Endogenous Sector–Biased Technological Change and Industrial Policy*

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Abstract

We build a model of structural transformation with endogenous sector–biased technological change. We show that if the return to specialization is larger in the goods sector than in the service sector, then the equilibrium has the following properties: aggregate growth is balanced; the service sector receives more innovation but the goods sector experiences more productivity growth; structural transformation takes place from goods to services. Compared to the efficient allocation the laissez–faire equilibrium has too much labor in the goods sector, implying that optimal industrial policy should aim to *increase*, not decrease, the pace of structural transformation.

Keywords: endogenous sector–biased technological change; horizontal innovation; industrial policy; structural transformation.
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1 Introduction

The reallocation of resources across broad sectors is one of the prominent features of modern economic growth: as economies develop and GDP per capita grows, the employment share of the goods–producing sector decreases and the employment share of the services–producing sector increases. A growing literature has been studying the driving forces and the consequences of this so–called structural transformation.[1] However, this literature has hardly contributed to the debate about whether industrial policies should protect industries in the shrinking goods sector and thereby slow down structural transformation. Many advanced economies have implemented such policies in the form of subsidies to agriculture, car manufacturing, heavy industry, and mining.

The likely reason for the silence of the literature on industrial policy issues is that it employs models in which equilibrium is efficient by construction and from the get go industrial policy can only do damage. The efficiency of equilibrium is owed to several strong assumptions: capital and labor can be reallocated between sectors without any frictions; technological progress at the sector level is exogenous (and so it is unaffected by policy). While relaxing either of these assumptions is potentially fruitful in the context of industrial policy, in this paper we focus on the second one. This choice is motivated by the fact that only the second assumption has implications for the long–run horizon that is typically the focus of studies on structural transformation.

We build a model of structural transformation in which technological progress is endogenous at the sector level and competition is imperfect. These features open the door for the possibility that industrial policy may bring about Pareto improvements. Our environment features innovators who optimally choose whether to innovate in the goods sector or in the service sector. We assume that innovation is equally costly in both sectors, that innovation is horizontal (i.e., takes the form of creating new varieties that are used as intermediate inputs either in the goods or the service sector), and that there is free entry into innovation. An innovator who creates a new variety needs to earn monopoly profits to recoup his innovation costs. We assume that innovators become monopolist producers for their varieties and we use the equilibrium concept of monopolistic competition, instead of perfect competition. Except for the fact that innovation in our environment is "directed" towards either sector, the way in which we model

[1]Herrendorf et al. (2014) offer a review of that literature. Contributions to it include Echevarria (1997), Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Foellmi and Zweimüller (2008), Buera and Kaboski (2009), Buera and Kaboski (2012), Herrendorf et al. (2013), Boppart (2014), Herrendorf et al. (2015), and Duernecker and Herrendorf (2015).

innovation is standard.[2]

We require our model to be consistent with the key stylized facts of growth and structural transformation: aggregate growth is balanced (i.e., aggregate variables grow at the same constant rates); structural transformation takes place underneath (i.e., labor is reallocated from the goods to the service sector); endogenous technological change is sector biased and leads to less productivity growth in the service sector than in the goods sector. This last feature is challenging to obtain because the usual market–size–effect implies that the expanding sector attracts more innovation and has stronger productivity growth than the shrinking goods sector; see e.g. Boppart and Weiss (2013). The exact opposite happens in the context of structural transformation: productivity growth is *less rapid* in the *expanding* service sector than in the *shrinking* goods sector.

The main contribution of this paper is to meet this challenge and develop a model of endogenous sector–biased technological change which is consistent with the three stylized facts described above. To achieve this, we need to assume one difference between the goods and the service sector: the return to specialization is larger in the goods sector than in the service sector.[3] We show that under this assumption, our model has a unique generalized balanced growth path with the desired properties: aggregate growth is balanced while structural transformation takes place underneath; the service sector receives more innovation than the goods sector but productivity growth is less rapid in the service sector.

Our model implies a surprising answer to the public–policy question posed above: industrial policy that protects employment in the goods sector makes matter worse (i.e., leads to a Pareto *deterioration*, instead of a Pareto improvement). The reason for this answer is that the laissez–faire equilibrium of our model has relatively too little innovation in the goods sector. This stems from the fact that innovators do not internalize the externality resulting from the returns to specialization, and that this externality is larger in the goods sector than in the service sector. Given the usual assumption that goods and services are complements in the utility function, too little innovation in the goods sector translates into too much labor in the goods sector compared to the service sector. In other words, in our model, optimal industrial policy should aim to reduce the size of the goods sector and to speed up structural transformation, instead of slowing

[2]Romer (1987) and Romer (1990) developed the first growth model with horizontal innovation. The literature that followed his seminal work has studied innovation and aggregate growth, but not the question towards which sector endogenous innovation is directed.

[3]For a Dixit–Stiglitz aggregtor, the return to specialization is defined as the elasticity of output with respect to the number of varieties while keeping the total quantity of inputs unchanged. Ethier (1982) and Benassy (1998) offer further discussion on the return to specialization.

it down.

Other recent work also studies structural transformation in the context of endogenous technological progress at the sector level; see for example Boppart and Weiss (2013), Struck (2014) and Hori et al. (2015). To the best of our knowledge, however, we are the first to develop a model of endogenous sector–biased technological change that is consistent with the three stylized facts described above. We are also the first to characterize the efficient allocation sufficiently so as to able to compare the efficient path with that of the laissez–faire equilibrium.

Our work is also related to the recent literature on directed technological change that developed from the seminal work of Acemoglu (2002, 2003, 2007). This literature asks why the skill premium and the supply of skilled workers have risen at the same time. It shows that market size effects play a key role in directing technological progress to the abundant factor of production. The novelty of our model compared to this literature is that the difference in the return to specialization overcomes the market–size effect so that productivity growth turns out to be stronger in the shrinking sector, which is what we see in the data.

The rest of the paper is organized as follows. Section 2 presents the model economy. Section 3 contains our results about equilibrium innovation and structural transformation. Section 4 contains our results about efficient innovation and structural transformation. Section 5 concludes. All proofs are delegated to the Appendix.

2 Model

2.1 Environment

2.1.1 Basic features

Time is continuous and runs forever. There is a measure one of identical households. In period t there are the following commodities: an investment good X_t, which we choose as the numeraire; consumption goods and services, C_{gt} and C_{st}; intermediate–good varieties $z_{jt}(i)$ for the production of C_{jt} where $j \in \{g, s\}$ and $i \in [0, A_{jt}]$. A_{jt} will be endogenously determined through innovation. We will use the notational convention that upper–case letters refer to sector–wide or economy–wide variables and lower–case letters to specific intermediate–good varieties.

2.1.2 Technology

The investment technology is of the AK form:

$$X_t = A_x K_{xt} \tag{1}$$

where A_x is TFP and K_x capital in the investment sector. An AK investment technology was suggested by Rebelo (1991) and has recently been used by Boppart (2014) in the context of structural transformation.

The consumption commodities are produced from intermediate goods according to the following technologies:

$$C_{jt} = A_{jt}^{\alpha_j - \frac{1}{\sigma-1}} \left(\int_0^{A_{jt}} [z_{jt}(i)]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where $\sigma > 1$ is the elasticity of substitution and $\alpha_j \in (0,1)$ is the so called return to specialization. This formulation follows Ethier (1982).

To understand the roles that σ and α_j play, we simplify by imposing symmetry across intermediate goods (which will hold in equilibrium). (2) then becomes:

$$C_{jt} = A_{jt}^{\alpha_j - \frac{1}{\sigma-1}} \left(\int_0^{A_{jt}} [z_{jt}(i)]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} = A_{jt}^{\alpha_j - \frac{1}{\sigma-1}} A_{jt}^{\frac{\sigma}{\sigma-1}} z_{jt} = A_{jt}^{\alpha_j} (A_{jt} z_{jt}) = A_{jt}^{\alpha_i} Z_{jt} \tag{3}$$

This equation shows that Ethier's formulation assigns two distinct roles to σ and α_j. Specifically, σ governs the complementarity between intermediate inputs, implying that it directly affects the marginal product of a variety and the monopoly power of the innovator. In contrast, α_j affects the marginal product of each variety only indirectly through the positive externality that results from the total stock of existing varieties. A particular innovator is too small to affect this stock.[4]

The literature refers to α_j as the return to specialization. To see the motivation for this choice of terminology, consider two economies in which the same total quantity of inputs is slit into fewer and into more intermediate inputs. In the former economy each intermediate input accounts for a larger share of the value of output than in the latter economy. Therefore,

[4] Benassy (1998) was the first to argue that σ and α play distinct roles in the context of economic growth. Acemoglu et al. (2007) show that σ and α have different implications for the types of contracts that are written in equilibrium. Epifani and Gancia (2009) and Beaudry et al. (2011) find that distinguishing between σ and α is crucial for the quantitative evaluation of their model.

each intermediate input is less specialized in the former economy. Applying this notion to our environment, A_{jt} measures the degree of specialization, with larger values corresponding to more specialization. (3) then implies that, in percentage terms, the return to specialization is given by the parameter α_j:

$$\frac{\partial \log(C_{jt})}{\partial \log(A_{jt})} = \alpha_j.$$

Therefore, α_j is commonly referred to as the return to specialization.

The intermediate goods technologies are of the Cobb–Douglas functional form:

$$z_{jt}(i) = \frac{1}{\theta^\theta(1-\theta)^{1-\theta}}[k_{jt}(i)]^\theta[l_{jt}(i)]^{1-\theta}, \tag{4}$$

where $k_{jt}(i)$ and $l_{jt}(i)$ denote capital and labor allocated to the production of variety i in sector j at time t. Dividing by $\theta^\theta(1-\theta)^{1-\theta}$ is a normalization that will simplify the algebra to come. Note that θ in (4) does not depend on the sector, that is, the capital share is the same in both sectors.[5]

Innovation is horizontal, i.e., it happens through the creation of new varieties of intermediate goods. We assume that creating the blueprint for a new intermediate–good variety requires one unit of the investment good. Our innovation technology is an example of the so called lab–equipment specification, which uses final goods as the input to innovation. An alternative specification of the innovation technology would be knowledge–based innovation, which uses labor as the input. The fact that labor is in fixed supply implies that there are two important differences between the two specifications. First, in the lab–equipment model there is less scope for monopoly power to distort the labor allocation. Second, to sustain growth the knowledge–based–innovation model needs spill–overs from past innovation on current innovation, which introduces a form of state dependence. We opt for a lab–equipment specification because it simpler in that it reduces the scope for monopoly distortions and it avoids the complications from state dependence. We conjecture that our model could be generalized to incorporate these two features.[6]

[5]While Valentinyi and Herrendorf (2008) show that in the data $\theta_g \neq \theta_s$, Herrendorf et al. (2015) show that Cobb–Douglas production functions with equal θ do a reasonable job at capturing the technological forces behind the postwar structural transformation in the US. Acemoglu and Guerrieri (2008) explore what happens when θ_j are sector specific.

[6]See Rivera-Batiz and Romer (1991) for further discussion about the two specifications.

2.1.3 Households

Households are endowed with one unit of time, an initial capital stock, $K_0 > 0$, and an initial stock of blueprints for intermediate–goods varieties, $A_0 > 0$.

Life–time utility is given by

$$\int_0^\infty \exp(-\rho t)\log(C_t)dt, \tag{5}$$

where $\rho > 0$ is the discount rate and the consumption aggregator is of the constant–elasticity–of–substitution form:

$$C_t = \left(\omega_g^{\frac{1}{\varepsilon}} C_{gt}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{\frac{1}{\varepsilon}} C_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where ω_j are positive weights and $\varepsilon \in [0,\infty)$ is the elasticity of substitution between the two consumption commodities (with $\varepsilon = 1$ being the Cobb–Douglas case, $\varepsilon \to 0$ being Leontief, and $\varepsilon \to \infty$ being perfect substitutes).

