movement to the left along the consumption function. There is a growing literature on the measurement of the MPCs on the left-hand side. The term on the right-hand side, on the other hand, is relatively unexplored.

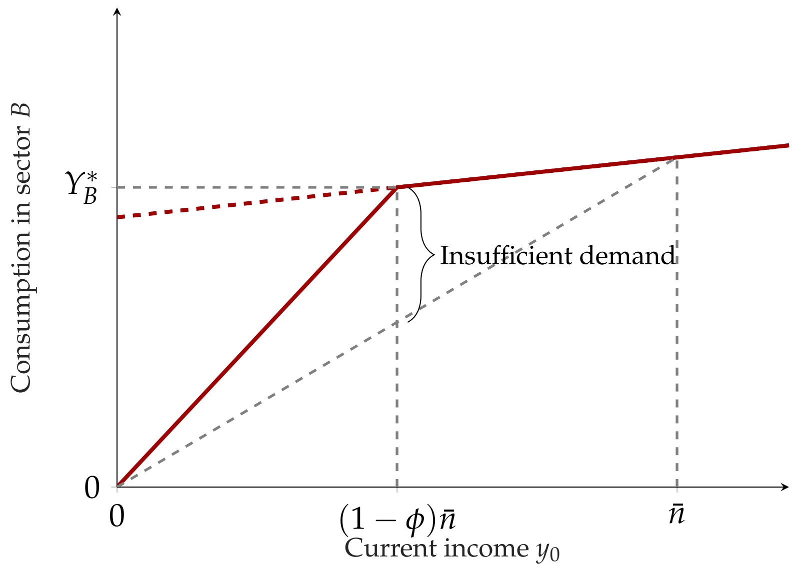


Figure 5: Consumption functions: incomplete markets. Consumption functions at $t = 0$. Dashed line: unconstrained consumer. Solid line: constrained consumer.

No Paradox of Toil. An interesting perspective on the shock analyzed here is that it does not feature the so-called "paradox of toil." Eggertsson (2010) points out that in the baseline new Keynesian model a negative supply shocks is expansionary at the zero lower bound. This effect comes from the fact that the shock causes higher inflation, moving the real interest rate down and thus stimulating demand. The type of sectoral shock considered here has opposite predictions, if condition (18) is satisfied. The temporary unavailability of the goods in sector A pushes up the "shadow" real interest rate, as argued on page 16. Moreover, the demand deficiency in sector B creates deflationary pressures in that sector. In our model the shock only lasts one period and deflation is not modeled, so this second channel does not act to further depress demand. But it would be easy to extend the model to allow for that amplification channel, which would go in the direction of causing a deeper recession.

3.3 Extending the Basic Model

Here we present three simple extensions of our basic model: allowing for an input-output network structure of production, allowing for labor mobility across sectors, and extending the range of shocks that can hit sector A.

3.3.1 Demand Chains

The analysis so far has emphasized the complementarity between the two sectors in consumer preferences. Introducing input-output relations allows for complementarity in production. The usual argument is that supply chain disruptions cause an amplification of supply shocks occurring upstream in the chain (sector A as input into sector B). In contrast, here we focus on downstream disruptions reducing demand for upstream sectors (sector B as input into sector A) and use the term "demand chain" to capture the mechanism we investigate, which amplifies the induced demand shortage.

In particular, we consider the possibility that goods produced in sector B are used as intermediate inputs by the shocked sector A. Intuitively, restaurants in sector A may buy goods and services, such as dishwashers and pest control, from sector B to produce their final good. When restaurants shut down, this impacts sector B separately from consumer responses. We investigate this idea with a simple input-output structure.

We continue to use the same preferences, but change the technology in sector A to the constant returns to scale technology

$$Y_A = F(X, N_A),$$

where X denotes good B used as intermediate input in sector A. Good B is still produced linearly from labor $Y_B = N_B$. We choose parameters so that $p_A^* = p_B^* = w_A^* = 1$, which is just a convenient normalization. This implies that, as in Section 3, the parameter ϕ in the utility function (10) gives the steady state share of A in aggregate consumption. Each worker supplies $\bar{n}$ units of labor and is fully specialized, with fraction $\tilde{\phi}$ working in sector A and fraction $1 - \tilde{\phi}$ in sector B. Let $x^* \equiv X^*/Y_B^*$ denote the steady state fraction of good B's output used as input in sector A. Accounting requires that for any parametrization of the model we have $\tilde{\phi} \leq \phi$ and $x^* < \phi$.[14]

Consider now the effect of a temporary shutdown of sector A. Skipping directly to the incomplete markets case with the real rate at $1/\beta - 1$, the recession in sector B is now given by

$$\frac{Y_{B0}}{Y_B^*} = (1 - (1-\mu)x^* - \mu\phi)\,(1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}\,. \tag{22}$$

With greater x^* the output drop is amplified and the condition for a Keynesian supply

[14] Total consumption equals total income and total income is $\bar{n}$ in steady state, as wages are 1 in both sectors. Since consumption of B goods is a share $1-\phi$ of total consumption, we have $c_B^* = (1-\phi)\,\bar{n}$. Moreover, from goods market clearing at full employment we have $c_B^* = Y_B^* - X^* = (1-x^*)\,(1-\tilde{\phi})\,\bar{n}$. It follows that $1-\phi = (1-x^*)\,(1-\tilde{\phi})$. The two inequalities follow from $x^* \geq 0$ and $\tilde{\phi} > 0$.

shock is relaxed, expanding the region in Figure 3. The expression in (22) reveals a useful intuition. The input share x^* acts as if it were a separate constrained agent group that loses x^* income and mechanically lowers its spending on A. The input-output from production has a marginal propensity to spend of one. In this sense, having demand chains is like having a higher fraction of constrained agents.

3.3.2 Labor Mobility

So far we have assumed workers cannot move between sectors. We now extend the analysis to allow a fraction λ of workers in each sector to freely move to the either sector. We revert to the model without demand chains (i.e. $x^* = 0$).

The total fall in output can be decomposed as a drop in natural output and an inefficient output gap due to the lack of demand. A fraction $(1-\lambda)\phi$ of workers in A cannot move to sector B, so natural output is now $Y_0^* = Y_{B0}^* = (1-(1-\lambda)\phi)\,\bar{n}$, and the ratio of natural output to steady state output

$$\frac{Y_0^*}{Y^*} = 1 - \phi + \lambda\phi < 1$$

is increasing in λ. Mobility makes the supply shock less severe. Equilibrium output can be shown to equal $Y_{B0} = (1-(1-\lambda)\phi\mu)\,(1-\phi)^{\frac{\sigma-1}{\epsilon-1}}\,\bar{n}$ so equilibrium output also rises with mobility.[15] However, the ratio of equilibrium to natural output

$$\frac{Y_0}{Y_0^*} = \frac{1-\phi\mu+\lambda\phi\mu}{1-\phi+\lambda\phi}\,(1-\phi)^{\frac{\sigma-1}{\epsilon-1}}$$

is decreasing in λ. Thus, $\lambda > 0$ expands the parameter space (ϵ, σ) where supply shocks are Keynesian relative to $\lambda = 0$.

Complete markets is equivalent to setting $\mu = 0$ in the above expressions. In this case natural output rises with mobility λ, while equilibrium output is constant. This illustrated in panel (a) of Figure (6). The gap between the two lines grows with λ. Turning to incomplete markets $\mu > 0$, natural output is rising in mobility, just as with complete markets. However, equilibrium output is now also rising in labor mobility. Intuitively, demand is now affected by mobility λ because workers moving out of A into B do not lose as much income. Panel (b) in Figure (6) illustrates this situation. According to our expressions the ratio of output to natural output falls with λ, so that the the gap between the two lines grows with λ. Summing up, labor mobility makes the size of the recession

[15] The fraction of workers in A that cannot move and are thus effectively constrained is $(1-\lambda)\,\mu\phi$. The

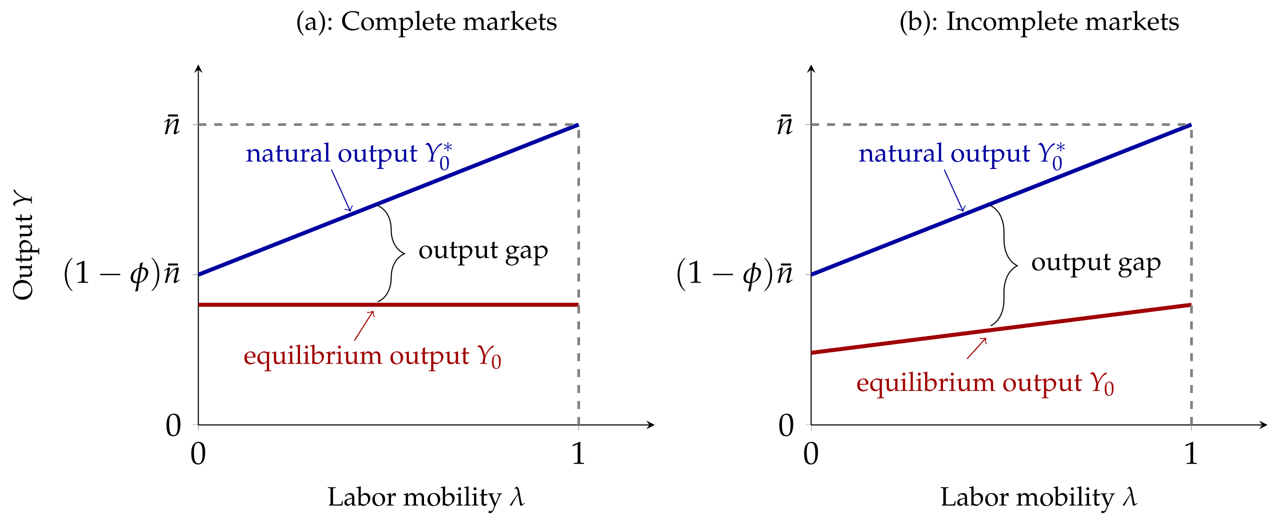


Figure 6: Labor mobility and the output gap.

smaller, by buffering income losses, but it raises the level of natural output by still more, thus making the demand deficiency more severe.