2.1.4 Resource constraints

The resource constraints are:

$$K_t = K_{xt} + K_{ct} = K_{xt} + (K_{gt} + K_{st}), \tag{7a}$$

$$K_{jt} = \int_0^{A_{jt}} k_{jt}(i)di, \quad j \in \{g, s\}, \tag{7b}$$

$$\dot{K}_t = A_x K_{xt} - (X_{gt} + X_{st}) - \delta K_t, \tag{7c}$$

$$\dot{A}_{jt} = X_{jt}, \quad j \in \{g, s\}, \tag{7d}$$

$$1 = L_{gt} + L_{st} \tag{7e}$$

$$L_{jt} = \int_0^{A_{jt}} l_{jt}(i)di, \quad j \in \{g, s\}. \tag{7f}$$

The first constraint says that the capital stocks must equal the sum of the capital allocated to the investment sector and the two consumption sectors. The second constraint imposes that the capital in each consumption sector is the sum of the capital stocks allocated to producing the different varieties of intermediate goods that are used in this sector. The third constraint is the law of motion for capital. It takes into account that the output of the investment sector can be used for building up the capital stock and for creating intermediate–goods new varieties. The investments in intermediate–goods varieties are denoted by X_{gt} and X_{st}. The fourth constraint is the law of motion for the stocks of intermediate–goods varieties. The last two constraints say that labor in the intermediate goods sectors has to add up to the total endowment of one.

Summarizing, our environment has the following key features: households own the production factors and derive utility from the two final consumption commodities; final–goods producers produce investment from capital and produce the two consumption commodities from intermediate goods; intermediate–goods producers produce intermediate goods from capital and labor; innovators create new intermediate–goods varieties from capital.

2.2 Equilibrium

In this subsection, we derive the conditions that characterize the equilibrium. We assume that there is perfect competition in the markets for the final goods and for the production factors capital and labor. We also assume that there is monopolistic competition in the markets for intermediate input varieties.

2.2.1 Production

The first–order condition to the problem of the investment firm is:

$$r_t = A_x, \tag{8}$$

where r_t denotes the rental price for capital. (8) shows that, as is usual for an AK investment technology, the rental price for capital equals the TFP of the investment technology. This will considerably simplify the analysis to come.

The problem of the firm in consumption sector j is given by:

$$\max_{C_{jt},\{z_{jt}(i)\}_{i=0}^{A_{jt}}} \left(P_{jt}C_{jt} - \int_0^{A_{jt}} p_{jt}(i) z_{jt}(i) di \right) \quad \text{s.t.} \quad A_{jt}^{\alpha_j - \frac{1}{\sigma-1}} \left(\int_0^{A_{jt}} [z_{jt}(i)]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \geq C_{jt}.$$

The first–order conditions imply the following inverse demand function:

$$p_{jt}(i) = P_{jt} A_{jt}^{\frac{\alpha_j(\sigma-1)-1}{\sigma}} \left(\frac{C_{jt}}{z_{jt}(i)} \right)^{\frac{1}{\sigma}}, \tag{9a}$$

where the sectoral price index is given as:

$$P_{jt} = A_{jt}^{-\alpha_j + \frac{1}{\sigma - 1}} \left(\int_0^{A_{jt}} [p_{jt}(i)]^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \tag{9b}$$

The inverse demand function states that the price of a variety relative to the sectoral price index is inversely related to the quantity of that variety relative to the sectoral output. This relationship is crucial for the producers of intermediate–good varieties because each will take into account how the price for the variety changes depending on the quantity he throws on the market.

The intermediate–goods producers rent capital and labor on competitive factor markets. The necessary conditions for the optimal level of factor inputs imply that the marginal costs of producing one unit of intermediate good depend on the rental prices of capital and labor in the Cobb–Douglas way:[7]

$$mc_t = r_t^{\theta} w_t^{1-\theta}. \tag{10}$$

Appendix A also shows that in equilibrium the capital–labor ratios are equalised across the consumption sectors, that the sectoral capital–labor ratios are equal to the aggregate capital–labor ratio in the consumption sectors, and that the capital–labor ratios are inversely related to the ratio of the rental prices for capital and labor:

$$K_{ct} = \frac{K_{jt}}{L_{jt}} = \frac{k_{jt}(i)}{l_{jt}(i)} = \frac{\theta}{1-\theta} \frac{w_t}{r_t}. \tag{11}$$

This feature is standard for Cobb–Douglas production functions with equal capital share parameters. It is crucial for being able to aggregate the two consumption sectors and to obtain a generalized balanced growth path along which consumption expenditure grow at the same rate as aggregate capital.[8]

The markets for intermediate goods are monopolistically competitive, that is, each intermediate–good producer is a monopolist for his variety but takes all aggregate variables as given. Specifically, an intermediate–good producer maximises profits while taking into account the demand function, (9a), and taking as given r_t, w_t, P_{jt}, and C_{jt}:

$$\max_{z_{jt}(i)} \left(p_{jt}(i) - r_t^{\theta} w_t^{1-\theta} \right) z_{jt}(i) \quad \text{s.t.} \quad p_{jt}(i) = P_{jt} A_{jt}^{\frac{\alpha_j(\sigma-1)-1}{\sigma}} \left(\frac{C_{jt}}{z_{jt}(i)} \right)^{\frac{1}{\sigma}}$$

[7] See the Appendix A for the derivation of the first order conditions for the monopolists.
[8] See Herrendorf et al. (2014) for further discussion.

The first–order condition implies that

$$p_{jt}(i) = \frac{\sigma}{\sigma - 1} r_t^{\theta} w_t^{1-\theta} \tag{12a}$$

$$z_{jt}(i) = A_{jt}^{\alpha_j(\sigma-1)-1} \left(\frac{P_{jt}}{p_{jt}(i)} \right)^{\sigma} C_{jt} \tag{12b}$$

$$\pi_{jt}(i) = \frac{1}{\sigma - 1} r_t^{\theta} w_t^{1-\theta} z_{jt}(i) \tag{12c}$$

Equation (12a) states that, as is standard, the monopolist sets his price as a markup $\sigma/(\sigma - 1)$ over his marginal cost $r_t^{\theta} w_t^{1-\theta}$. (12b) says that the supply of an intermediate good variety depends both on its relative prices and on sector output. (12c) says that equilibrium profits of the monopolist are the product of the markup, the marginal costs, and the scale of production. (12a)–(12c) have the implication that the equilibrium will be symmetric across the intermediate–goods producers of sector j:

$$p_{jt}(i) = p_{jt}, \tag{13a}$$

$$z_{jt}(i) = z_{jt}, \tag{13b}$$

$$\pi_{jt}(i) = \pi_{jt}, \tag{13c}$$

implying that sector level aggregates can be written as $Z_{jt} = A_{jt} z_{jt}$ and $\Pi_{jt} = A_{jt} \pi_{jt}$. Symmetry is important for being able to aggregate the production functions for specific varieties to a sectoral production function for total output in each consumption sector. Specifically, using (11) and that $z_{jt}(i) = z_{jt}$, we get

$$z_{jt} = \frac{1}{\theta^{\theta}(1 - \theta)^{1-\theta}} K_{ct}^{\theta} l_{jt}(i). \tag{14}$$

Hence, we also have symmetry with respect to labor: $l_{jt}(i) = l_{jt}$. Using this and the production function for C_{jt}, we can derive the total equilibrium output in consumption sector j:

$$C_{jt} = \frac{1}{\theta^{\theta}(1 - \theta)^{1-\theta}} A_{jt}^{\alpha_j} K_{ct}^{\theta} L_{jt}. \tag{15}$$

where we used that our notational convention implies that $L_{jt} = A_{jt} l_{jt}$. (15) shows that the TFP of the sectoral production function is the product of the normalizing constant $1/\theta^{\theta}(1 - \theta)^{1-\theta}$ and of $A_{jt}^{\alpha_j}$. Therefore, sectoral TFP will increase when innovation creates more varieties of intermediate goods, which we argued above corresponds to more specialization. The strength of this effect depends on the parameter α_j, which measures the return to specialization. This will play a key role in what is to come.

2.2.2 Innovation

There are infinitely many potential innovators and there is free entry into innovation. Entrants receive a patent for the blueprint of producing a new variety. For simplicity, we assume the patent lasts forever. The households fund the entry cost in exchange for receiving the ownership to the patent. Free entry implies that the present value of the monopoly profits from new variety i' in sector j equals the entry cost of one:

$$v_{jt}(i') = \int_t^{\infty} \exp(-r(\tau - t))\pi_{j\tau}(i')d\tau = 1, \quad \forall i' \in [A_{jt}, A_{jt} + X_{jt}]. \tag{16}$$

Note that since the entry costs are equal for both sectors, potential entrants are indifferent between entering either one of them. Since we have shown that $\pi_{j\tau}(i) = \pi_{j\tau}$ for all existing varieties $i \in [0, A_{j\tau}]$ and for all $\tau \geq t$, (16) implies that the value of existing and new patents must be the same:

$$v_{jt}(i) = v_{jt}(i') = 1. \tag{17}$$

Hence, there are no capital gains in equilibrium,

$$\dot{v}_{jt}(i) = 0. \tag{18}$$

2.2.3 Households

Recall from the previous subsection that, in equilibrium, profits for all varieties are the same in each sector, the value of all patents equals one, and there are no capital gains. Using these equilibrium properties, we can write the problem of the representative household as:

$$\max_{\{C_{gt}, C_{st}, K_t, A_{gt}, A_{st}\}_{t=0}^{\infty}} \int_0^{\infty} \exp(-\rho t) \log\left(\left[\omega_g^{\frac{1}{\varepsilon}} C_{gt}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{\frac{1}{\varepsilon}} C_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) dt$$

$$\text{s.t. } P_{gt}C_{gt} + P_{st}C_{st} + \dot{K}_t + \dot{A}_{gt} + \dot{A}_{st} = (r_t - \delta)K_t + \pi_{gt}A_{gt} + \pi_{st}A_{st} + w_t. \tag{19}$$

Note that in the absence of capital gains the rate of return on holding patents equals profits, π_{jt}.

As usual, the Euler equation is one of the dynamic necessary conditions for a solution to the household problem:

$$\frac{\dot{E}_t}{E_t} = r_t - \delta - \rho = \pi_{jt} - \rho, \tag{20}$$

where $E_t \equiv P_{gt}C_{gt} + P_{st}C_{st}$ denotes consumption expenditure. Using that $r_t = A_x$, the Euler equation implies that equilibrium profits are given by:

$$\pi_{jt} = A_x - \delta. \tag{21}$$

We denote the growth rate of E_t by

$$\gamma \equiv \frac{\dot{E}_t}{E_t} = A_x - \delta - \rho.$$

To ensure that our model economy has an equilibrium with positive growth, i.e., $\gamma > 1$, we assume that the TFP of the investment technology is large enough:

Assumption 1 $A_x > \delta - \rho$.

Let $A_t \equiv A_{gt}+A_{st}$. Then the transversality condition is the other dynamic necessary condition for a solution of the intertemporal problem:[9]

$$\lim_{t\to\infty} \exp(-\rho t)\frac{K_t}{E_t} = \lim_{t\to\infty} \exp(-\rho t)\frac{A_t}{E_t} = 0. \tag{22}$$

The following static first–order condition is also necessary for a solution to the household problem:

$$\frac{E_{st}}{E_{gt}} = \frac{\omega_s}{\omega_g}\left(\frac{P_{st}}{P_{gt}}\right)^{1-\varepsilon}. \tag{23a}$$

This equation determines the direction of structural transformation. Specifically, to replicate that the service sector has both a rising relative price and a rising expenditure share, one needs that the final consumption goods are complements in the utility function:[10]

Assumption 2 $\varepsilon \in [0, 1)$.

[9]Note that the usual transversality condition refers to $K_t + A_t$. Since both parts are non–negative, the usual transversality condition implies two separate conditions for K_t and A_t.

[10]This was one of the main observations of Ngai and Pissarides (2007).