3.3.3 Keynesian spillovers of arbitrary shocks to gains from trade

So far, we have focused on a supply shock to sector A. What if, instead, the shock was not one to supply but to demand for sector A goods?

To study this case, we introduce a preference shifter θ_t into the utility function of households,

$$U\left(c_{At}, c_{Bt}\right) = \frac{\sigma}{\sigma - 1}\left(\phi^{\frac{1}{\epsilon}}\left(\theta_t c_{At}\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}\frac{\sigma-1}{\sigma}}$$

shifting the utility households receive from consuming good A. A value $\theta_t < 1$ can capture the associated health risk with consuming good A. For simplicity, we consider again an extreme shock, $\theta_0 = 0$, which hits at date $t = 0$, but the results hold for arbitrary $\theta_0 < 1$.

Due to the shock, households set all consumption of good A to zero, $c_{A0} = 0$. How does this affect consumption of good B, and the natural interest rate? This depends on how the marginal utility with respect to good B changes in period $t = 0$ relative to the steady state. Since $c_{A0} = 0$ for both demand and supply shocks, however, the expressions for the natural interest rate, both with complete and incomplete markets, are identical to the ones in Section (3). Therefore, our analysis can equally well be understood as a characterization

remaining fraction of workers $1 - \phi\mu + \lambda\phi\mu$ consume (19) on average, yielding the desired expression.

of arbitrary shocks to the gains from trade, hitting demand, supply, or a combination of the two, which have an asymmetric effect on some sectors of the economy.

4 Fiscal Policy: Multipliers and Social Security

We now turn to fiscal policy. We discuss the size of the multiplier in our environment and the role for social insurance, including optimal policy.

4.1 The Muted Power of Stimulus Dollar-for-Dollar

To consider the effects of fiscal stimulus, we introduce a stylized government sector. The government chooses paths of government spending G_t, government debt D_t, and lump-sum transfers (or taxes) T_{jt} targeted to sector j workers, subject to the flow budget constraint

$$G_t + \phi T_{At} + (1-\phi)\, T_{Bt} + (1 + r_{t-1}) D_{t-1} = D_t$$

We assume that in the initial steady state, $G_t = T_{At} = T_{Bt} = D_t = 0$.[16]

We consider two stimulus policies. The first is traditional government spending: the government purchases an amount G_0 of sector B goods at date zero. The second is a targeted transfer program: the government chooses a positive transfer T_{A0} to sector A workers. One may think of this roughly as unemployment insurance benefits. For convenience, we assume that both interventions are financed by a permanent tax on B sector workers $T_{Bt} = T_B < 0$ for all $t \geq 1$, but discuss other financing choices briefly.

Proposition 5. *Consider the incomplete market economy and suppose condition (18) is satisfied. Suppose the central bank keeps the interest rate at $r_0 = 1/\beta - 1$ and the government chooses small spending and transfers G_0, T_{A0}, T_B that satisfy the government intertemporal budget constraint*

$$(1-\beta)\,(G_0 + \phi T_{A0}) + \beta\,(1-\phi)\, T_B = 0.$$

Then equilibrium output in sector B is given by

$$Y_{B0} = G_0 + \mu\phi T_{A0} + (1-\mu\phi)\,(1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}\, Y_B^*. \tag{23}$$

In particular, there is a unit government spending multiplier and a sector-A transfer multiplier equal to μ.

[16]The budget constraints for a worker i in sector j becomes $c_{it} + a_{it} \leq w_{jt} n_{it} + (1 + r_{t-1}) a_{it-1} + T_{jt}$.

This is a striking result. The average MPC in the economy is

$$\overline{mpc} = (1 - \mu\phi) \times mpc^U + \mu\phi \times mpc^C, \tag{24}$$

where $mpc^C = 1$ is the MPC of constrained agents and

$$mpc^U = \frac{1-\beta}{1-\beta+\beta\left(1-\phi\right)^{-\frac{\sigma-1}{\epsilon-1}}}$$

is the MPC of unconstrained agents. This latter MPC is relatively small and vanishes as $\beta \to 1$ and is further reduced due to the unavailability of A goods when $\sigma > 1$.[17] A traditional Keynesian-cross argument would suggest a multiplier of $1/\left(1-\overline{mpc}\right) \geq 1$, which can be large if there is a large fraction of constrained consumers. Various recent papers show that Keynesian-cross arguments can indeed be used in environments with heterogeneity and incomplete markets (Galí et al. 2007, Farhi and Werning 2016, Auclert et al. 2018, Bilbiie 2019). What is different here?

The crucial difference is that sector A is shut down, so no agent can spend on sector A. This severs the usual virtuous cycle behind the Keynesian multiplier. The usual multiplier emerges because any initial stimulus increases income, which would then be spent on the margin; this spending creates still higher income and so on. However, when sector A is shut down, money spent by agents or the government flows into the pockets of sector B workers, not of sector A workers with higher MPCs, as illustrated in Figure 1. Thus, traditional fiscal stimulus is less effective dollar-for-dollar. To see this in more detail, note that the expression that captures correctly the partial equilibrium response of aggregate spending to an increase in output Y_{B0} is not (24) but

$$(1-\mu\phi) \times mpc^U \times \frac{dy^U}{dY_{B0}} + \mu\phi \times mpc^C \times \frac{dy^C}{dY_{B0}} = \frac{1-\beta}{1-\beta+\beta\left(1-\phi\right)^{-\frac{\sigma-1}{\epsilon-1}}} \tag{25}$$

where dy^U/dY_{B0} and dy^C/dY_{B0} are the sensitivities of the incomes of each group to aggregate date-0 output Y_{B0}. These two quantities are equal to $dy^U/dY_{B0} = 1/\left(1-\mu\phi\right)$ and $dy^C/dY_{B0} = 0$ in the model, so the weight on constrained agents goes to zero. The observation that the aggregate effect of shocks depends on the joint distribution of MPCs and income sensitivities is in line with Patterson (2019). However, the pattern of correlation be-

[17] See the derivations of the consumption functions in Appendix (A.2). The notation mpc used here denotes the marginal propensity to consume for an infinitesimal income change, while the notation MPC used in (21) denotes the marginal propensity to consume for a discrete income change.

tween MPCs and sensitivities is negative due to the asymmetric supply shock, as opposed to positive as estimated by Patterson (2019) during normal times, leading to a dampened effect of fiscal stimulus.[18]

The expression in (25) is small, if β is close to 1, but not zero, suggesting a small but positive consumption response to a G_0 shock. Why do we get a unit multiplier in Proposition 5 instead? The answer has to do with the Ricardian behavior of unconstrained agents, who anticipate higher taxes in the future to finance G_0. The assumption that all future taxes fall on sector B workers implies that this Ricardian effect exactly cancels the multiplier effect. This argument is similar to that in Woodford (2011) for the representative agent case.[19]

4.2 The Importance of Social Insurance

While transfers to sector A workers have lower multipliers dollar-for-dollar than usual, this does not imply that they should not be used. Indeed, targeted transfers serve an important role as social insurance and may provide the best form of stimulus for the economy. They form an integral part of the optimal policy response. To see this, consider a specific policy that transfers income to sector A workers with replacement rate ρ, so that

$$T_{A0} = \rho \bar{n} \geq 0$$

at $t = 0$. The transfer is paid for by issuing debt and this debt is serviced with constant future taxes on A and B agents: $T_{At} = T_A \leq 0$ and $T_{Bt} = T_B \leq 0$ for $t = 1, 2, \ldots$, setting the fraction of tax revenue coming from A agents to some value $\zeta \in [0, 1)$, that is, setting

$$\frac{\phi T_A}{(1-\phi)\, T_B} = \frac{\zeta}{1-\zeta}.$$

Previously, we focused for simplicity on $\zeta = 0$. Now we allow $\zeta > 0$ to discuss transfers more generally. A natural benchmark sets $\zeta = \phi$, in which case A and B agents pay the same tax.

[18] If we assume a share μ of agents to be exogenously hand-to-mouth, fiscal multipliers would be larger. Our assumption of a kinked consumption function captures nonlinearities present in richer models of consumption behavior.

[19] With different assumptions on the distribution of taxes in future periods, we can get a multiplier slightly above 1. For example, if we assume that all agents, not only B agents, pay a permanent tax T from period $t = 1$ onwards we obtain a multiplier for G_0 (with zero transfers at $t = 0$) of $1 + (1-\phi)^{\frac{\sigma-1}{\epsilon-1}} \mu \phi \frac{1-\beta}{\beta}$ instead of 1.

For present purposes, it is useful to also allow the central bank to adjust the interest rate upwards, but not downwards.

Proposition 6. *Suppose the interest rate is set at $r_0 = 1/\beta - 1$ or is raised to obtain full employment if possible. Suppose condition (18) is satisfied, so there is a demand deficiency at zero transfers. There exist two cutoffs ρ', ρ'' that satisfy $\rho' \in (0,1)$ and $\rho' \leq \rho''$ (with strict inequality if $\zeta > 0$) such that:*

1. *Aggregate output Y_0 is increasing in ρ for $\rho < \rho'$.*
2. *Aggregate output Y_0 is constant at the complete markets equilibrium level for $\rho \geq \rho'$.*
3. *The economy reaches full insurance when $\rho = \rho''$; moreover, $\rho'' \leq 1$ for $\zeta = \phi$.*
4. *The optimum for a utilitarian planner is to set $\rho = \rho''$.*

In a nutshell, this proposition states that targeted transfers play a dual role, with no tradeoff. First, they get us closer to full employment, sometimes all the way to full employment. Second, they also remove intertemporal inefficiencies due to borrowing constraints, and can take us all the way to full insurance. Moreover, a utilitarian planner would always prefer full insurance, precisely because of the "double whammy": it offers maximal macro stimulus while at the same time giving the best interpersonal micro allocation.