Note that (23a) implies that the expenditure share of sector j, χ_{jt}, is a function of just the prices. Denoting the expenditure share by χ_{jt} and sectoral expenditure by $E_{jt} \equiv P_{jt}C_{jt}$, we have:

$$\chi_{jt} \equiv \frac{E_{jt}}{E_t} = \frac{\omega_j P_{jt}^{1-\varepsilon}}{\omega_g P_{gt}^{1-\varepsilon} + \omega_s P_{st}^{1-\varepsilon}}. \tag{23b}$$

3 Equilibrium Innovation and Structural Transformation

3.1 Preliminary remarks

Imposing the standard notion of balanced growth is too strong in our context, because it would imply that all ratios are constant which would rule out structural transformation. We follow Kongsamut et al. (2001) and employ the concept of a generalized balanced growth (GBGP henceforth), which is an equilibrium path along which the real interest rate is constant. The AK technology in the investment sector gives us the constant real interest rate and the existence of a generalized balanced growth path "for free". We want three additional properties from the generalized balanced growth path:

- balanced growth happens on the aggregate, i.e., aggregate variables grow at constant rate;
- structural transformation from goods to service takes place underneath;
- the expanding service sector has slower TFP growth than the shrinking goods sector.

Developing and solving a model with these three properties is challenging and constitutes the main contribution of this paper. We are now ready to state our first result.

Lemma 1 *In each consumption sector, the payments to capital and labor, the profits, and the number of varieties are proportional to the expenditures on the consumption good, and the proportionality factors are independent of the sector:*

$$\Pi_{jt} = \frac{1}{\sigma} E_{jt} \tag{24a}$$

$$w_t L_{jt} = \frac{\sigma - 1}{\sigma}(1-\theta) E_{jt} \tag{24b}$$

$$r_t K_{jt} = \frac{\sigma - 1}{\sigma}\theta E_{jt} \tag{24c}$$

$$A_{jt} = \frac{1}{\sigma(r_t - \delta)} E_{jt} \tag{24d}$$

Proof: See Appendix B.

Three remarks about this lemma follows. First, for $\sigma \to \infty$, the factor payments converge to the perfect–competition case: $w_t L_{jt} = (1-\theta)E_{jt}$ and $r_t K_{jt} = \theta E_{jt}$. Second, (24b) and (24d) imply that in our model relative sectoral labor, relative expenditures, and relative varieties are equal:

$$\frac{L_{gt}}{L_{st}} = \frac{E_{gt}}{E_{st}} = \frac{A_{gt}}{A_{st}} \tag{25}$$

The first equality is a standard property of structural transformation models with Cobb–Douglas production functions that have equal capital share parameters. The second equality reflects the standard property of endogenous innovation models that larger markets attract more innovation. Lastly, summing over the two sectors, Lemma 1 implies that Π_t, w_t, K_{ct} and A_t are proportional to E_t, which we know grows at rate γ. This insight is crucial for the being able to establish the next proposition that shows that all aggregate variables grow at the same rate γ.

3.2 Balanced growth and structural transformation

Proposition 1 (Balanced Growth) *Along the GBGP, all aggregate variables expressed in units of the numeraire grow at the same rate:*

$$\gamma = \frac{\dot{E}_t}{E_t} = \frac{\dot{\Pi}_t}{\Pi_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{K}_{ct}}{K_{ct}} = \frac{\dot{A}_t}{A_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{X}_t}{X_t}.$$

Proof: See Appendix C.

Given that our model economy has a generalized balanced growth path with the desired aggregate properties, we now turn our attention to whether it can account for changes in the stylized facts on sectoral relative prices, expenditure, and labor allocations. It turns out that this requires an additional assumption:

Assumption 3 $\alpha_s < \alpha_g < 1-\theta$

This assumption says that the return to specialization is larger in the goods sector than in the service sector and that both returns are bounded from above by the labor share parameter. While it is hard to measure the return to specialization in the data, there is some support for Assumption 3. Specifically, taking as given existing estimates for the growth rate of varieties (which in itself is hard to measure), Acemoglu et al. (2007) calibrate a value of 0.25 for aggregate α. Setting the labor share to two thirds, this leaves ample room for $\alpha_s < \alpha_g < 1-\theta$, which is all we will need. The logic of the calibration of Acemoglu et al. (2007) also provides a justification for imposing that $\alpha_s < \alpha_g$, because it points out that the value of α is directly linked to productivity growth. Since we know that productivity growth is stronger in the goods sector than in the service sector, this logic suggests to impose that $\alpha_g > \alpha_s$. The next proposition shows that Assumption 3 is required for our model to be consistent with both the three stylized facts of structural transformation described above and the additional fact the relative price of investment has been falling in the post–war U.S.[11]

Proposition 2 (Structural Transformation) *Suppose that Assumptions 1–2 hold. If and only if Assumption 3 holds the GBGP has the following properties:*

(i) relative innovation and relative TFP evolve in opposite directions: A_{st}/A_{gt} grows and $A_{st}^{\alpha_s}/A_{gt}^{\alpha_g}$ falls;

(ii) there is structural transformation from the goods to the service sector: L_{st}/L_{gt} and χ_{st}/χ_{gt} grow without bound;

(iii) the price of services relative to goods, P_{st}/P_{gt}, grows without bound.

(iv) the price of consumption relative to investment, P_t, grows without bound.

Proof: See Appendix D.

A key feature of the last proposition is that while variety growth is weaker in the goods sector,

$$\frac{\dot{A}_{gt}}{A_{gt}} < \frac{\dot{A}_{st}}{A_{st}},$$

TFP growth is stronger in the goods sector than in the service sector:

$$\alpha_g \frac{\dot{A}_{gt}}{A_{gt}} > \alpha_s \frac{\dot{A}_{st}}{A_{st}}.$$

The reason for this is that the difference in the returns to specialization overturns the difference in variety growth. This surprising result is the main novelty of this paper.

It is crucial for result of Proposition 2 that the growth rates of TFP, $\alpha_j \dot{A}_{jt}/A_{jt}$, are not only endogenous but also are *falling* over time in either sector. This is different from Ngai and Pissarides (2007) who assumed that the growth rates of $\alpha_j \dot{A}_{jt}/A_{jt}$ are exogenous and *constant*.

[11]Note that Parts (i)–(iii) of the Proposition can be obtained also under the weaker assumption $\alpha_s < \alpha_g$. The additional assumption $\alpha_j < 1 - \theta$ is only required for Part (iv).

Not restricting the growth rates of sectoral TFPs to be constant in our model turns out to be much more important than one might initially realize. To see this, note that Appendix D derives the following analytical solutions for the sectoral TFP growth rates (see equations (D.23a) and (D.23b)):

$$\alpha_g \frac{\dot{A}_{gt}}{A_{gt}} = \frac{\alpha_g[1 + (1-\varepsilon)\alpha_s]}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma, \tag{26a}$$

$$\alpha_s \frac{\dot{A}_{st}}{A_{st}} = \frac{\alpha_s[1 + (1-\varepsilon)\alpha_g]}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma. \tag{26b}$$

These equations show that if we restricted $\alpha_j \dot{A}_{jt}/A_{jt}$ to be constant, then the share of service labor in total labor, L_{st}, would be constant and there would not be any structural transformation. These equations imply that the difference in the TFP growth rates of goods and services declines over time; see equation (D.24) in the Appendix for the proof. This is consistent with the evidence of Triplett and Bosworth (2003) that the difference in the TFP growth rates of goods and services has declined in the U.S. since 1995.

3.3 Intuition

Equations (26a)–(26b) can also be used to build intuition for the behavior of the growth rates of varieties and of TFPs in our model. To this end, it is instructive to start with the special case $\alpha_g = \alpha_s$. (26a)–(26b) then imply that sector TFPs grow at the common rate $\alpha\gamma$, varieties grow at the common rate γ, L_{st} is constant, and there is not structural transformation. This makes intuitive sense, as $\alpha_g = \alpha_s$ implies that the technologies of the consumption sectors are identical. Now consider the case $\alpha_g > \alpha_s$. Given Assumption 2, we have $\varepsilon < 1$ and we can say three things: first, the growth rates of varieties and of TFP are decreasing as the sectoral labor share of services increases; TFP grows more strongly in the goods sector; varieties grow more strongly in the service sector. The reason for why the relative growth rates of TFP and varieties move in opposite directions lies in the way in which the market–size effect works when $\varepsilon < 1$: the larger return to specialization implies that TFP grows more strongly in the goods sector; labor gets reallocated to the sector with the slower productivity growth, namely, services; the resulting increase in the market size of services leads to more innovation in the service sector.

3.4 Discussion

Our work is also closely related to the recent literature on directed technological change that developed from the seminal work of Acemoglu (2002, 2003, 2007). This literature asks why some production factors benefit more from technological change than others. It shows that technological change is biased in favor of more abundant production factors, and if the elasticity of substitution between different factors is sufficiently high, then the relative factor price raises in response to an increase in the relative factor supply. These insights have been applied to understand the puzzling fact that the skill premium and the supply of skilled workers have risen at the same time in recent times. There are important similarities between the factor–biased technological change studied in this literature and the sector–biased technological change studied here. In both cases technological change is biased because of a market–size effect and the elasticity of substitution plays a key role in determining the consequence of the bias. There are also important differences. In our model, the elasticity applies to the sector outputs in the utility function whereas in the canonical model of factor–biased technological change it applies to production factors. Moreover, in our model, the difference in the return to specialization has to overcome the market–size effect so that productivity growth is stronger in the shrinking sector. Lastly, our model exhibits a generalised balanced growth path with different and changing growth rates of sectoral technological change. In contrast, the canonical model of factor–biased technological change exhibits a balanced growth path with equal and constant growth rates of factor–augmenting technological change.

In our model, the difference in the returns to specialization is responsible for the endogenous sector bias of technological change. The way in which we set up our model economy implies that this is an exogenous feature of technology. Grossman and Helpman (1991) offer a alternative specification that makes the consumption aggregator part of preferences. The difference in α_j can then be interpreted as reflecting a difference in the taste for variety, that is, households have a stronger taste for goods varieties than for service varieties.

There are at two other possible differences between the consumption sectors that could potentially lead to endogenous sector–biased technological change. First, the entry costs may differ between the two sectors, capturing that it is harder to innovate in one sector. It turns out that this introduces a level difference between the stocks of varieties in the two consumption sectors, but does not lead to differential growth rates of productivity and structural transformation. Second, the markups may be different between the two sector, capturing different degrees of monopoly power. For example, it is sometimes argued that since many services cannot be traded, there is more monopoly power associated with them. It turns out that assuming different markups precludes BGP of aggregate variables, as it implies that the sectoral composition matters for aggregate variables. And, again, it does not lead to structural transformation.

4 Efficient Innovation and Structural Transformation

To answer the question whether the laissez faire equilibrium is efficient, we solve the planner problem:

$$\max_{\left\{\substack{C_{gt},C_{st},z_{gt}(i),z_{st}(i),\\ k_{gt}(i),k_{st}(i),l_{gt}(i),l_{st}(i),\\ X_{gt},X_{st},K_t,K_{ct},A_{gt},A_{st}}\right\}_{t=0}^{\infty}} \int_0^\infty \exp(-\rho t)\log\left(\omega_g^{\frac{1}{\varepsilon}} C_{gt}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{\frac{1}{\varepsilon}} C_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} dt \tag{27a}$$

$$\text{s.t}\quad C_{jt} = A_{jt}^{\alpha_j-\frac{1}{\sigma-1}}\left(\int_0^{A_{jt}} [z_{jt}(i)]^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}},\ j\in\{g,s\}, \tag{27b}$$

$$z_{jt}(i) = \frac{1}{\theta^\theta(1-\theta)^{1-\theta}}[k_{jt}(i)]^\theta[l_{jt}(i)]^{1-\theta},\ j\in\{g,s\},\ i\in[0,A_{jt}], \tag{27c}$$

$$K_{ct} = \int_0^{A_{gt}} k_{gt}(i)di + \int_0^{A_{st}} k_{st}(i)di, \tag{27d}$$

$$1 = \int_0^{A_{gt}} l_{gt}(i)di + \int_0^{A_{st}} l_{st}(i)di, \tag{27e}$$

$$\dot{K}_t = A_x(K_t - K_{ct}) - (X_{gt}+X_{st}) - \delta K_t, \tag{27f}$$

$$\dot{A}_{jt} = X_{jt}\ j\in\{g,s\}. \tag{27g}$$

Appendix E shows that for $\alpha_j \in (0,1)$, the planner problem is well defined and there is a solution exists. Appendix E also contains the first–order conditions to this problem. Trivially, a GBGP again exists because the marginal product of capital in investment production still equals A_x. Beyond that, it is somewhat challenging to contrast the paths in laissez–faire equilibrium and under the planner problem. We start by characterizing the case in which the laissez–faire equilibrium is efficient:

Proposition 3 (Efficient Innovation) *Suppose Assumptions 1–2 hold and $\alpha_g = \alpha_s = 1/(\sigma-1)$. Then the GBGP is efficient.*

Proof: See Appendix F.