Despite the virtues of full insurance, the proposition also clarifies that in order to minimize the output gap it is sufficient to have replacement rates lower than full insurance and lower than 100%. In other words, if the only concern is restoring full employment, then lower levels of transfers will do. One reason for this result is that although agents may be constrained, the natural allocation also calls for them to consume less than previously. Thus, full 100% replacement rates are not required.

To unpack what is behind this proposition, it is useful to consider two cases separately.

First, suppose $\sigma > \epsilon$. There exists a threshold $\tilde{\rho}$ such that for $\rho \geq \tilde{\rho}$ no agent is constrained in equilibrium. Given that our model (Gorman) aggregates, we must then reach the complete markets aggregate outcome. Since $\sigma > \epsilon$ and $1 + r_0 = 1/\beta$, Proposition 3 implies that we are below full employment. Lowering transfers below $\tilde{\rho}$ will reduce aggregate employment, but going above the threshold will not increase it further. Thus, in this case $\rho' = \tilde{\rho}$. Note that increasing in ρ above ρ' does not increase aggregate output, but affects the individual distribution of consumption.

Next, consider the case $\sigma < \epsilon$. In this case, there is still a threshold $\tilde{\rho}$ such that for $\rho \geq \tilde{\rho}$ no one is constrained and the economy reaches the complete markets aggregate allocation.

However, now at $\rho = \tilde{\rho}$ the central bank must be setting the real rate at its natural level above $1/\beta - 1$, due to Proposition 3. Therefore, if we keep $r_0 = 1/\beta - 1$, at $\rho = 0$ there is a demand deficiency and at $\rho = \tilde{\rho}$ there is excess demand. A continuity argument implies that there is a $\rho \in (0, \tilde{\rho})$ at which the economy reaches full employment. The cutoff ρ' is precisely that value.

Finally, note that in the benchmark case where the taxes to finance the debt are equalized for both groups, full insurance requires a replacement rate of 100%, $\rho = 1$, if we are at full employment. However, the full insurance replacement rate is lower if full employment is not achievable, in the case $\sigma > \epsilon$. Moreover, even when full employment is possible, the lowest replacement rate that reaches full employment is below 100%. More generally, the maximal stimulus is attained at a replacement rate below 100%.

To see why this is the case, suppose full employment is feasible and suppose equal taxes, $\zeta = \phi$. At full insurance, $\rho = 1$, both A and B workers are consuming the same in all periods and have the same income from $t = 1, 2, \ldots$ Thus, they must have the same income at $t = 0$. It follows that both agents are strictly saving in equilibrium. Thus, we can lower the replacement rate (and lower taxes) and, although savings may fall for group B, no agents will become borrowing constrained.

One can see that if $\zeta = 0$, then even the full insurance replacement rate is below 100%, so that $\rho'' < 1$ and $\rho' < \rho''$ just reinforces this conclusion for ρ'. For $\zeta > \phi$, on the other hand, the full insurance replacement rate is above 100%. The only way to provide a net transfer to A workers that makes them whole, if they are paying most of the taxes, is to provide them with a replacement rate above 100%.[20] However, even in that case we have $\rho' < 1$. Indeed, when $\zeta \to 1$ the government is basically providing a loan to A workers. As a result, the replacement rate is below $1 - \phi$.

5 Labor Hoarding vs Job Match Destruction

We now turn to an extension in which we model how firms and workers match to produce output. This allows us to discuss how firms in sector A can provide insurance by retaining workers on the payroll even if they are temporarily unproductive. Moreover, it allows us to discuss potential medium run effects of the shutdown, if destroyed matches are hard to rebuild.[21]

[20] As $\zeta \to 1$, the full insurance replacement rate diverges, $\rho'' \to \infty$. In the limit, as A workers pay all the taxes, they require an ever expanding transfer, that is mostly saved to pay ever increasing taxes, so that B workers contribute some given amount with their vanishing fraction of taxes.

[21] A model that also emphasizes the medium-run effects of job destructions is Gregory et al. (2020).

5.1 Generalizing the Model

For this extension, we generalize the model of the previous sections, allowing for a continuum of monopolistically competitive firms to produce each sector's output,

$$Y_{At} = \left(\int_0^{\phi} y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \qquad Y_{Bt} = \left(\int_{\phi}^{1} y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

Each firm i is matched with a (representative) worker i who works in that sector. Thus, there are ϕ firms and workers in sector A and $1-\phi$ firms and workers in sector B.

Due to monopolistic competition, each firm charges a markup $\frac{1}{\epsilon-1}$ over marginal cost, so that the real wage is given by

$$w = 1 - \epsilon^{-1}$$

Moreover, firms are subject to the following friction: they can either pay workers for their full hours $\overline{n}$, even if the hours they actually work n_{it} is below $\overline{n}$, or not at all. When a firm decides not to pay and thus lay off their worker, we assume that it loses some future match value with the worker, as replacing the worker costs time. For tractability, we assume that the firm loses all its future match value, and is immediately replaced by an entrant in the next period, hiring the laid off worker.

This implies that the value of firm i is given by

$$V_{it} = \max \left\{ y_{it} - w\overline{n} + \frac{1}{1+r_t} V_{it+1}, 0 \right\}$$

In words, firm i chooses to lay off its worker when its continuation value is below zero, but otherwise keeps paying its worker in full. In steady state, the value of a firm is the same for all firms and given by

$$V^* = \frac{\epsilon^{-1}\overline{n}}{1-\beta}$$

We assume all firms are owned by unconstrained workers.

We explore this model in two steps. In this section, we focus on the behavior of firms in sector A, and accordingly assume that sector B firms never shut down, which is satisfied if demand in sector B is close to its steady state value. In the next section, we allow for firms in sector B to shut down, as well. We focus once more on a shock that stops all economic activity in sector A.

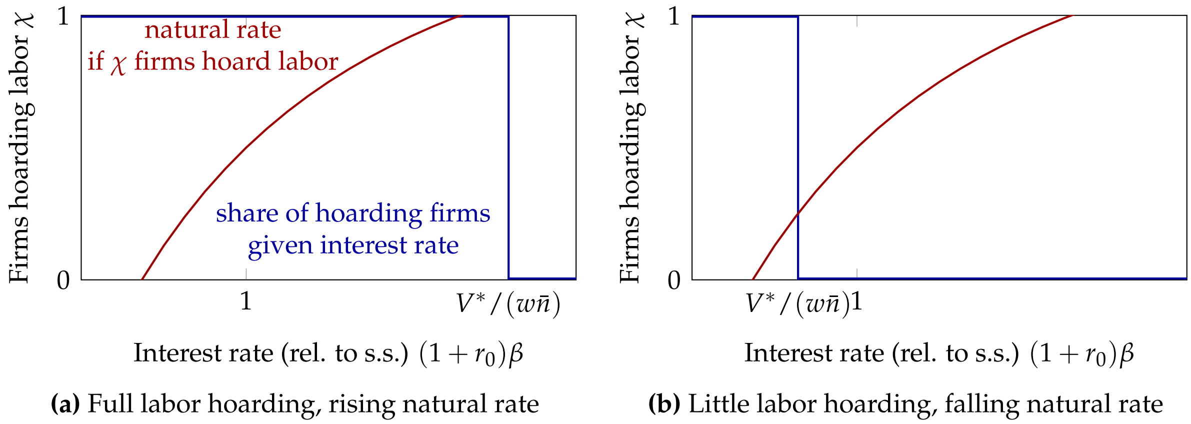


Figure 7: Endogenous labor hoarding

5.2 Insurance Through Labor Hoarding

In this model, firms in sector A may decide to hoard their workers to preserve future match values, and thus implicitly provide insurance to their workers. In particular, this happens when the firm value is positive despite zero current output, that is,

$$V_0 = -w\bar{n} + \frac{1}{1+r_0}V^* > 0. \tag{26}$$

When, instead, V_0 is negative, workers lose their jobs in sector A, as we have previously assumed. Finally, when V_0 is equal to zero, an arbitrary share $\chi \in [0,1]$ of firms lay off their workers, where χ will be determined in equilibrium.

Insurance provided to sector A workers also alters the real interest rate r_0. In particular, following similar steps in those in Section 3, we find that with χ firms hoarding, the natural rate is given by

$$1 + r_0 = \beta^{-1}(1-\phi)^{\frac{1}{\sigma}\frac{\sigma-\epsilon}{\epsilon-1}}\,(1-w\mu\phi)^{\frac{1}{\sigma}}\underbrace{\left(\frac{1-\phi}{1-\phi-\chi w\mu\phi}\right)^{\frac{1}{\sigma}}}_{>1} \tag{27}$$

Except for the last term, the expression is virtually identical to (17). The last term is greater than 1 whenever $\chi > 0$, capturing the fact that labor hoarding not only acts as insurance for sector A workers, but also stimulates demand. As illustrated in Figure 7, the intersection (26) and (27) describes the equilibrium interest rate r_0 and the equilibrium share of labor hoarding firms in the economy. One can distinguish two possible cases.

Panel (a) shows a situation where at the natural interest rate, firms are fully hoarding labor. In this case, a monetary authority only needs to implement the natural rate; insurance will happen endogenously, without fiscal policy, due to labor hoarding.

Panel (b) shows a situation where firms are not all hoarding labor at the natural interest rate. In that case, there is a role for monetary policy to set the interest rate below the natural rate, not for output stabilization, but for insurance purposes.

5.3 Liquidity Problems and Policy Proposals

What happens if firms are liquidity constrained? If firms have some finite amount of liquidity at their disposal, say, because they cannot borrow nor issue equity and have limited past accumulated profits at their disposal, then they no longer maximize the present value of profits in an unconstrained fashion. This distorts firm decisions towards laying workers off, since the current period loss cannot be financed.

In this case, policies that directly affect the liquidity of firms or that insure firms for their loss in revenue, may restore the preferable outcome. In the model, this could be accomplished by a transfer to firms. In practice, these policies could be implemented in a number of ways and through a combination of fiscal and monetary branches of the government.