To develop intuition for this result, one needs to understand why there are no distortions if $\alpha_g = \alpha_s = 1/(\sigma-1)$. There are two reasons for this. First, in this case the growth rate of capital accumulation is not distorted and K_t is growing at rate γ in both the laissez–faire equilibrium and the planner problem; see the Appendix F for the proof. Given the same initial K_0, this implies that the total capital stock and the capital allocated to investment must be the same. Second, the labor allocation is not distorted because labor is used only in the intermediate–good sectors, which both are subject to the *same* monopoly distortion. This leaves the possibility of distortions to the allocation between intermediate goods production and innovation of the capital that is not used for investment. To study this allocation, note that the innovation decision is characterized by the following conditions:[12]

$$\text{Laissez faire:} \quad \frac{1}{A_x - \delta}\frac{1}{\sigma - 1}\frac{A_x k_j}{\theta} = 1$$

$$\text{Planner problem:} \quad \frac{1}{A_x - \delta}\alpha_j\frac{A_x k_j^*}{\theta} = 1$$

where a superscript star denotes the solution to the planner problem. The right–hand side equals the marginal cost of innovation, which is one in both cases because it takes one unit of investment to create a new variety. The left–hand sides equal the marginal benefit of innovation. In the laissez faire equilibrium, the marginal benefit equals the present discounted value of the stream of profits, that is, the inverse of the interest rate, $1/(A_x - \delta)$, times the period profits, $[1/(\sigma - 1)]A_x k_j/\theta$; see (12c). Since $A_x k_j/\theta$ is the quantity of intermediate goods in both cases, the term $1/(\sigma - 1)$ can be interpreted as the private rate of return on producing one unit of intermediate good. Under the planner problem, the social rate of return, α_j, replaces $1/(\sigma - 1)$ in the expressions for the marginal benefit. Since, $1/(\sigma - 1) = \alpha_g = \alpha_s$, the private and social rates of return are equal in the two cases, implying that the same amount of capital is split in the same ratio between innovation and the production of intermediate goods.

Although the allocations are the same in the laissez–faire equilibrium and the planner problem, and in both cases the reduction in capital used in production is the same compared to perfect competition, the economic reasons are completely different. The monopolist restricts capital used in production in order to increase the price for his product and reap the monopoly profits, but he does not take into account the externality from the returns to specialization. The planner restricts capital used in production in order to be able to achieve the higher innovation level that internalize the externality resulting from the return to specialization, but he does not think in terms of monopoly profits and restricting production. If $1/(\sigma - 1) = \alpha_g = \alpha_s$, then both considerations turn out to lead to the exact same split of capital between production and

[12] See Appendix F, equations (F.56d) and (F.57d).

innovation. In contrast, if $1/(\sigma - 1) < \alpha_g = \alpha_s$, then the monopolist chooses inefficiently high capital in production because his monopoly power is "too small". If $1/(\sigma - 1) > \alpha_g = \alpha_s$, then the monopolist chooses inefficiently low capital in production because his monopoly power is "too large".

If $\alpha_s < \alpha_g$, then the GBGP is always inefficient irrespective of the value of σ. Saying more is somewhat challenging because the planner problem does not exhibit balanced growth on the aggregate. The reason for this is that the strength of the externality of sectoral innovation differs between the two sectors when $\alpha_s < \alpha_g$. As a result, the efficient growth rate of total innovation, A_t, depends on its composition. For our purpose, we can avoid this issue and make statements that are conditional on a given $A_t = A_{gt} + A_{st}$ that is reached at some points by both the laissez–faire equilibrium allocation and the efficient allocation. To tie this in with the analysis done so far, note that the state of our economy can be written as K_t, A_{gt}, A_{st} or as $K_t, A_t, A_{gt}/A_{st}$. While the former is what we have done so far, the latter is more useful now. Denoting again the efficient allocation by a superscript star, we can show the following result:

Proposition 4 (Inefficient Innovation) *Suppose Assumptions 1–3 hold. Then the following is true:*

- *For any interior equilibrium path and any interior solution path to the planner problem, there is an* $\underline{A} \in (0, \infty)$ *so that each path reaches all* $A \in (\underline{A}, \infty)$.[13]
- *For all* $A \in (\underline{A}, \infty)$*, we have:*

$$\frac{A_g^*(A)}{A_s^*(A)} > \frac{A_g(A)}{A_s(A)}, \tag{28a}$$

$$\frac{L_g^*(A)}{L_s^*(A)} < \frac{L_g(A)}{L_s(A)}, \tag{28b}$$

$$\frac{\dot{A}_j^*(A)}{A_j^*(A)} < \frac{\dot{A}_j(A)}{\dot{A}_j(A)}, \quad j \in \{g, s\}. \tag{28c}$$

Proof: See Appendix G.

Proposition 4 shows that for the same measure of total varieties the planner is already further ahead in the process of structural transformation in that he allocates more labor to the service sector compared to the equilibrium with the same value of A. A key implication of this is that industrial policies that slow down structural transformation by protecting the goods sector from shrinking will reduce welfare, instead of increasing it.

[13]By interior, we mean that the solutions to all problems are interior.

5 Conclusion

We have built a multisector model with endogenous sector–biased technological change that results from the purposeful innovations of new intermediate–good varieties. We have shown that our model exhibits generalized balanced growth on the aggregate and structural transformation from the goods sector to the service sector underneath, while endogenous labor productivity growth is stronger in the shrinking goods sector than the expanding service sector. Compared to the efficient allocation, the laissez–faire equilibrium of our model directs too little innovation to the goods sector. Given the usual assumption that goods and services are complements in the utility function, the laissez–faire equilibrium then has the goods sector employ too much labor. This suggests that, if anything, industrial policy should aim to reduce the size of the goods sector, instead of maintaining it.

We view our model as a benchmark that serves as a useful first step to think about questions related to optimal endogenous sector–biased technological change. We acknowledge several potential limitations of this benchmark. Specifically, investment is produced from capital but not labor, implying that there is no structural transformation in investment; creating new varieties requires the input of capital but not labor. In the future, we plan to generalizes the model by relaxing these limitations. We also acknowledge that we have ignored another possible reason for policy interventions, namely that the reallocation of capital and labor from the goods sector to the service sector is costly. One reason may be that skills or capital are sector specific and thus cannot be costlessly transferred between sectors. While our hunch is that such considerations are not of first–order importance over the long horizons on which the literature on structural transformation focuses, we nonetheless think that it is interesting to analyze them, and if only because they feature prominently in the policy debate. We leave this as a topic for future research.

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Appendices: Derivations and Proofs

A Intermediate–goods producers' first–order conditions

Minimizing the cost $r_t k_{jt}(i) + w_t l_{jt}(i)$ of producing a given quantity of intermediate good, $z_{jt}(i)$, subject to the technology constraint

$$\frac{1}{\theta^\theta(1-\theta)^{1-\theta}}[k_{jt}(i)]^\theta[l_{jt}(i)]^{1-\theta} \geq z_{jt}(i)$$

gives the following conditional factor–demand functions:

$$k_{jt}(z_{jt}(i)) = \theta\left(\frac{w_t}{r_t}\right)^{1-\theta} z_{jt}(i), \tag{A.1a}$$

$$l_{jt}(z_{jt}(i)) = (1-\theta)\left(\frac{w_t}{r_t}\right)^{-\theta} z_{jt}(i). \tag{A.1b}$$

The marginal costs of producing one unit of intermediate good then follow by calculating $r_t k_{jt}(1) + w_t l_{jt}(1)$. Moreover, (A.1a)–(A.1b) imply that the capital–labor ratios are equalized across the two consumption sectors.

Turning to the markets for intermediate goods, each innovator is the monopolist producer of the new variety he created. Monopolistic competition implies that each producer maximises its profits subject to the demand function and taking as given all aggregate variables:

$$\max_{z_{jt}(i)}\left\{\left(p_{jt}(i) - r_t^\theta w_t^{1-\theta}\right) z_{jt}(i)\right\} \quad \text{s.t.} \quad p_{jt}(i) = P_{jt} A_{jt}^{\frac{\alpha_j(\sigma-1)-1}{\sigma}}\left(\frac{C_{jt}}{z_{jt}(i)}\right)^{\frac{1}{\sigma}}.$$

The first–order condition with respect to $z_{jt}(i)$ is

$$\frac{\sigma-1}{\sigma} P_{jt} A_{jt}^{\frac{\alpha_j(\sigma-1)-1}{\sigma}}\left(\frac{C_{jt}}{z_{jt}(i)}\right)^{\frac{1}{\sigma}} = r_t^\theta w_t^{1-\theta}. \tag{A.2}$$

Together with the demand function for $z_{jt}(i)$ and the marginal cost of producing one unit of intermediate goods $r_t^\theta w_t^{1-\theta}$, equation (A.2) implies price, quantity, and profits of intermediate good i in sector j:

$$p_{jt}(i) = \frac{\sigma}{\sigma-1} r_t^\theta w_t^{1-\theta}, \tag{A.3a}$$

$$z_{jt}(i) = A_{jt}^{\alpha_j(\sigma-1)-1}\left(\frac{\sigma-1}{\sigma}\right)^\sigma\left(\frac{P_{jt}}{r_t^\theta w_t^{1-\theta}}\right)^\sigma C_{jt}, \tag{A.3b}$$

$$\pi_{jt}(i) = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\frac{P_{jt}}{r_t^{\theta}w_t^{1-\theta}}A_{jt}^{\alpha_j-\frac{1}{\sigma-1}}\right)^{\sigma-1}E_{jt}. \tag{A.3c}$$

B Proof of Lemma 1

Equations (9b) and (A.3a) imply that the sectoral price level is inversely related to the number of varieties:

$$P_{jt} = \frac{\sigma}{\sigma-1}r_t^{\theta}w_t^{1-\theta}A_{jt}^{-\alpha_j}. \tag{B.4}$$

This equation, $\Pi_{jt} = A_{jt}\pi_{jt}$ and (A.3c) imply (24a) in the main text. Next, using (B.4) and (15), we get

$$E_{jt} = P_{jt}C_{jt} = \frac{\sigma}{\sigma-1}\frac{r_t^{\theta}w_t^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}K_{ct}^{\theta}L_{jt}. \tag{B.5}$$

Note that (11) implies that $K_{ct} = \frac{K_{jt}}{L_{jt}} = \frac{\theta}{1-\theta}\frac{w_t}{r_t}$. Together with (B.5) this gives (24b) and (24c) in the main text. Integrating (21) across varieties yields

$$\Pi_{jt} = A_{jt}(r_t-\delta).$$

Combining this with (24a), we get (24d) in the main text. Note that this implies that

$$\frac{A_{st}}{A_{gt}} = \frac{E_{st}}{E_{gt}}. \tag{B.6}$$

C Proof of Proposition 1

C.1 $\frac{\dot{E}}{E} = \frac{\dot{A}}{A} = \frac{\dot{K}_c}{K_c} = \frac{\dot{w}}{w} = \gamma$

The Euler equation implies that:

$$\gamma \equiv \frac{\dot{E}_t}{E_t} = A_x - (\delta+\rho). \tag{C.7}$$

Since we assume that $A_x > \delta + \rho$, we have: $\dot{E}/E = \gamma > 0$. Next note that if we sum up (24a)–(24d) over j, we obtain

$$\Pi_t = \frac{1}{\sigma} E_t, \tag{C.8a}$$
$$w_t L_t = \frac{\sigma - 1}{\sigma}(1-\theta) E_t, \tag{C.8b}$$
$$r_t K_{ct} = \frac{\sigma - 1}{\sigma} \theta E_t, \tag{C.8c}$$
$$A_t = \frac{1}{\sigma} \frac{1}{r_t - \delta} E_t \tag{C.8d}$$

for the aggregate variables. (C.8b)–(C.8d) together with $\dot{E}/E = \gamma > 0$ and $r_t = A_x$ imply that $\dot{w}/w = \dot{K}_c/K_c = \dot{A}/A = \gamma$.