This discussion lends support to policy proposals at at the outset of the economic crises in March 2020 generated by the COVID-19 pandemic in the US and Europe. For example, Hamilton and Veuger (2020) propose emergency loans for small and medium sized firms most affected by liquidity problems facilitated by the Fed, complimented with tax credits on the fiscal side. Saez and Zucman (2020) propose an ambitious insurance policy, or "buyer of last resort", whereby the government makes up for any loss in revenue by in effect buying up the missing demand. Hanson et al. (2020) propose a "business continuity insurance" policy, providing financial support to businesses to meet fixed obligations. Through the lens of our model, this would help businesses remain alive and able to hire workers.

Even some policy proposals aimed at paying workers directly, through unemployment benefits, emphasize the importance of preserving matches. For example, Dube (2020) calls for incentivizing temporary layoffs, so called furloughs, and the use of worker-sharing provisions, to keeps workers on payroll and allows workers to return easily after the shutdowns.[22]

[22] Giupponi and Landais (2018) provide some evidence on related policies in Europe and study optimal policy in a model of labor hoarding and work-sharing.

5.4 Slow Recoveries

What happens if workers that are let go at $t = 0$ cannot immediately find a new job at $t = 1$? To be concrete, consider a simple case: suppose no matches can be created at $t = 1$, but they can be costlessly created at $t = 2$. Then, if the interest rate is not low enough to induce hoarding, the economy will suffer a recession over both periods $t = 0, 1$, effectively prolonging the duration of the shock. In period $t = 0$ the shock is exogenous, but in period $t = 1$ it results from the loss of job matches. Through an expectations channel, this may also make the recession at $t = 0$ deeper.

The assumption that matches cannot be created at $t = 1$ but can be costlessly recreated at $t = 2$ is extreme, but we expect similar conclusions in a more elaborate model of search and vacancies, where job matches are created in a costly and incremental manner over time.

6 Business Exit Cascades

In the previous section we studied the incentives of businesses in sector A to keep workers employed. Now we shift our focus on sector B and ask what determines the incentives of businesses in that sector to keep employing workers and to remain open, instead of shutting down and laying off workers. Importantly, if businesses shut down, this reduces demand for all other open businesses, increasing their incentive to shut down as well.

We continue to work with the disaggregated firm-level perspective from the previous section: both sectors consist of a continuum of firms, each employing one worker, who can be laid off at the expense of future surplus.

We make two changes relative to the previous section. First, we fix the real interest rate $1 + r_0$, for convenience at the steady state value of β^{-1}, focusing on the implications for aggregate demand Y_{B0} in sector B.

Second, in order to model businesses exits in a more continuous fashion, we assume that there is a distribution of outstanding (net) liabilities across firms which we denote by ζ. In particular, the value of a firm i is then given by

$$V_{i0} = y_0 - w\overline{n} - \zeta_i + \frac{1}{1 + r_0} V^* \tag{28}$$

where $y_0 = Y_{B0}/(1 - \phi)$ is the demand per business in sector B. We assume each firm's net liability ζ_i lies within an interval $[\underline{\zeta}, \overline{\zeta}]$ and is distributed among sector B firms according to a cdf $F\left(\frac{\zeta-\underline{\zeta}}{\overline{\zeta}-\underline{\zeta}}\right)$ where F is a cdf on $[0,1]$. The boundaries $\underline{\zeta}$ and $\overline{\zeta}$ are chosen such that, for simplicity, all firms close when they face zero demand $y_0 = 0$,

$$-w\overline{n} - \underline{\zeta} + \frac{1}{1+r_0}V^* = 0$$

and all firms open with steady state demand $y_0^* = \overline{n}$,

$$(1-w)\overline{n} - \overline{\zeta} + \frac{1}{1+r_0}V^* = 0.$$

6.1 The Business Exit Multiplier

This structure implies that there is now an endogenous mass of firms that is active in the economy, which we denote by $1-\Phi$. Previously, Φ was equal to ϕ, as only sector A was shut down. In this section, however, the share of active firms depends on demand for sector B goods,

$$1 - \Phi = (1-\phi)F\left(\frac{Y_{B0}}{Y_{B0}^*}\right) \tag{29}$$

But, as we have shown in the previous sections, demand for sector B goods precisely depends on the share of open businesses. In particular, a mass Φ of workers will be out of work in equilibrium. Based on the Euler equation underlying (27) (with zero labor hoarding among closed businesses) we find

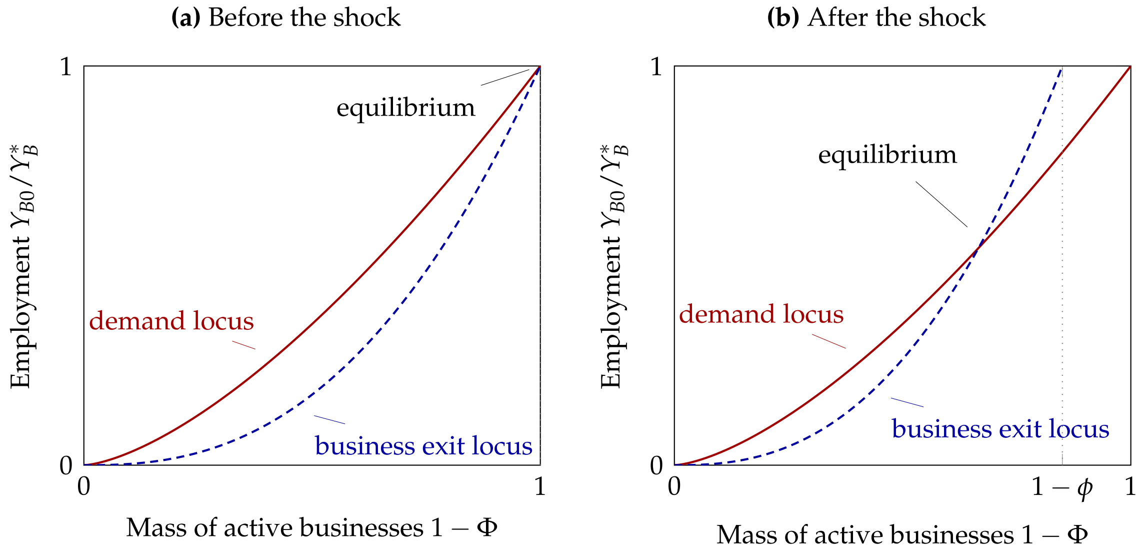
$$\frac{Y_{B0}}{Y_B^*} = (1-\Phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}\left(1 - w\mu\Phi\right) \tag{30}$$

Under the condition for Keynesian supply shocks, (18) (here with $w\mu$ instead of μ) this is decreasing in Φ, capturing that demand falls with more businesses exits.

The relationship between (29) and (30) is illustrated in Figure 8. The horizontal axis represents the mass of active businesses $1-\Phi$. The vertical axis represents the demand relative to potential Y_{B0}/Y_B^*. Under the condition for Keynesian supply shocks, both (29) and (30) describe positively sloped curves in Figure 8. We call (30) the "demand locus", as it describes the demand Y_{B0} for sector B goods, taking as given how many businesses are still open and workers employed, $1-\Phi$. We call (29) the "(business) exit locus" as it describes the mass of businesses exiting and workers laid off given demand Y_{B0}.

When there is no shock, $\phi = 0$, the two curves necessarily intersect at coordinates $1-\Phi = 1$ and $Y_{B0}/Y_B^* = 1$ (Panel a). However, a positive $\phi > 0$ shifts the exit locus to the

Figure 8: The business exit multiplier



left (Panel b). Interestingly, this shift raises the mass of inactive businesses by more than just ϕ, as additional workers laid off by exiting businesses also stop consuming. There is a cascade of business exits and lay-offs that generates a "business exit multiplier".[23]

6.2 Policies

We use the framework laid out here to discuss the effectiveness of two policies.

Profit subsidy / employer-side payroll tax cut. The first policy is a profit subsidy, which, as in Section 4, we assume is paid for by employed agents. The subsidy raises profits by $1+\tau$ for some $\tau > 0$. In our model, this is similar to an employer-side cut in payroll taxes. Such a subsidy enters the business exit locus (29), modifying it to

$$1-\Phi = (1-\phi)\, F\left((1+\tau)\frac{Y_{B0}}{Y_B^*}\right)$$

and thus shifting the locus to the right. This mitigates some of the consequences of the shock, both in terms of business activity and demand.

[23] Figure 8 shows that one can easily get multiple equilibria in this setting, when both curves intersect multiple times. We plan to investigate this case in future research.

Monetary policy. We model monetary policy as a change in the real interest rate $1 + r_0$ away from $1/\beta$. This clearly affects the demand locus (30) through the Euler equation. In particular, we have that

$$\frac{Y_{B0}}{Y_B^*} = (1 - \Phi)^{\frac{\sigma-\epsilon}{\epsilon-1}} (1 - w\mu\Phi) \left(\beta(1 + r_0)\right)^{-1/\sigma}.$$

Accommodative monetary policy shifts the demand locus up, thereby also reducing the number of business exits in the economy.

Aside from the standard intertemporal substitution channel, however, there is another transmission mechanism that is active here. In particular, observe that, just like in Section 5, the future value of a match is discounted less now in (28), encouraging firms to hoard labor. This modifies the exit locus to

$$1 - \Phi = (1 - \phi) F \left(\frac{Y_{B0}}{Y_B^*} + \left(\frac{1}{\beta (1 + r_0)} - 1 \right) \frac{\beta V^*}{Y_B^*} \right).$$

It shows that monetary policy shifts the exit locus to the right, contributing to the recovery. Observe that this channel of monetary policy depends on the future value V^* of employment relationships. Viewed through this lens, the channel might have important distributional consequences, encouraging labor hoarding particularly for workers with a large future match surplus. We leave such an exploration for future work.

7 Optimal Combined Shutdown and Macro Policy

Up to now we have taken the supply shock as given, just assuming certain sectors are temporarily shut down. We now consider a setting where we model more explicitly health concerns in a pandemic, both at the private and at the social level, and think about optimal policy in terms of Pigouvian interventions and macro stabilization.