C.2 $\dot{K}/K = \gamma$

Rewriting (7c) gives:

$$\frac{\dot{K}_t}{K_t} = (A_x - \delta) - A_x \frac{K_{ct}}{K_t} - \frac{X_t}{K_t}. \tag{C.9}$$

Using (7d), this can be rewritten as

$$\frac{\dot{K}_t}{K_t} = (A_x - \delta) - A_x \frac{K_{ct}}{K_t} - \frac{\dot{A}_t}{K_t} = (A_x - \delta) - \left(A_x - \frac{\dot{A}_t}{K_{ct}}\right) \frac{K_{ct}}{K_t}. \tag{C.10}$$

To get an expression for $\dot{A}_t/K_{ct}$, we start by deriving the relationship between A_t and K_{ct}. Combining (C.8c) and (24d) with $r_t = A_x$, we have

$$A_{jt} = \frac{1}{\sigma - 1} \frac{1}{\theta} \frac{A_x}{A_x - \delta} \chi_{jt} K_{ct}. \tag{C.11}$$

Summing over j leads to

$$A_t = A_{gt} + A_{st} = \frac{1}{\sigma - 1} \frac{1}{\theta} \frac{A_x}{A_x - \delta} K_{ct}. \tag{C.12}$$

Differentiating (C.12) with respect to time and using that $\gamma = \dot{K}_{ct}/K_{ct}$ gives:

$$\dot{A}_t = \frac{1}{\sigma - 1} \frac{1}{\theta} \frac{A_x}{A_x - \delta} \dot{K}_{ct} = \frac{1}{\sigma - 1} \frac{\gamma}{\theta} \frac{A_x}{A_x - \delta} K_{ct}.$$

Substituting this into (C.10) for $\dot{A}_t$, we obtain:

$$\frac{\dot{K}_t}{K_t} = (A_x - \delta) - A_x \frac{(\sigma-1)\theta(A_x-\delta)+\gamma}{(\sigma-1)\theta(A_x-\delta)} \frac{K_{ct}}{K_t}. \tag{C.13}$$

Next define:

$$\frac{K_c}{K} \equiv \frac{\rho}{A_x} \frac{\theta(A_x-\delta)}{\theta(A_x-\delta)+\gamma/(\sigma-1)}. \tag{C.14}$$

We show now that $\dot{K}_t/K_t = \gamma$

(i) Suppose that $\dot{K}_T/K_T > \gamma$ for some $T > 0$.
Then $K_{cT}/K_T < K_c/K$ and is falling for all $t > T$. This implies that $\dot{K}_t/K_t > \gamma$ for all $t > T$ and $\lim_{t\to\infty} \dot{K}_t/K_t = A_x - \delta$. This will violate the transversality condition:

$$\lim_{t\to\infty} \exp(-\rho t)\frac{K_t}{E_t} = 0.$$

(ii) To see this, use $\dot{E}/E = A_x - \delta - \rho$ to rewrite the condition as

$$\lim_{t\to\infty} \frac{K_t}{\exp((A_x-\delta)t)} = 0,$$

where we used that E_t is growing at rate γ. $\lim_{t\to\infty} \dot{K}_t/K_t = A_x - \delta$ implies that

$$\lim_{t\to\infty} \frac{K_t}{\exp((A_x-\delta)t)} > 0,$$

which violates the transversality condition. Therefore, it must be that $\dot{K}/K \leq \gamma$.

(iii) Suppose that $\dot{K}_t/K_t < \gamma$ for some $t = T$.
As K_{ct} is growing at rate γ, K_{cT}/K_T is rising. (C.13) implies that then K_{ct}/K_t is rising for all $t > T$ and that $\dot{K}_t/K_t$ is falling for all $t > T$. Hence, $\dot{K}_t/K_t < \dot{K}_T/K_T < \gamma$ for all $t > T$, which implies that $\lim_{t\to\infty}(K_{ct}/K_t) = \infty$ and $\lim_{t\to\infty} \dot{K}_t/K_t = -\infty$. This violates the resources constraint.

(iv) Taking the previous cases together, it follows that $\dot{K}_t/K_t = \gamma$.

C.3 $\dot{X}/X = \dot{Y}/Y = \gamma$

(C.9) and $\dot{K}/K = \dot{K}_c/K_c = \gamma$ implies $\dot{X}/X = \gamma$. Since $\dot{E}/E = \dot{K}/K = \gamma$ too, this implies that $\dot{Y}/Y = \gamma$.

C.4 $\dot{A}_t/A_t = \gamma$

Along the generalised balanced growth path, (C.14) has to be satisfied. Substituting out K_{ct} using (C.12) leads to a condition that A_t/K_t have to satisfy along the GBGP:

$$\frac{A_t}{K_t} = \frac{\rho/(\sigma - 1)}{\theta(A_x - \delta) + \gamma/(\sigma - 1)}. \tag{C.15}$$

For a given initial capital stock K_0, the aggregate initial number of varieties, A_0, that puts the economy onto the GBGP is determined by (C.15).

Finally, note that the initial A_0 determined by K_0 also determine A_{j0}. This is because (23a), (B.4), and (B.6) imply

$$\frac{A_{st}}{A_{gt}} = \frac{E_{st}}{E_{gt}} = \frac{\omega_s}{\omega_g}\left(\frac{P_{st}}{P_{gt}}\right)^{1-\varepsilon} = \frac{\omega_s}{\omega_g}\frac{A_{gt}^{\alpha_g(1-\varepsilon)}}{A_{st}^{\alpha_s(1-\varepsilon)}}.$$

Together with $A_{g0} + A_{s0} = A_0$, this equation determines the unique A_{g0} and A_{s0} that go along with the initial K_0.

D Proof of Proposition 2

D.1 Evolution of A_{st}/A_{gt} and $A_{st}^{\alpha_s}/A_{gt}^{\alpha_g}$

D.1.1 Evolution of A_{st}/A_{gt}

We start deriving the growth rate of the varieties. (24d) and (B.6) implies that

$$A_x - \delta = \frac{1}{\sigma}\frac{\chi_{jt}E_t}{A_{jt}} = \frac{1}{\sigma}\frac{L_{jt}E_t}{A_{jt}}, \tag{D.16}$$

where we used that $\chi_{jt} = L_{jt}$ (see (25)). Taking logs and differentiating with respect to time, we get

$$\frac{\dot{L}_{jt}}{L_{jt}} = \frac{\dot{\chi}_{jt}}{\chi_{jt}} = \frac{\dot{A}_{jt}}{A_{jt}} - \frac{\dot{E}_t}{E_t} = \frac{\dot{A}_{jt}}{A_{jt}} - \gamma. \tag{D.17}$$

Substituting (B.4) into (23a), we find:

$$L_{jt} = \chi_{jt} = \frac{\omega_j A_{jt}^{(\varepsilon-1)\alpha_j}}{\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}}. \tag{D.18}$$

Taking logs and differentiating with respect to time, we find:

$$\frac{\dot{L}_{jt}}{L_{jt}} = -(1-\varepsilon)\alpha_j \frac{\dot{A}_{jt}}{A_{jt}} + \frac{(1-\varepsilon)\alpha_g \omega_g A_{gt}{}^{(\varepsilon-1)\alpha_g}}{\omega_g A_{gt}{}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}{}^{(\varepsilon-1)\alpha_s}} \frac{\dot{A}_{gt}}{A_{gt}} + \frac{(1-\varepsilon)\alpha_s \omega_s A_{st}{}^{(\varepsilon-1)\alpha_s}}{\omega_g A_{gt}{}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}{}^{(\varepsilon-1)\alpha_s}} \frac{\dot{A}_{st}}{A_{st}}. \tag{D.19}$$

Using (D.18) and (D.17), this equation can be rewritten to:

$$[1 + (1-\varepsilon)\alpha_j] \frac{\dot{A}_{jt}}{A_{jt}} = \gamma + (1-\varepsilon)\alpha_g L_{gt} \frac{\dot{A}_{gt}}{A_{gt}} + (1-\varepsilon)\alpha_s L_{st} \frac{\dot{A}_{st}}{A_{st}}. \tag{D.20}$$

Since the right–hand side is the same for both $j \in \{g, s\}$, this implies that the left–hand sides must be same too:

$$[1 + (1-\varepsilon)\alpha_g] \frac{\dot{A}_{gt}}{A_{gt}} = [1 + (1-\varepsilon)\alpha_s] \frac{\dot{A}_{st}}{A_{st}}. \tag{D.21}$$

Setting $j = s$ in (D.20), and using (D.21) to substitute out the growth rate of A_{gt} yield

$$[1 + (1-\varepsilon)\alpha_s] \frac{\dot{A}_{st}}{A_{st}} = \gamma + \frac{(1-\varepsilon)\alpha_g}{1 + (1-\varepsilon)\alpha_g} [1 + (1-\varepsilon)\alpha_s] L_{gt} \frac{\dot{A}_{st}}{A_{st}} + (1-\varepsilon)\alpha_s L_{st} \frac{\dot{A}_{st}}{A_{st}}.$$

Rearranging this yields

$$\begin{aligned} [1 + (1-\varepsilon)\alpha_s][1 + (1-\varepsilon)\alpha_g] \frac{\dot{A}_{st}}{A_{st}} &= [1 + (1-\varepsilon)\alpha_g]\gamma \\ &+ (1-\varepsilon)\alpha_g [1 + (1-\varepsilon)\alpha_s][1 - L_{st}] \frac{\dot{A}_{st}}{A_{st}} + (1-\varepsilon)\alpha_s [1 + (1-\varepsilon)\alpha_g] L_{st} \frac{\dot{A}_{st}}{A_{st}}, \end{aligned} \tag{D.22}$$

which can be rewritten as

$$\begin{aligned} [1+(1-\varepsilon)\alpha_s] \frac{\dot{A}_{st}}{A_{st}} + (1-\varepsilon)\alpha_g [1 + (1-\varepsilon)\alpha_s] \frac{\dot{A}_{st}}{A_{st}} &= [1 + (1-\varepsilon)\alpha_g]\gamma \\ &+ (1-\varepsilon)\alpha_g [1 + (1-\varepsilon)\alpha_s] \frac{\dot{A}_{st}}{A_{st}} - (1-\varepsilon)\alpha_g L_{st} \frac{\dot{A}_{st}}{A_{st}} - (1-\varepsilon)^2 \alpha_g \alpha_s L_{st} \frac{\dot{A}_{st}}{A_{st}} \\ &+ (1-\varepsilon)\alpha_s L_{st} \frac{\dot{A}_{st}}{A_{st}} + (1-\varepsilon)^2 \alpha_g \alpha_s L_{st} \frac{\dot{A}_{st}}{A_{st}}, \end{aligned}$$

implying

$$\left[1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}\right]\frac{\dot{A}_{st}}{A_{st}} = [1 + (1-\varepsilon)\alpha_g]\gamma.$$

The last equation can be solved for the growth rate of A_{st}. Using (D.21), we also obtain an expression for the growth rate of A_{gt}:

$$\frac{\dot{A}_{st}}{A_{st}} = \frac{1 + (1-\varepsilon)\alpha_g}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma, \tag{D.23a}$$

$$\frac{\dot{A}_{gt}}{A_{gt}} = \frac{1 + (1-\varepsilon)\alpha_s}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma. \tag{D.23b}$$

The growth rates of both A_{gt} and A_{st} vary continuously in L_{st} with finite limits at $L_{st} = 0$ and $L_{st} = 1$ under our assumptions that $\varepsilon \in [0, 1)$ and $\alpha_g > \alpha_s$. Hence to determine $\lim_{t\to\infty} A_{st}/A_{st}$, it is sufficient to check whether the differences between the growth rates of A_{st} and A_{gt} are strictly positive for all $L_{st} \in [0, 1]$. Equations (D.23a) and (D.23b) imply that

$$\frac{\dot{A}_{st}}{A_{st}} - \frac{\dot{A}_{gt}}{A_{gt}} = \frac{(1-\varepsilon)(\alpha_g - \alpha_s)}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma > 0, \tag{D.24}$$

which is indeed strictly positive for all $L_{st} \in [0, 1]$ since $\alpha_g - \alpha_s > 0$ by assumption. Hence, A_{st}/A_{gt} rises over time with $\lim_{t\to\infty} A_{st}/A_{gt} = \infty$.

D.1.2 Evolution of $A_{st}^{\alpha_s}/A_{gt}^{\alpha_g}$

We now turn to the evolution $A_{st}^{\alpha_s}/A_{gt}^{\alpha_g}$. Using the same argument as before, to determine $\lim_{t\to\infty} A_{st}^{\alpha_s}/A_{gt}^{\alpha_g}$, it is sufficient to check whether the differences between the growth rates of $A_{st}^{\alpha_s}$ and $A_{gt}^{\alpha_g}$ are strictly negative for all $L_{st} \in [0, 1]$. Equations (D.23a) and (D.23b) imply that

$$\alpha_s\frac{\dot{A}_{st}}{A_{st}} - \alpha_g\frac{\dot{A}_{gt}}{A_{gt}} = \frac{\alpha_s - \alpha_g}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma. \tag{D.25}$$

which is indeed strictly negative for all $L_{st} \in [0, 1]$ since $\alpha_s < \alpha_g$ by assumption. Hence $A_{st}^{\alpha_s}/A_{gt}^{\alpha_g}$ falls over time with $\lim_{t\to\infty} A_{st}^{\alpha_s}/A_{gt}^{\alpha_g} = 0$.

D.2 Evolution of L_{st}/L_{gt} and χ_{st}/χ_{gt}

Since $\chi_{st} = L_{st}$ and $\chi_{gt} = 1 - \chi_{st} = 1 - L_{st}$, it is sufficient to charterize the evolution of L_{st}/L_{gt}. We start by deriving the growth rate of L_{jt}. Using (D.19), we get

$$\frac{\dot{L}_{jt}}{L_{jt}} = -(1-\varepsilon)\alpha_j \frac{\dot{A}_{jt}}{A_{jt}} + (1-\varepsilon)\alpha_g L_{gt} \frac{\dot{A}_{gt}}{A_{gt}} + (1-\varepsilon)\alpha_s L_{st} \frac{\dot{A}_{st}}{A_{st}}.$$

Setting $j = g$ ($j = s$) and plugging in (D.23a) and (D.23b) for the growth rates A_{gt} and A_{st}, we find

$$\frac{\dot{L}_{gt}}{L_{gt}} = -\frac{(1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma, \tag{D.26a}$$

$$\frac{\dot{L}_{st}}{L_{st}} = \frac{(1-\varepsilon)(\alpha_g - \alpha_s)(1 - L_{st})}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma. \tag{D.26b}$$

The growth rates of both L_{gt} and L_{st} vary continuously in L_{st} with finite limits at $L_{st} = 0$ and $L_{st} = 1$ under our assumptions that $\varepsilon \in [0, 1)$ and $\alpha_g > \alpha_s$. Hence to determine $\lim_{t\to\infty} L_{st}/L_{st}$, it is sufficient to check whether the differences between the growth rates of L_{st} and $:_{gt}$ are strictly positive for all $L_{st} \in [0, 1]$. Combining the last two equations gives:

$$\frac{\dot{L}_{st}}{L_{st}} - \frac{\dot{L}_{gt}}{L_{gt}} = \frac{(1-\varepsilon)(\alpha_g - \alpha_s)}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma > 0, \tag{D.27}$$

which is indeed strictly positive for all $L_{st} \in [0, 1]$ since $\alpha_g - \alpha_s > 0$ by assumption. Hence, L_{st}/L_{gt} rises over time with $\lim_{t\to\infty} L_{st}/L_{gt} = \infty$. Since $L_{gt}, L_{st} \in (0, 1)$, this must mean $\lim_{t\to\infty} L_{gt} = 0$ and $\lim_{t\to\infty} L_{st} = 1$. Since $\chi_{st} = L_{st}$ and $\chi_{gt} = 1 - \chi_{st} = 1 - L_{st}$, we also have $\lim_{t\to\infty} \chi_{st}/\chi_{gt} = \infty$, and $\lim_{t\to\infty} \chi_{gt} = 0$ and $\lim_{t\to\infty} \chi_{st} = 1$.

D.3 Evolution of P_{st}/P_{gt}

(D.18) implies that

$$\chi_{gt} = \frac{\omega_g}{\omega_g + \omega_s (P_{st}/P_{gt})^{1-\varepsilon}}.$$

Together with $\dot{\chi}_g/\chi_g < 0$ and $\lim_{t\to\infty} \chi_{gt} = 0$, the equation implies that P_s/P_g grows and $\lim_{t\to\infty} P_{st}/P_{gt} = \infty$.

D.4 Evolution of P_{st}/P_{gt}

P_t is defined by

$$P_t \equiv \left(\omega_g P_{gt}^{\frac{1}{1-\varepsilon}} + \omega_s P_{st}^{\frac{1}{1-\varepsilon}}\right)^{\frac{1}{1-\varepsilon}}.$$

This implies that

$$\frac{\dot{P}_t}{P_t} = (1-\chi_{st})\frac{\dot{P}_{gt}}{P_{gt}} + \chi_{st}\frac{\dot{P}_{st}}{P_{st}}.$$

It is necessary and sufficient for P_t to grow without bound for all $\chi_{st} \in [0,1]$ that P_{gt} grows without bound since $\lim_{t\to\infty} P_{st}/P_{gt} = \infty$. It follows from (B.4) that

$$\frac{\dot{P}_{gt}}{P_{gt}} = (1-\theta)\gamma - \alpha_g \frac{\dot{A}_{gt}}{A_{gt}} > [(1-\theta) - \alpha_g]\gamma. \tag{D.28}$$

The last inequality follows from the fact that the growth rate of A_{gt} is bounded from above by γ (see (D.23b)). To see this, observe that under our assumptions of $\alpha_g > \alpha_s$ and $\varepsilon \in [0,1)$, (D.23b) implies that the growth rate of A_{gt} is monotonically decreasing in L_{st}. In addition, (D.23b) also implies

$$\lim_{L_{st}\to 0} \frac{\dot{A}_{gt}}{A_{gt}} = \gamma > \frac{\dot{A}_{gt}}{A_{gt}} > \lim_{L_{st}\to 1} \frac{\dot{A}_{st}}{A_{st}} = \frac{1+(1-\varepsilon)\alpha_s}{1+(1-\varepsilon)\alpha_g}\gamma.$$

Finally, the inequality (D.28) implies that $(1-\theta) > \alpha_g$ is a sufficient condition for P_t to be rising.

E Efficient allocation

We characterise the solution to the planner problem in two steps.

(i) Given A_{gt}, A_{st} and K_{ct}, we characterise the static efficient allocation of K_{ct} and L_t across sectors and varieties, and the implied efficient levels of C_{gt}, C_{st} and C_t.

(ii) We characterise the dynamic efficient allocation of $\{K_t, K_{ct}, A_{gt}, A_{st}\}_{t=0}^{\infty}$.

E.1 Efficient static allocation

E.1.1 Static planner problem

$$
\max_{\substack{C_{gt},C_{st},z_{gt}(i),z_{st}(i),\\ k_{gt}(i),k_{st}(i),l_{gt}(i),l_{st}(i)}} \left(\omega_g^{\frac{1}{\varepsilon}} C_{gt}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{\frac{1}{\varepsilon}} C_{st}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}
$$

$$
\begin{aligned}
\text{s.t.} \qquad C_{jt} &= A_{jt}^{\alpha_j+1-\frac{\sigma}{\sigma-1}} \left(\int_0^{A_{jt}} [z_{jt}(i)]^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}, \qquad j \in g, s,\\
z_{jt}(i) &= \frac{1}{\theta^\theta(1-\theta)^{1-\theta}} [k_{jt}(i)]^\theta [l_{jt}(i)]^{1-\theta}, \qquad j \in g, s,\ i \in [0, A_{jt}],\\
K_{ct} &= \int_0^{A_{gt}} k_{gt}(i)di + \int_0^{A_{st}} k_{st}(i)di,\\
1 &= \int_0^{A_{gt}} l_{gt}(i)di + \int_0^{A_{st}} l_{st}(i)di.
\end{aligned}
$$

The first–order conditions to the static planner problem are

$$
C_{jt}: \quad \lambda_{jt}^c = \omega_j^{\frac{1}{\varepsilon}} \left(\frac{C_t}{C_{jt}}\right)^{\frac{1}{\varepsilon}}, \tag{E.30a}
$$

$$
z_{jt}(i): \quad \lambda_{jt}^z(i) = \lambda_{jt}^c A_{jt}^{(\alpha_j+1)\frac{\sigma-1}{\sigma}-1} \left(\frac{C_{jt}}{z_{jt}(i)}\right)^{\frac{1}{\sigma}}, \tag{E.30b}
$$

$$
k_{jt}(i): \quad \lambda_t^k = \lambda_{jt}^z(i) \frac{\theta}{\theta^\theta(1-\theta)^{1-\theta}} [k_{jt}(i)]^{\theta-1} [l_{jt}(i)]^{1-\theta}, \tag{E.30c}
$$

$$
l_{jt}(i): \quad \lambda_t^l = \lambda_{jt}^z(i) \frac{1-\theta}{\theta^\theta(1-\theta)^{1-\theta}} [k_{jt}(i)]^{\theta} [l_{jt}(i)]^{-\theta}, \tag{E.30d}
$$

where $\lambda_{jt}^c, \lambda_{jt}^z(i), \lambda_t^k, \lambda_t^l$ are the shadow prices associated with the respective constraints.

E.1.2 Efficient allocation of capital across sectors and varieties

(E.30c) and (E.30d) imply that the capital–labor ratios are equalised across sectors:

$$
K_{ct} = \frac{K_{jt}}{L_{jt}} = \frac{\int_0^{A_{jt}} k_{jt}(i)di}{\int_0^{A_{jt}} l_{jt}(i)di} = \frac{k_{jt}(i)}{l_{jt}(i)} = \frac{\theta}{1-\theta} \frac{\lambda_t^l}{\lambda_t^k}. \tag{E.31}
$$

It follows then from (E.30c) that

$$
\lambda_{jt}^z(i) = \lambda_{jt}^z = \lambda_t^z.
$$

where the last equality follows from the fact that the capital–labor ratios are equalized. If $\lambda^z_{jt}(i) = \lambda^z_{jt}$, then (E.30b) implies that

$$z_{jt}(i) = z_{jt}.$$

If $z_{jt}(i) = z_{jt}$ and the capital-labor ratios are equalised, then the production technologies imply that

$$k_{jt}(i) = k_{jt}, \qquad l_{jt}(i) = l_{jt}.$$

It follows now that the efficient $z_{jt}(i) = z_{jt}$ is given by

$$z_{jt} = \frac{1}{\theta^\theta (1-\theta)^{1-\theta}} K^\theta_{ct} l_{jt}, \tag{E.32}$$

which in turn implies that the efficient sector consumption is

$$C_{jt} = \frac{A^{\alpha_j}_{jt}}{\theta^\theta (1-\theta)^{1-\theta}} K^\theta_{ct} A_{jt} l_{jt} = \frac{A^{\alpha_j}_{jt}}{\theta^\theta (1-\theta)^{1-\theta}} K^\theta_{ct} L_{jt}. \tag{E.33}$$