To this end, let us modify the consumers' objective function to include a health component. To keep things simple, assume the health component is additive and does not directly affect the consumers' capacity to work. In particular, we modify the two sector model of Section 3, introducing the utility function

$$\sum_{t=0}^{\infty} \beta^t \left(U \left(c_{At}, c_{Bt} \right) + h_t \right),$$

where

$$h_t = H\left(c_{At}, n_{At}, Y_{At}, \xi_t\right)$$

is the consumer's health. The parameter ξ_t is the underlying shock and can take two values: $\underline{\xi}$ in normal times and $\overline{\xi}$ when there is an ongoing epidemic. When $\xi_t = \underline{\xi}$, the function H is just a constant. When $\xi_t = \overline{\xi}$, the function H is decreasing in c_{At}, n_{At} and Y_{At}. Agents have a higher probability of being infected if they consume or work more in sector A, and if aggregate activity is higher in that sector. The variables c_{At} and n_{At} are chosen by individual consumers, while the level of activity in sector 1, Y_{At}, is determined in equilibrium and is taken as given by individual consumers. The presence of Y_{At} captures the basic externality of an epidemic: more interactions in sector A cause a faster spread of the epidemic and so increase the probability of being infected for each individual. The rest of the model is identical to that in Section 3.

As in the rest of the paper, the shock is temporary and unexpected, so $\xi_0 = \overline{\xi}$ and $\xi_t = \underline{\xi}$ for $t = 1, 2, ...$ The government has access to two type of policies: a public health policy, in the form of a stay-at-home order which makes production and consumption impossible in sector A; and macroeconomic policies, which include a choice of the real rate r_0 and fiscal policy, as described in Section 4.

To discuss interactions between the two types of policies, we begin by considering partial interventions and their effect, and then we consider the case in which all policies are set optimally. Our results are organized around three remarks. We start with an elementary observation.

Remark 1. Involuntary unemployment is not necessarily socially inefficient in our model.

To make this point, consider an economy in which there is no stay-at-home policy in place, so both sectors are potentially active, despite the shock $\xi_0 = \overline{\xi}$. Even absent containment policies, private motives will still induce a contraction in activity in sector A, as people try to avoid contagion by reducing consumption and labor supply. This contraction in activity may result in involuntary unemployment in sector A. To show that in a simple case, consider the complete market economy with nominal wage rigidities. Suppose $\sigma = \epsilon$ and suppose the central bank keeps the interest rate unchanged, so sector B is at full employment and $Y_{B0} = (1 - \phi)\, \bar{n}$. In equilibrium there is unemployment in sector A, that is,

$$c_{A0} = Y_{A0} < \phi \overline{n},$$

if the following two conditions are satisfied

$$U_{c_A}\left(Y_{A0},(1-\phi)\,\bar{n}\right)+H_{c_A}\left(Y_{A0},Y_{A0},Y_{A0},\overline{\xi}\right)=U_{c_A}\left(c_1^*,c_2^*\right),$$

and

$$U_{c_A}\left(Y_{A0},(1-\phi)\,\bar{n}\right)+H_{c_A}\left(Y_{A0},Y_{A0},Y_{A0},\overline{\xi}\right)+H_{n_A}\left(Y_{A0},Y_{A0},Y_{A0},\overline{\xi}\right)>0. \tag{31}$$

The first condition is the Euler equation in terms of good A and it implies $Y_{A0}<\phi\overline{n}$ because consumers try to avoid consuming good A due to $H_{c_A}<0$. The second condition is the optimality condition for labor supply and implies that it is optimal for the consumers to supply $n_{A0}=\bar{n}$ as the private benefit from consumption, captured by the first two terms, exceeds the private cost of working, captured by the last term. The expression (31) can be interpreted as a Keynesian wedge, as the only disutility from work in our model comes from health costs.

Once we take into account the public health side of the problem, the presence of unemployment may not be socially inefficient, as agents do not internalize the externality in H. That is, it is possible that the two conditions above are satisfied and, at the same time,

$$U_{c_A}\left(Y_{A0},(1-\phi)\,\bar{n}\right)+H_{c_A}\left(Y_{A0},Y_{A0},Y_{A0},\overline{\xi}\right)+H_{n_A}\left(Y_{A0},Y_{A0},Y_{A0},\overline{\xi}\right)+H_{Y_A}\left(Y_{A0},Y_{A0},Y_{A0},\overline{\xi}\right)<0. \tag{32}$$

The last inequality implies that reducing activity in sector A below Y_{A0} increases social welfare. If the only policy tool available is the monetary policy rate r_0, increasing r_0 above $1/\beta-1$ increases social welfare, even though it further depresses employment in sector A and causes unemployment in sector B. The reason is that the Keynesian wedge, captured by the first three terms in (32), is more than compensated by the Pigouvian wedge, captured by the last term. This point is related to the general observation made in one-sector macro-epidemiological models like Eichenbaum et al. (2020), that reducing total activity can be socially desirable in order to slow down infections.

In the example above, there is a trade-off between public health objectives and aggregate demand stabilization. That happens in an example in which there are is no stay-at-home policy in place. Once such policy is allowed, are the social welfare benefits of macro stabilization smaller or larger? That is, are the public health policies and macro policies complements or substitutes? The next remark shows that in our context they are complements.

As in the rest of the paper, assume that under a stay-at-home policy sector A is completely shut down. Then the equilibrium analysis of Section 3 applies, as h_t is a constant for individual optimization, because either $\xi_t = \underline{\xi}$, or $\xi_t = \overline{\xi}$ with $c_{A0} = n_{A0} = Y_{A0} = 0$.

Remark 2. Under the conditions of Propositions 3 and 4 there are complementarities between public health policies and aggregate demand stabilization.

The basic reason for this remark is that public health policies can produce a Keynesian supply shock and macro policies can be helpful to correct the effects of the latter.

Consider again the example above and now suppose the government shuts down sector A. Consider the complete market economy, with nominal rigidities. Suppose that $\sigma > \epsilon$, so we have an inefficient recession in sector A. Lowering r_0 (assuming we do not hit the ZLB) allows the government to reach a first best efficient allocation if the following condition is satisfied at the full employment allocation

$$U_{c_A}\left(0,(1-\phi)\,\bar{n}\right) + H_{c_A}\left(0,0,0,\overline{\xi}\right) + H_{n_A}\left(0,0,0,\overline{\xi}\right) + H_{Y_A}\left(0,0,0,\overline{\xi}\right) < 0. \tag{33}$$

This condition implies that a corner solution with a complete shutdown of sector A, combined with an appropriately expansive monetary policy to maintain full employment in sector B, reach a socially efficient outcome. Since activity in sector B has not negative effects on public health, it is optimal to keep that sector at full employment. At the same time, (33) implies that the public health benefits of keeping sector A closed are large enough, given the shock $\overline{\xi}$, that it is optimal to have zero activity in that sector.

What happens when markets are incomplete? Now the social planner has to take into account three possible sources of inefficiency: inefficiency due to lack of insurance, inefficiency due to the public health externality, inefficiency due to involuntary unemployment. The next remark shows that if the government has sufficient tools it can deal with all of them and restore first best efficiency. In discussing the remark, we will show that again there are relevant complementarities between the tools used. In particular, social insurance policies that help relieve the first inefficiency can ameliorate the dilemma between the other two.

Remark 3. In the incomplete markets economy, a combination of public health policies, social insurance policies, and monetary policy can achieve the first best for a utilitarian social planner.

For this example, we need the incomplete market version of the two sector model, with nominal wage rigidities. Suppose that parameters are such that a stay-at-home order produces a Keynesian supply shock, so there is inefficient unemployment in sector B. Suppose also that monetary policy is constrained by a ZLB constraint and suppose that this constraint is binding in equilibrium. Suppose also that the ZLB constraint is not binding with complete markets. We know such a configuration is possible possible by Proposition 4.

Consider first what happens if the only policy tools available are a stay-at-home order and monetary policy, and monetary policy is at the ZLB. Suppose we can relax slightly the containment policy and increase output in sector A by dY_A. Consider the marginal benefit of this increase, for a utilitarian social planner. The effects on the consumption component of utility is

$$\int_0^1 \left[U_{c_A}\left(0, c_{iB0}\right) \partial c_{iA0} + U_{c_B}\left(0, c_{iB0}\right) \partial c_{iB0}\right] di$$

where ∂c_{ij0} is the effect of dY_A on the consumption of consumer i of good j, in general equilibrium. This effect can be large when the fraction of constrained agents μ is large for two reasons: for distributional reasons, as there are consumers with zero consumption of both goods who gain from being able to earn income in sector A, and for the presence of inefficient involuntary unemployment in sector B. Hence it is possible that relaxing the containment policy may be desirable, as a second best way of correcting these two inefficiencies.

Consider next what happens if the government can also use the transfers T_{A0}, T_{B0} introduced in Section 4. Namely, the government reallocates income from sector B workers to sector A workers, so as to equalize their after-transfer incomes. This policy is clearly desirable from an insurance viewpoint. Moreover, given this policy, the constraint on monetary policy is no longer binding, as we are now effectively in the complete markets economy. And finally, if condition (33) is satisfied, a complete shut down of Sector A is now optimal.

In the example just discussed, there is a combination of policies that achieves the first best allocation: a stay-at-home policy that shuts down sector A, a social insurance policy that compensates the workers in sector A, and a monetary policy that hits the natural rate. The fact that the social insurance policy makes it easier to achieve the demand stabilization objective is not surprising per se: it is an example of a fiscal policy that makes it easier to do monetary policy. The novel observation is that this type of fiscal policy also makes it less costly for the government to impose a larger supply shock on the economy, that is, it makes it easier to pursue public health objectives.