E.1.3 Efficient allocation of labor across sectors and varieties

Combining (E.32) and (E.33) leads to

$$\frac{C_{jt}}{z_{jt}} = A^{\alpha_j+1}_{jt}. \tag{E.34}$$

Substituting this into (E.30b), we obtain

$$\lambda^z_t = \lambda^c_{jt} A^{\alpha_j}_{jt},$$

or

$$\lambda^c_{jt} = A^{-\alpha_j}_{jt} \lambda^z_t. \tag{E.35}$$

It follows from (E.30a) and (E.35) that

$$\left(\frac{\omega_g}{\omega_s}\frac{C_{st}}{C_{gt}}\right)^{\frac{1}{\varepsilon}} = \frac{\lambda^c_{gt}}{\lambda^c_{st}} = \frac{A^{-\alpha_g}_{gt}}{A^{-\alpha_s}_{st}}.$$

Using the expression for the efficient sector output, (E.33), this can be rewritten as

$$\left(\frac{\omega_g}{\omega_s}\frac{A_{st}^{\alpha_s} L_{st}}{A_{gt}^{\alpha_g} L_{gt}}\right)^{\frac{1}{\varepsilon}} = \frac{A_{gt}^{-\alpha_g}}{A_{st}^{-\alpha_s}} \tag{E.36}$$

implying

$$\frac{\omega_g}{\omega_s}\frac{A_{st}^{(1-\varepsilon)\alpha_s}}{A_{gt}^{(1-\varepsilon)\alpha_g}} = \frac{L_{gt}}{L_{st}} = \frac{1-L_{st}}{L_{st}}. \tag{E.37}$$

Hence the efficient labor allocation is given by

$$L_{jt} = \frac{\omega_j A_{jt}^{-(1-\varepsilon)\alpha_j}}{\omega_g A_{gt}^{-(1-\varepsilon)\alpha_g} + \omega_s A_{st}^{-(1-\varepsilon)\alpha_s}}. \tag{E.38}$$

E.1.4 Efficient level of C_t

Substituting (E.38) into (E.33) yields the efficient sector consumption

$$C_{jt} = \frac{1}{\theta^\theta (1-\theta)^{1-\theta}} \frac{\omega_j A_{jt}^{\varepsilon\alpha_j}}{\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}} K_{ct}^{\theta}. \tag{E.39}$$

Substituting now equation (E.39) into the aggregator for C_t, (2), yields the efficient aggregate consumption

$$C_t = \frac{1}{\theta^\theta (1-\theta)^{1-\theta}} \left(\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}\right)^{\frac{1}{\varepsilon-1}} K_{ct}^{\theta}. \tag{E.40}$$

E.2 Dynamic efficient allocation

E.2.1 Dynamic planner problem

$$\max_{\{C_t, X_{gt}, X_{st}, K_t, K_{ct}, A_{gt}, A_{st}\}_{t=0}^{\infty}} \int_0^\infty \exp(-\rho t)\log(C_t)dt \tag{E.41a}$$

$$\text{s.t} \qquad C_t = \frac{1}{\theta^\theta (1-\theta)^{1-\theta}} \left(\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}\right)^{\frac{1}{\varepsilon-1}} K_{ct}^{\theta} \tag{E.41b}$$

$$\dot{K}_t = A_x(K_t - K_{ct}) - (X_{gt} + X_{st}) - \delta K_t \tag{E.41c}$$

$$\dot{A}_{jt} = X_{jt}, \qquad j \in \{g, s\}. \tag{E.41d}$$

Note that (E.41b) implies that the social production function of C_t is concave in A_{jt} if $\alpha_j \in [0,1]$.

The first–order conditions for the intertemporal planner problem are

$$C_t: \quad \frac{1}{C_t} = \mu_t^c, \tag{E.42a}$$

$$X_{jt}: \quad \mu_t^k = \mu_{jt}^A, \tag{E.42b}$$

$$K_{ct}: \quad \mu_t^c \theta \frac{C_t}{K_{ct}} = \mu_t^k A_x, \tag{E.42c}$$

$$K_t: \quad -\frac{\partial}{\partial t}\left(\mu_t^k \exp(-\rho t)\right) = \mu_t^k \exp(-\rho t)(A_x - \delta), \tag{E.42d}$$

$$A_{jt}: \quad -\frac{\partial}{\partial t}\left(\mu_{jt}^A \exp(-\rho t)\right) = \mu_t^c \exp(-\rho t) C_t \frac{\alpha_j \omega_j A_{jt}^{(\varepsilon-1)\alpha_j - 1}}{\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}}, \tag{E.42e}$$

where $\mu_t^c \exp(-\rho t), \mu_t^k \exp(-\rho t), \mu_{jt}^A \exp(-\rho t)$ are the present value shadow prices associated with the respective constraint. Since $\mu_t^c C_t = 1$, the previous system of equation can be restated as

$$\frac{1}{C_t} = \mu_t^c, \tag{E.43a}$$

$$\mu_t^k = \mu_{jt}^A, \tag{E.43b}$$

$$\frac{\theta}{K_{ct}} = \mu_t^k A_x, \tag{E.43c}$$

$$\rho - \frac{\dot{\mu}_t^k}{\mu_t^k} = A_x - \delta, \tag{E.43d}$$

$$\rho - \frac{\dot{\mu}_{jt}^A}{\mu_{jt}^A} = \frac{1}{\mu_{jt}^A} \frac{\alpha_j \omega_j A_{jt}^{(\varepsilon-1)\alpha_j - 1}}{\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}}. \tag{E.43e}$$

E.2.2 Efficient growth rates of $\mu_t^k, \mu_{gt}^A, \mu_{st}^A$

(E.43d) implies that the shadow price of capital, μ_t^k, falls at a constant rate

$$-\frac{\dot{\mu}_{jt}^k}{\mu_{jt}^k} = \gamma = A_x - \delta - \rho. \tag{E.44}$$

It follows now from (E.43b) that $\mu_t^k = \mu_{gt}^A = \mu_{st}^A$, hence μ_{jt}^A also falls at rate γ.

E.2.3 Efficient growth rate of K_{ct}

(E.43c) together with the fact that μ_t^k grows at rate γ implies that K_{ct} grows at rate γ.

E.2.4 Efficient growth rates of A_{jt}

Since $\mu^A_{gt} = \mu^A_{st}$, and they grow at a constant rate, it follows from (E.43e) that

$$\alpha_g \omega_g A_{gt}^{(\varepsilon-1)\alpha_g - 1} = \alpha_s \omega_s A_{st}^{(\varepsilon-1)\alpha_s - 1} \tag{E.45}$$

implying

$$[1 + (1-\varepsilon)\alpha_g]\frac{\dot{A}_{gt}}{A_{gt}} = [1 + (1-\varepsilon)\alpha_s]\frac{\dot{A}_{st}}{A_{st}}. \tag{E.46}$$

The left–hand side of (E.43e) is constant over time. Hence, taking logs and the derivatives with respect to time of the right hand side yields:

$$\begin{aligned} 0 = & -\frac{\dot{\mu}^A_{jt}}{\mu^A_{jt}} + [(\varepsilon-1)\alpha_j - 1]\frac{\dot{A}_{jt}}{A_{jt}} \\ & - \frac{(\varepsilon-1)\alpha_g \omega_g A_{gt}^{(\varepsilon-1)\alpha_g}}{\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}}\frac{\dot{A}_{gt}}{A_{gt}} - \frac{(\varepsilon-1)\alpha_s \omega_s A_{st}^{(\varepsilon-1)\alpha_s}}{\omega_g A_{gt}^{(\varepsilon-1)\alpha_g} + \omega_s A_{st}^{(\varepsilon-1)\alpha_s}}\frac{\dot{A}_{st}}{A_{st}}, \end{aligned}$$

which can be rewritten to:

$$[1 + (1-\varepsilon)\alpha_j]\frac{\dot{A}_{jt}}{A_{jt}} = \gamma + (1-\varepsilon)\alpha_g L_{gt}\frac{\dot{A}_{gt}}{A_{gt}} + (1-\varepsilon)\alpha_s L_{st}\frac{\dot{A}_{st}}{A_{st}}, \tag{E.47}$$

where we used (E.38) and the fact that μ^A_{jt} falls at rate γ. Substituting (E.46) into this equation for $j = g$ and solving for $\dot{A}_{st}/A_{st}$, we get the same expression for the growth rate as in the laissez faire equilibrium (compare (D.20) and (E.47)). Consequently, the efficient growth rate of $\dot{A}_{st}/A_{st}$ if and only if the L_{jt} are the same in the laissez faire equilibrium.

E.2.5 Efficient paths of A_t, A_{gt}, A_{st}

First we derive an expression for X_{jt} which amounts to deriving an expression for $\dot{A}_{jt}$ as $\dot{A}_{jt} = X_{jt}$. Combining (E.43b) and (E.43c) leads to

$$\mu^A_{jt} = \frac{\theta}{A_x K_{ct}}.$$

This equation together with (E.38) and (E.43e) imply that

$$A_{jt} = \frac{1}{\theta}\frac{A_x}{A_x - \delta}\alpha_j K_{ct} L_{jt}, \tag{E.48}$$

where we used that μ_{jt}^A grows at rate γ. Summing over j and rearranging leads to

$$A_t = A_{gt} + A_{st} = \frac{1}{\theta}\frac{A_x}{A_x - \delta}(\alpha_g L_{gt} + \alpha_s L_{st})K_{ct}. \tag{E.49}$$

Comparing (C.11) with (E.48), and (C.12) with (E.49), while taking into account that $\chi_{jt} = L_{jt}$ under laissez faire, we conclude that both the number of varieties per sector and in the whole economy are different across the laissez faire allocation and the solution to the planner problem.

Next we derive an expression for $\dot{A}_{gt} + \dot{A}_{st}$. Taking the derivative of (E.49) with respect to time yields

$$\dot{A}_{gt} + \dot{A}_{st} = \frac{1}{\theta}\frac{A_x}{A_x - \delta}\left((\alpha_g \dot{L}_{gt} + \alpha_s \dot{L}_{st}) + (\alpha_g L_{gt} + \alpha_s L_{st})\frac{\dot{K}_{ct}}{K_{ct}}\right)K_{ct}. \tag{E.50}$$

Note that the expression for the efficient labor allocation, (E.38), and that of the expenditure share in laissez faire equilibrium, (D.18), are the same. Hence we can use (D.26b) to derive an expression for the efficient law of motion of $\dot{L}_{st}$,

$$\dot{L}_{st} = \frac{(1-\varepsilon)(\alpha_g - \alpha_s)L_{st}(1-L_{st})}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}\gamma. \tag{E.51}$$

Now using that $L_{gt} = 1 - L_{st}$ and $-\dot{L}_{gt} = \dot{L}_{st}$, equation (E.50) can be restated as

$$\dot{A}_{gt} + \dot{A}_{st} = \frac{1}{\theta}\frac{A_x}{A_x - \delta}\left(-\frac{(1-\varepsilon)(\alpha_g - \alpha_s)^2 L_{st}(1-L_{st})}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}} + \alpha_g - (\alpha_g - \alpha_s)L_{st}\right)\gamma K_{ct}.$$

This can be rewritten as

$$\frac{\dot{A}_{gt} + \dot{A}_{st}}{K_{ct}} = \frac{\gamma}{\theta}\frac{A_x}{A_x - \delta}\frac{\alpha_g[1 + (1-\varepsilon)\alpha_s] - (\alpha_g - \alpha_s)L_{st}}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}}. \tag{E.52}$$

Not that the term in the square bracket equals α_g for $L_{st} = 0$, and α_s for $L_{st} = 1$, and the expression is monotonically decreasing in L_{st} as $\alpha_g - \alpha_s > 0$.

E.2.6 Efficient path of K_t

The resource constraint (E.41c) can be rewritten

$$\dot{K}_t = (A_x - \delta)K_t - \left(A_x - \frac{\dot{A}_{gt} + \dot{A}_{st}}{K_{ct}}\right)K_{ct}, \tag{E.53}$$

where we used that $\dot{A}_{jt} = X_{jt}$.