8 Concluding Discussion

The contribution of this paper has been to explore the conditions under which a shock to the (safe) production of goods—which we call "supply shock" for simplicity—in some sectors of the economy spills over into an inefficient demand shock in other sectors—a "Keynesian supply shock". We do not attempt to take a hard stand on whether these conditions are met. Indeed, our analysis lays out the land and is useful for the debate regardless of which case applies. That said, the conditions do appear plausible in some scenarios.

Our analysis and results transcend the ongoing pandemic, but a natural question to ask is whether the COVID-19 shock had features similar to those of a Keynesian supply shock. We do not have a definite answer to this question, but offer a few points. At a broad macro level, the CPI has been falling for three months, despite monetary easing. Meanwhile, real-time data is accumulating on the spending and employment patters across sectors and households. Our very tentative reading of this evidence is that it is broadly consistent with mechanisms emphasized in our theory, such as the complementarity generating co-movement and the importance of the income channel, as present in our incomplete markets cases. For example, Cajner et al. (2020) documents employment losses in almost every sector of the economy, even in non-contact intensive ones (Dingel and Neiman, 2020). The income channel has been relatively muted, due to the introduction of unprecedented income-support policies (e.g. Ganong et al., 2020). However, the time series patterns for spending are suggestive of a responses to stimulus checks (Chetty et al., 2020; Cox et al., 2020).[24,25] Less is known for now on the repercussions of the pandemic to businesses, another aspect of our analysis.[26] Liquidity programs for businesses may have also muted the shock to firms.

Our paper emphasizes that these policies mitigate or reverse demand deficiencies that would be present without them. In this way it provides foundations for the strong policy actions undertaken. In any case, we intend to explore these and other questions with the help of this developing evidence in greater depth in future work.

[24]Our simple demand system has homothetic preferences and, thus, abstracts from luxuries vs. essentials which explains the larger drop in spending by the rich during the pandemic. A straightforward extension of our framework can acknowledge this fact, without affecting our main results.

[25]Chetty et al. (2020) provide some suggestive evidence for an income channel, showing that workers that work in higher income areas, were hit more by the reduction in non-essential spending, and these workers had a greater reduction in spending than workers of similar income working in other areas. A vast pre-pandemic literature estimates and surveys MPCs (see e.g. Johnson et al. 2006).

[26]Bartik et al. (2020) provide an early survey of small businesses.

References

Alvarez, Fernando, David Argente, and Francesco Lippi, "A Simple Planning Problem for COVID-19 Lockdown," March 2020. mimeo.

Andersen, Asger Lau, Emil Toft Hansen, Niels Johannesen, and Adam Sheridan, "Consumer responses to the COVID-19 crisis: Evidence from bank account transaction data," *Available at SSRN 3609814*, 2020.

Atkeson, Andrew, "What Will Be the Economic Impact of COVID-19 in the US? Rough Estimates of Disease Scenarios," Working Paper 26867, National Bureau of Economic Research March 2020.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub, "The intertemporal keynesian cross," Technical Report, National Bureau of Economic Research 2018.

Baldwin, Richard and Beatrice Weder di Mauro, *Economics in the Time of COVID-19*,, VoxEU CEPR Press, 2020.

Balleer, Almut, Peter Zorn, Sebastian Link, and Manuel Menkhoff, "Demand or Supply? Price Adjustment during the Covid-19 Pandemic," 2020.

Baqaee, David Rezza and Emmanuel Farhi, "Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis," 2020.

Bartik, Alexander W, Marianne Bertrand, Zoë B Cullen, Edward L Glaeser, Michael Luca, and Christopher T Stanton, "How Are Small Businesses Adjusting to COVID-19? Early Evidence from a Survey," Working Paper 26989, National Bureau of Economic Research April 2020.

Bayer, Christian, Benjamin Born, Ralph Luetticke, and Gernot J Müller, "The Coronavirus Stimulus Package: How large is the transfer multiplier?," 2020.

Beaudry, Paul and Franck Portier, "Understanding noninflationary demand-driven business cycles," *NBER Macroeconomics Annual*, 2014, *28* (1), 69–130.

Berger, David, Kyle Herkenhoff, and Simon Mongey, "An SEIR Infectious Disease Model with Testing and Conditional Quarantine," Working Paper 2020-25, Becker Friedman Institute for Economics 2020.

Bilbiie, Florin O, "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," *Journal of economic theory*, 2008, *140* (1), 162–196.

_ , "The new Keynesian cross," *Journal of Monetary Economics*, 2019.

Brinca, Pedro, Joao B Duarte, and Miguel Faria e Castro, "Measuring Sectoral Supply and Demand Shocks during COVID-19," *FRB St. Louis Working Paper*, 2020, (2020-011).

Caballero, Ricardo J and Alp Simsek, "A Model of Asset Price Spirals and Aggregate Demand Amplification of a" Covid-19" Shock," Technical Report, National Bureau of Economic Research 2020.

Cajner, Tomaz, Leland D Crane, Ryan A Decker, John Grigsby, Adrian Hamins-Puertolas, Erik Hurst, Christopher Kurz, and Ahu Yildirmaz, "The US labor market during the beginning of the pandemic recession," Technical Report, National Bureau of Economic Research 2020.

Carvalho, Vasco M, Stephen Hansen, Álvaro Ortiz, Juan Ramon Garcia, Tomasa Rodrigo, Sevi Rodriguez Mora, and Pep Ruiz de Aguirre, "Tracking the Covid-19 crisis with high-resolution transaction data," 2020.

Challe, Edouard, "Uninsured unemployment risk and optimal monetary policy in a zero-liquidity economy," *American Economic Journal: Macroeconomics*, 2020, *12* (2), 241–83.

Chetty, Raj, John N Friedman, Nathaniel Hendren, and Michael Stepner, "Real-Time Economics: A New Platform to Track the Impacts of COVID-19 on People, Businesses, and Communities Using Private Sector Data," Technical Report, Mimeo 2020.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc, "International risk sharing and the transmission of productivity shocks," *The Review of Economic Studies*, 2008, *75* (2), 443–473.

Cox, Natalie, Peter Ganong, Pascal Noel, Joseph Vavra, Arlene Wong, Diana Farrell, and Fiona Greig, "Initial Impacts of the Pandemic on Consumer Behavior: Evidence from Linked Income, Spending, and Savings Data," 2020.

Danieli, Ana and Jane Olmstead-Rumsey, "Sector-specific shocks and the expenditure elasticity channel during the covid-19 crisis," *Available at SSRN 3593514*, 2020.

Dingel, Jonathan I and Brent Neiman, "How many jobs can be done at home?," Technical Report, National Bureau of Economic Research 2020.

Dube, Arin, “Filling the Holes in Family and Business Budgets: Unemployment Benefits and Work Sharing in the Time of Pandemics,” *Economics for Inclusive Prosperity*, March 2020, (24).

Eggertsson, Gauti B, “The paradox of toil,” Staff Report 433, FRB of New York 2010.

Eichenbaum, Martin S, Sergio Rebelo, and Mathias Trabandt, “The Macroeconomics of Epidemics,” Working Paper 26882, National Bureau of Economic Research March 2020.

Farhi, Emmanuel and Iván Werning, “Fiscal multipliers: Liquidity traps and currency unions,” in “Handbook of Macroeconomics,” Vol. 2, Elsevier, 2016, pp. 2417–2492.

Faria e Castro, Miguel, “Fiscal Multipliers and Financial Crises,” Working Papers 2018-23, Federal Reserve Bank of St. Louis October 2018.

—, “Fiscal Policy during a Pandemic,” Working Papers 2020-006, Federal Reserve Bank of St. Louis March 2020.

Fornaro, Luca and Martin Wolf, “Covid-19 Coronavirus and Macroeconomic Policy,” Technical Report, CEPR March 2020.

Galí, Jordi, J David López-Salido, and Javier Vallés, “Understanding the effects of government spending on consumption,” *Journal of the european economic association*, 2007, *5* (1), 227–270.

Ganong, Peter, Pascal J Noel, and Joseph S Vavra, “US Unemployment Insurance Replacement Rates During the Pandemic,” Working Paper 27216, National Bureau of Economic Research May 2020.

Gatti, Domenico Delli, Mauro Gallegati, Bruce C Greenwald, Alberto Russo, and Joseph E Stiglitz, “Mobility constraints, productivity trends, and extended crises,” *Journal of Economic Behavior & Organization*, 2012, *83* (3), 375–393.

Giupponi, Giulia and Camille Landais, “Subsidizing Labor Hoarding in Recessions: The Employment & Welfare Effects of Short Time Work,” CEP Discussion Papers dp1585, Centre for Economic Performance, LSE December 2018.

Gopinath, Gita, “Limiting the Economic Fallout of the Coronavirus with Large Targeted Policies,” *IMFblog*, 2020, (March 9).

Gourinchas, Pierre-Olivier, "Flattening the Pandemic and Recession Curves," 2020. mimeo Berkeley University.

Gregory, Victoria, Guido Menzio, and David G Wiczer, "Pandemic Recession: L or V-Shaped?," Technical Report, National Bureau of Economic Research 2020.

Hamilton, Steven and Stan Veuger, "How to Help American Businesses Endure and Jobs Survive," 2020. mimeo George Washington University.

Hanson, Samuel G., Jeremy C. Stein, Adi Sunderam, and Eric Zwick, "Business Continuity Insurance: Keeping America's Lights On During the Pandemic," *Working Paper*, 2020.

Huber, Kilian, "Disentangling the effects of a banking crisis: Evidence from German firms and counties," *American Economic Review*, 2018, *108* (3), 868–98.

Johnson, David S, Jonathan A Parker, and Nicholas S Souleles, "Household expenditure and the income tax rebates of 2001," *American Economic Review*, 2006, *96* (5), 1589–1610.

Jorda, Oscar, Sanjay S. Singh, and Alan M. Taylor, "Longer-run economic consequences of pandemics," March 2020. mimeo.

Kaplan, Greg, Benjamin Moll, and Gianluca Violante, "Pandemics according to HANK," *Powerpoint presentation, LSE*, 2020, *31.*

Matsuyama, Kiminori, "A ricardian model with a continuum of goods under nonhomothetic preferences: Demand complementarities, income distribution, and north-south trade," *Journal of political Economy*, 2000, *108* (6), 1093–1120.

Mitman, Kurt, Stanislav Rabinovich et al., "Optimal Unemployment Benefits in the Pandemic," Technical Report, Institute of Labor Economics (IZA) 2020.

Murphy, Kevin M, Andrei Shleifer, and Robert W Vishny, "Industrialization and the big push," *Journal of political economy*, 1989, *97* (5), 1003–1026.

Patterson, Christina, "The Matching Multiplier and the Amplification of Recessions," in "2019 Meeting Papers" number 95 Society for Economic Dynamics 2019.

Ravn, Morten O and Vincent Sterk, "Job uncertainty and deep recessions," *Journal of Monetary Economics*, 2017, *90*, 125–141.

Rowe, Nick, "Blog Post: Relative supply shocks, Unobtainium, Walras' Law, and the Coronavirus," *Worthwhile Canadian Initiative A mainly Canadian economics blog*, March 2020.

Saez, Emmanuel and Gabriel Zucman, "Keeping Business Alive: The Government as Buyer of Last Resort," 2020. mimeo Berkeley.

Sahm, Claudia, "U.S. policymakers need to fight the coronavirus now," 2020.

Werning, Iván, "Incomplete markets and aggregate demand," Technical Report, National Bureau of Economic Research 2015.

Woodford, Michael, "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, January 2011, *3* (1), 1–35.

_ , "Pandemic Shocks, Effective Demand, and Stabilization Policy," July 2020. mimeo Columbia University.

A Appendix

A.1 Non-homothetic preferences

This section considers a simple class of non-homothetic preferences that is used for two purposes: to extend, using a limit argument, Propositions3 and 4 to the case $\epsilon \leq 1$; to generalize the analysis in Section 3 and show that Keynesian supply shocks can also arise also with $\sigma < 1$.

For some given small $\underline{c}$ define

$$f(c) = \frac{1}{\epsilon}\underline{c}^{1-\frac{1}{\epsilon}} + \left(1 - \frac{1}{\epsilon}\right)\underline{c}^{-\frac{1}{\epsilon}}c$$

for $c \leq \underline{c}$ and $f(c) = c^{1-\frac{1}{\epsilon}}$ for $c > \underline{c}$. Notice that this function is continuous and differentiable and has derivative $f'(c) = \left(1 - \frac{1}{\epsilon}\right)\underline{c}^{-\frac{1}{\epsilon}}$ if $c < \underline{c}$ and $f'(c) = \left(1 - \frac{1}{\epsilon}\right)c^{-\frac{1}{\epsilon}}$ if $c \geq \underline{c}$.

Let the per-period utility function be

$$U(c_A, c_B) = \frac{\sigma}{\sigma - 1}\left[\phi^{\frac{1}{\epsilon}} f(c_A) + (1-\phi)^{\frac{1}{\epsilon}} f(c_B)\right]^{\frac{\epsilon}{\epsilon-1}\frac{\sigma-1}{\sigma}}.$$

Notice that if c_A and c_B are both above the level $\underline{c}$ the preferences are locally CES. Assume that this is the case in steady state, that is, assume $\underline{c} < \min\{\phi\bar{n}, (1-\phi)\,\bar{n}\}$. However, when $c_A = 0$, after the shock to sector A, we have

$$U_{c_B}(0, c_B) = \frac{\sigma}{\sigma - 1}\left[\phi^{\frac{1}{\epsilon}} \frac{1}{\epsilon} \underline{c}^{1-\frac{1}{\epsilon}} + (1-\phi)^{\frac{1}{\epsilon}} c_B^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}\left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)} (1-\phi)^{\frac{1}{\epsilon}} c_B^{-\frac{1}{\epsilon}}.$$

This expression is well defined for any $\epsilon \in (0, \infty)$.

Consider the economy with incomplete markets. Substituting the full employment levels of consumption in the two sectors in periods 0 and 1 implies that the natural rate is

$$1 + r_0 = \frac{1}{\beta} g(\underline{c}),$$

where

$$g(\underline{c}) \equiv \left(\phi^{\frac{1}{\epsilon}} \frac{1}{\epsilon}\left(\frac{\underline{c}}{\bar{n}}\right)^{1-\frac{1}{\epsilon}} + (1-\phi)(1-\mu\phi)^{\frac{1}{\epsilon}-1}\right)^{\frac{\epsilon}{\epsilon-1}\left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)} (1-\mu\phi)^{\frac{1}{\epsilon}}.$$

We first analyze what happens when $\underline{c} \to 0$. If $\epsilon > 1$ the analysis is as in the main text as $g(\underline{c})$ is well defined at $\underline{c} = 0$. If $\epsilon < 1$ we have

$$\lim_{\underline{c}\to 0}\left(\phi^{\frac{1}{\epsilon}} \frac{1}{\epsilon}\left(\frac{\underline{c}}{\bar{n}}\right)^{1-\frac{1}{\epsilon}} + (1-\phi)(1-\mu\phi)^{\frac{1}{\epsilon}-1}\right)^{\frac{\epsilon}{\epsilon-1}} \to 0,$$

which implies the following result.

Lemma 1. *If $\epsilon < 1$ we have $\lim_{\underline{c}\to 0} g(\underline{c}) = 0$ if $\sigma > \epsilon$ and $\lim_{\underline{c}\to 0} g(\underline{c}) = +\infty$ if $\sigma < \epsilon$.*

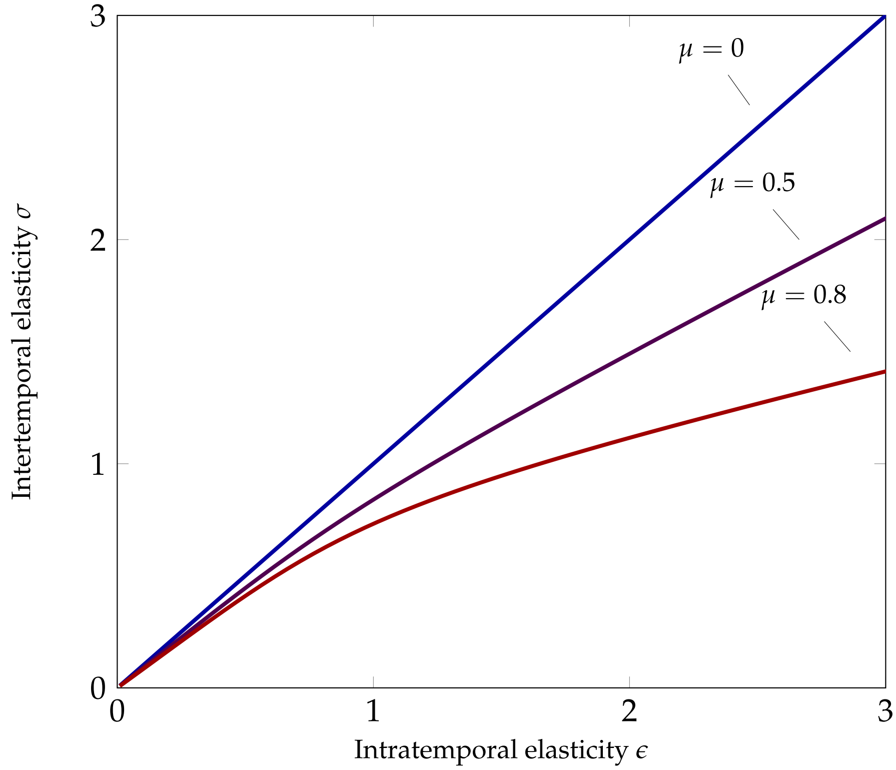
Consider now extending Proposition 3 to the region $\epsilon < 1$. The economy with complete markets yields an expression for the real rate equal to the case with incomplete markets and $\mu = 0$. Using the lemma above then shows that inequality (14) also applies in the case $\epsilon < 1$.

Consider next extending Proposition 4 to the region $\epsilon < 1$. Using the lemma above, when $\epsilon < 1$ the inequality (18) is replaced by $\sigma > \epsilon$, as shown in Figure 3.

Finally, consider the preferences above away from the limit $\underline{c} \to 0$. Figure 9 shows the boundary of the Keynesian supply shock region, analogous to those plotted in Figures 2 and 3, for an example with parameters $\phi = 0.5$ and $\underline{c}/\bar{n} = 0.1$. Given the expressions above, the boundary is given by the pairs ϵ, σ that satisfy

$$\frac{\epsilon}{\epsilon - 1}\left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right)\log\left(\phi^{\frac{1}{\epsilon}} \frac{1}{\epsilon}\left(\frac{\underline{c}}{\bar{n}}\right)^{1-\frac{1}{\epsilon}} + (1-\phi)(1-\mu\phi)^{\frac{1}{\epsilon}-1}\right) + \frac{1}{\epsilon}\log(1-\mu\phi) = 0.$$

Figure 9: When are supply shocks Keynesian? Non homothetic preferences



It is possible to show that the boundary shifts downwards as μ increases, that is, more market incompleteness increases the KSS region, extending the result of Proposition 4. In the figure, this is illustrated by showing the boundary for three values of μ. When $\mu = 0$, we obtain the same result as under complete markets and homothetic preferences and the boundary is just $\sigma = \epsilon$. For larger values of μ the boundary shifts down, and, in particular, it also includes pairs with $\sigma < 1$ and $\rho > 1$.

A.2 Consumption functions

In this section we derive the individual consumption functions used for the plots in Figures 4 and 5, and derive condition (21).

Given consumer preferences the marginal utility of good B is

$$U_{c_B}\left(c_{At}, c_{Bt}\right) = c_t^{\frac{1}{\epsilon} - \frac{1}{\sigma}} \left(1 - \phi\right)^{\frac{1}{\epsilon}} c_{Bt}^{-\frac{1}{\epsilon}},$$

where

$$c_t = \left(\phi^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)^{\frac{1}{\epsilon}} c_{Bt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

For unconstrained consumers the Euler equation with $c_{A0} = 0$ takes the following form

$$c_0^{\frac{1}{\epsilon}-\frac{1}{\sigma}} (1-\phi)^{\frac{1}{\epsilon}} c_{B0}^{-\frac{1}{\epsilon}} = \beta (1+r_0) c_1^{\frac{1}{\epsilon}-\frac{1}{\sigma}} (1-\phi)^{\frac{1}{\epsilon}} c_{B1}^{-\frac{1}{\epsilon}},$$

then using that

$$c_{1B} = (1-\psi) c_1$$

we obtain

$$c_0^{\frac{1}{\epsilon}-\frac{1}{\sigma}} (1-\phi)^{\frac{1}{\epsilon}} c_{B0}^{-\frac{1}{\epsilon}} = \beta (1+r_0) c_1^{-\frac{1}{\sigma}},$$

where

$$c_0 = \left((1-\phi)^{\frac{1}{\epsilon}} c_{B0}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

Taking into account that $c_t = c_1$for $t = 1, 2, \ldots$, the intertemporal budget constraint is

$$c_{B0} + \frac{\beta}{1-\beta} c_1 = y_0 + \frac{\beta}{1-\beta} \bar{n}.$$

where we have also used that $1 + r_0 = 1/\beta$. Solving, we obtain the consumption function

$$c_{B0} = \frac{(1-\beta) y_0 + \beta \bar{n}}{1 - \beta + \beta (1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}. \tag{34}$$

For constrained consumers we can follow similar steps, allowing for the Euler equation to hold as an inequality, and obtain

$$c_{B0} = \min \left\{ y_0, \frac{(1-\beta) y_0 + \beta \bar{n}}{1 - \beta + \beta (1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} \right\}. \tag{35}$$

The consumption functions before the shock can be derived in similar manner and are

$$c_{Bt} = (1-\phi) \left((1-\beta) y_t + \beta \bar{n}\right) \tag{36}$$

for unconstrained consumers, and

$$c_{Bt} = (1-\phi) \min \left\{ y_t, \left((1-\beta) y_t + \beta \bar{n}\right) \right\} \tag{37}$$

for constrained consumers. Notice that in the last expression the factor $(1-\phi)$ appears before the min operator, because before the shock the consumers allocate a fraction ϕ of their spending to good A, whether or not the constraint is binding. Expressions (34)-(37) are used to obtain the plots in Figures 4 and 5.

Suppose now that the income of the consumers in sector B remains at $\bar{n}$. The total change in consumption following the shock is

$$\left(\frac{(1-\beta)(1-\phi)+\beta(1-\mu\phi)}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1-\phi) \right) \bar{n}. \tag{38}$$

Notice that this expression is negative iff condition ((18)) holds.

The expression above can be decomposed in three terms:

1. The shift in the consumption function for given income $\bar{n}$:
$$\left(\frac{1}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1-\phi) \right) \bar{n};$$

2. The change in consumption of the unconstrained consumers hit by the shock, due to the income reduction $(-\bar{n})$:
$$(1-\mu)\phi \left(\frac{1-\beta}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} \right) (-\bar{n});$$

3. The change in consumption of the constrained consumers hit by the shock, due to the income reduction $(-\bar{n})$:
$$\mu\phi \left(\frac{1}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} \right) (-\bar{n}).$$

The marginal propensities to consume are

$$MPC^{S,U} = \frac{1-\beta}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

and

$$MPC^{S,C} = \frac{1}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

for the two groups, with

$$\overline{MPC}^{A} \equiv (1-\mu)\, MPC^{S,U} + \mu MPC^{S,C}.$$

Moreover the reduction in consumption in the A sector is equal to $\phi\bar{n}$, so

$$\left[\frac{\Delta c_B}{\Delta c_A}\right]^{shutdown} = \frac{\left(\frac{1}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1-\phi)\right)\bar{n}}{\phi\bar{n}}.$$

We conclude that the expression in (38) is negative iff

$$\left[\frac{\Delta c_B}{\Delta c_A}\right]^{shutdown} \phi\bar{n} + \mu\phi MPC^{S,U}(-\bar{n}) + (1-\mu)\,\phi MPC^{S,C}(-\bar{n}) < 0$$

which is equivalent to (21) in the main text.

A.3 Proof of Proposition 6

When $1 + r_0 = 1/\beta$, to satisfy the government budget constraint we need the following negative transfers (hence, taxes):

$$T_A = -\zeta\frac{1-\beta}{\beta}T_{A0},$$
$$T_B = -(1-\zeta)\frac{\phi}{1-\phi}\frac{1-\beta}{\beta}T_{A0}.$$

Define the net present value of lifetime transfers to A and B workers:

$$\hat{T}_A = T_{A0} + \frac{\beta}{1-\beta}T_A = (1-\zeta)\,T_{A0},$$
$$\hat{T}_B = -(1-\zeta)\,\frac{\phi}{1-\phi}T_{A0}.$$

From A.2, the consumption function of unconstrained consumers working in sector j is

$$c_{B0} = \frac{(1-\beta)\left(n_{j0} + \hat{T}_j\right) + \beta\bar{n}}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}},$$

where

$$n_{A0} = 0, \quad n_{B0} = \frac{Y_{B0}}{1-\phi}.$$

For constrained consumers it is

$$c_{B0} = \min\left\{ \frac{(1-\beta)\left(n_{j0} + \hat{T}_j\right) + \beta\bar{n}}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}, n_{j0} + T_{j0} \right\}.$$

We can then compute the cutoff $\tilde{T}_{A0}$ at which shocked and constrained consumers shift from a binding to a non-binding constraint. The cutoff satisfies

$$\frac{(1-\beta)(1-\zeta)T_{A0} + \beta\bar{n}}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} = T_{A0},$$

and is thus

$$\tilde{T}_A \equiv \frac{\beta\bar{n}}{\zeta(1-\beta)+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} \leq (1-\phi)^{\frac{\sigma-1}{\epsilon-1}}\bar{n} < \bar{n}. \tag{39}$$

The last inequality follows because we are assuming $\epsilon > 1$ and condition (18) requires $\sigma > 1$.

Define the full insurance transfer

$$T''_A \equiv \frac{Y_B^{CM}}{1-\zeta}$$

where Y_B^{CM} is the equilibrium output under complete markets which is equal to $(1-\phi)^{\frac{\sigma-1}{\epsilon-1}}\bar{n}$ if $\sigma > \epsilon$ and equal to $(1-\phi)\bar{n}$ if $\sigma \leq \epsilon$ because the central bank will raise rates in that case. Notice that if $\zeta = \phi$ we have $T''_A = (1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}\bar{n} < \bar{n}$ if $\sigma > \epsilon$ and $T''_A = \bar{n}$ if $\sigma \leq \epsilon$.

Using the expressions above, it is easy to show that

$$\tilde{T}_A \leq T''_A, . \tag{40}$$

with equality only if $\zeta = 0$.

Consider the case $\sigma \geq \epsilon$. The good market equilibrium condition then takes the following form if $T_{A0} < \tilde{T}_A$

$$Y_{B0} = \mu\phi T_{A0} + \frac{(1-\beta)(Y_{B0} - (1-\zeta)\phi\mu T_{A0}) + \beta(1-\mu\phi)\bar{n}}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}},$$

where the first term on the right-hand side is the demand of shocked and constrained agents and the second term is the demand of all other agents. If $T_{A0} \geq \tilde{T}_A$ the good market equilibrium condition becomes

$$Y_{B0} = \mu\phi \frac{(1-\beta)(1-\zeta) T_{A0} + \beta\bar{n}}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} + \frac{(1-\beta)(Y_{B0} - (1-\zeta)\phi\mu T_{A0}) + \beta(1-\mu\phi)\bar{n}}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

which simplifies to

$$Y_{B0} = \frac{(1-\beta) Y_{B0} + \beta\bar{n}}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

and gives $Y_{B0} = (1-\phi)^{\frac{\sigma-1}{\epsilon-1}} \bar{n} = Y_B^{CM}$ and is independent of T_{A0}.

The case $\sigma < \epsilon$ is similar, except that the economy reaches full employment in sector B when the transfer is

$$T_{A0} = \bar{T}_{A0} \equiv \frac{(1-\phi)^{-\frac{\sigma-\epsilon}{\epsilon-1}} - (1-\mu\phi)}{\zeta(1-\beta)+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} \frac{\beta\bar{n}}{\mu\phi} < \tilde{T}_A.$$

Notice that $\bar{T}_{A0} > 0$ because condition (18) is assumed and the inequality $\bar{T}_{A0} < \tilde{T}_A$ can be easily checked when $\sigma < \epsilon$. Recall that we assumed that whenever there is excess demand the central bank adjusts the interest rate to attain full employment. Therefore, in this case output is increasing in T_{A0} for $T_{A0} < \bar{T}_{A0}$ and is constant at full employment for $T_{A0} \geq \bar{T}_{A0}$. Once the economy is at full employment, changes in transfers in general affect the natural interest rate, but that does not affect the computation of the cutoffs.

We can then define the cutoffs for the replacement rate

$$\rho' = \frac{\min\{\tilde{T}_{A0}, \bar{T}_{A0}\}}{\bar{n}}, \quad \rho'' = \frac{T_A''}{\bar{n}},$$

and check that all the properties in the proposition are satisfied. In particular $\rho' < 1$ follows from (39). Also, $\rho' \leq \rho''$, with equality only if $\zeta = 0$ follows from (40). The cutoff $\tilde{\rho}$ used in the text to explain the argument is simply $\tilde{T}_{A0}/\bar{n}$.

Showing that the economy reaches full insurance at ρ'' is easy because aggregate income is Y_B^{CM} and agents in sector A receive a present value transfer equal to Y_B^{CM} (and their borrowing constraint is not binding). Showing that a utilitarian planner would choose ρ'' is also easy because aggregate output cannot be increased above Y_B^{CM} (given the assumption that the real rate cannot fall below $1/\beta - 1$) and the transfer ρ'' allocates that output equally across agents.