Plugging (E.52) into (E.53) for $\dot{A}_{gt} + \dot{A}_{st}$, we get

$$\dot{K}_t = (A_x - \delta)K_t - A_x \left[1 + \frac{\gamma}{\theta(A_x - \delta)} \frac{\alpha_g[1 + (1-\varepsilon)\alpha_s] - (\alpha_g - \alpha_s)L_{st}}{1 + (1-\varepsilon)\alpha_s + (1-\varepsilon)(\alpha_g - \alpha_s)L_{st}} \right] K_{ct}. \tag{E.54}$$

The term in the squared bracket on the right hand side of (E.54) declining in L_{st}, and for $L_{st} = 0$ it equals to

$$1 + \frac{\gamma\alpha_g}{\theta(A_x - \delta)},$$

and for $L_{st} = 1$ it equals to

$$1 + \frac{\gamma\alpha_s}{\theta(A_x - \delta)}.$$

- Suppose that
$$\frac{K_{cT}}{K_T} \leq \frac{\rho}{A_x} \frac{\theta(A_x - \delta)}{\theta(A_x - \delta) + \gamma\alpha_g}$$
for some T. (E.54) implies that $\gamma_T(K) \leq \gamma$ for $L_{st} = 0$. Since the right hand side of (E.41c) is increasing in L_{st}, and L_{st} is monotonically increasing over time, we have $\gamma_t(K) < \gamma$ for all $t > T$. As K_{ct} is growing at rate γ, it follows that $\lim_{t\to\infty}(K_{ct}/K_t) = \infty$ implying that $\lim_{t\to\infty} \gamma_t(K) = -\infty$. This violates the resources constraint.

- Suppose that
$$\frac{K_{cT}}{K_T} \geq \frac{\rho}{A_x} \frac{\theta(A_x - \delta)}{\theta(A_x - \delta) + \gamma\alpha_s}$$
for some T. (E.54) implies that $\gamma_T(K) \geq \gamma$ for $L_{st} = 1$. Since the right hand side of (E.41c) is increasing in L_{st}, we $\gamma_t(K) > \gamma$ for all L_{st} and hence for all $t > T$ with $\lim_{t\to\infty} \gamma_t(K) = A_x - \delta$. But that would violate the transversality condition.

This implies that

$$\frac{\rho}{A_x} \frac{\theta(A_x - \delta)}{\theta(A_x - \delta) + \gamma\alpha_g} \leq \frac{K_{c0}}{K_0} \leq \frac{\rho}{A_x} \frac{\theta(A_x - \delta)}{\theta(A_x - \delta) + \gamma\alpha_s}. \tag{E.55}$$

Since the problem is concave, it has a solution such that for a given K_0 there is a K_{c0} so that K_{c0}/K_0 satisfies (E.55), $\gamma_t(K) < \gamma$ and $\gamma_t(K)$ monotonically rising over time with $\lim_{t\to\infty} \gamma_t(K) = \gamma$. The reason why $\gamma_t(K) < \gamma$ for all $t \in [0, \infty)$ is that if there is a t such that $\gamma_t(k) > \gamma$, then

$$\frac{K_{ct+\Delta t}}{K_{t+\Delta t}} < \frac{K_{ct}}{K_t}$$

since K_{ct} grows at rate γ. But then it follows from (E.41c) that $\gamma_{t+\Delta t}(K) > \gamma_t(K) > \gamma$, and

$\lim_{t\to\infty} \gamma_t(K) = A_x - \delta$ which would violate the transversality condition. $\gamma_t(K) > \gamma$ keeps rising because both K_{ct}/K_t and the term in the squared bracket is falling.

F Proof of Proposition 3

First we collect the conditions that characterize the laissez faire and the efficient allocation. Second, we show that the two sets of conditions are equivalent if $\alpha_g = \alpha_s = 1/(\sigma - 1)$.

F.1 The laissez faire equilibrium allocation

In addition to consumption aggregator, (6), the resource constraints, and the transversality condition, the following conditions characterize the laissez faire allocation:

$$K_{ct} = \frac{k_{jt}}{l_{jt}}, \tag{F.56a}$$

$$z_{jt} = \frac{1}{\theta^\theta (1-\theta)^{1-\theta}} k_{jt}^\theta l_{jt}^{1-\theta}, \qquad j \in \{g, s\} \tag{F.56b}$$

$$C_{jt} = A_{jt}^{\alpha_j + 1} z_{jt}, \qquad j \in \{g, s\} \tag{F.56c}$$

$$1 = \frac{1}{A_x - \delta} \frac{1}{\sigma - 1} \frac{A_x k_{jt}}{\theta}, \qquad j \in \{g, s\} \tag{F.56d}$$

$$\frac{C_{st}}{C_{gt}} = \frac{\omega_s}{\omega_g} \frac{A_{st}^{\varepsilon \alpha_s}}{A_{gt}^{\varepsilon \alpha_g}}, \tag{F.56e}$$

$$\frac{\dot{K}_{ct}}{K_{ct}} = A_x - \delta - \rho, \tag{F.56f}$$

Note that we eliminated all the relative prices to make it comparable with the efficient allocation. (F.56a) states that the capital–labor ratios are equalised across sectors, and varieties. (F.56b) is the technology for intermediate goods. (F.56c) is obtained from equation (2), from the technology of the sector consumption for $z_{jt}(i) = z_{jt}$. Conditions (24c) and (24d) of Lemma 1 imply (F.56d) when one takes into account that $K_{jt} = A_{jt} k_{jt}$. (F.56e) follows the household first order condition, (23a), after we substituted out relative prices with (B.4). We get (F.56f) from the Euler–equation after substituting out E_t with (C.8c).

F.2 The efficient allocation

In addition to consumption aggregator, (6), the resource constraints and the transversality condition, the following conditions characterize the efficient allocation:

$$K_{ct} = \frac{k_{jt}}{l_{jt}}, \tag{F.57a}$$

$$z_{jt} = \frac{1}{\theta^{\theta}(1-\theta)^{1-\theta}} k_{jt}^{\theta} l_{jt}^{1-\theta}, \qquad j \in \{g, s\} \tag{F.57b}$$

$$C_{jt} = A_{jt}^{\alpha_j+1} z_{jt}, \qquad j \in \{g, s\} \tag{F.57c}$$

$$1 = \frac{1}{A_x - \delta} \alpha_j \frac{A_x k_{jt}}{\theta}, \qquad j \in \{g, s\} \tag{F.57d}$$

$$\frac{C_{st}}{C_{gt}} = \frac{\omega_s}{\omega_g} \frac{A_{st}^{\varepsilon\alpha_s}}{A_{gt}^{\varepsilon\alpha_g}}, \tag{F.57e}$$

$$\frac{\dot{K}_{ct}}{K_{ct}} = A_x - \delta - \rho, \tag{F.57f}$$

(F.57a) states that the capital–labor ratios are equalised across sectors, and varieties as we have shown for the efficient allocation (see (E.31)). (F.57b) is the technology for intermediate goods. (F.57c) is obtained from equation (2), from the technology of the sector consumption for $z_{jt}(i) = z_{jt}$. (F.57d) follows from (E.48) when one takes into account that $K_{jt} = K_{ct} L_{jt}$ and $K_{jt} = A_{jt} k_{jt}$. Equation (F.57e) follows from the efficient marginal rate of substitution, (E.36) together with (E.33). Equation (F.57f) follows from combining (E.43c) and (E.44).

F.3 Comparing the laissez faire and efficient allocation

Comparing the laissez faire allocation, (F.56a)–(F.56f), with the efficient allocation, (F.57a)–(F.57f), we can see that only equations (F.56d) and (F.57d) are different. However, if

$$\alpha_g = \alpha_s = \frac{1}{\sigma - 1},$$

the two equations become equivalent.

Now consider a laissez–faire equilibrium and a solution to the planner problem that start with the same K_0, A_{go}, A_{s0}. We know from (F.56d) and (F.57d) that $k_{jt} = k_{jt}^*$ are the same. Since $A_{jo} = A_{j0}^*$, we also have $K_{jt} = K_{jt}^*$ and $K_{ct} = K_{ct}^*$. Using (F.56a) and (F.57a), it follows that $l_{jt} = l_{jt}^*$ and $L_{jt} = L_{jt}^*$. In addition, (F.56f) and (F.57f) imply that K_{ct} and K_{ct}^* both grow at rate γ. We know from Proposition 1 that K_t and A_t also grow at rate γ.

It remains to show that A^*_t and K^*_t also growth at rate γ. For $\alpha_g = \alpha_s$, (E.49) implies that A^*_t grows at the same rate as K^*_{ct}, that is, γ. Hence, $X_{gt} + X_{st} = X^*_{gt} + X^*_{st}$. Since $K_0 = K^*_0$, $K_{c0} = K^*_{c0}$, and $X_{g0} + X_{s0} = X^*_{g0} + X^*_{s0}$, we have $K_{x0} = K^*_{x0}$ and $\dot{K}_0 = \dot{K}^*_0$. Moving the same argument forward implies that $K_{xt} = K^*_{xt}$, $\dot{K}_t = \dot{K}^*_t$, and $K_t = K^*_t$.

G Proof of Proposition 4

The first step is to notice if a value $\underline{A}_1$ is part of a GBGP equilibrium, then all $A \in \{\underline{A}_1, \infty)$ are part of that equilibrium. The reason for this is that the equilibrium path is interior and smooth and that A_t converges to ∞ as time goes to infinity. Similarly, if a value $\underline{A}_2$ is part of a solution path to the planner problem, then all $A \in \{\underline{A}_2, \infty)$ are part of that solution path. Now define $\underline{A} \equiv \max\{\underline{A}_1, \underline{A}_2\}$.

The second step is to note that the private and the social marginal rate of substitution between the A_j equal the marginal rate of transformation between them. Since it takes one unit of investment to create a new variety in either sector, the marginal rate of transformation equals one. Hence, we have:

$$\frac{\omega_g (A_g)^{(\varepsilon-1)\alpha_g - 1}}{\omega_s (A_s)^{(\varepsilon-1)\alpha_s - 1}} = 1 = \frac{\alpha_g \omega_g (A^*_g)^{(\varepsilon-1)\alpha_g - 1}}{\alpha_s \omega_s (A^*_s)^{(\varepsilon-1)\alpha_s - 1}}. \tag{G.58}$$

Using that $A = A_g + A_S = A^*_g + A^*_s$, the previous equation implies:

$$\frac{(A_s)^{1+(1-\varepsilon)\alpha_s}}{(A - A_s)^{1+(1-\varepsilon)\alpha_g}} > \frac{(A^*_s)^{1+(1-\varepsilon)\alpha_s}}{(A - A^*_s)^{1+(1-\varepsilon)\alpha_g}}. \tag{G.59}$$

Since the ratios are monotonically increasing in A_s and A^*_s, (G.59) implies that $A_s > A^*_s$. Since $A = A_g + A_S = A^*_g + A^*_s$, this implies that $A_g < A^*_g$. In other words, for the same given state variable, A, the planner allocates more varieties to the goods sector and fewer varieties to the service sector. The intuitive reason for this is that the planner's innovation choices takes into account that the positive externality from the return to specialization is larger in the goods than in the service sector.

(25) and (G.58) imply that

$$\frac{L_{st}}{L_{gt}} = \frac{\omega_g (A_g)^{(1-\varepsilon)\alpha_g}}{\omega_s (A_s)^{(1-\varepsilon)\alpha_s}}.$$

Recalling (E.37), we also have:

$$\frac{L_{st}^*}{L_{gt}^*} = \frac{\omega_g (A_g^*)^{(1-\varepsilon)\alpha_g}}{\omega_s (A_s^*)^{(1-\varepsilon)\alpha_s}}.$$

Combining the last two equations with the facts that $A_g < A_g^*$ and $A_s > A_s^*$, we get:

$$\frac{L_{st}(A)}{L_{gt}(A)} < \frac{L_{st}^*(A)}{L_{gt}^*(A)} \quad \forall A \in (\underline{A}, \infty). \tag{G.60}$$

In other words, for any value of total varieties that lies on both paths, the planner problem allocates more labor to services than the GBGP. In lose terms, this means that equilibrium structural transformation takes place inefficiently slowly.

This leaves to show that for a given $A \in (\underline{A}, \infty)$ variety growth is smaller under then planner problem than in the laissez faire equilibrium, which again says that the planner is already further ahead in the process of structural transformation. To see why this is the case, note that, as Appendix E.2.4 shows, the functional form for $\dot{A}_{jt}^*/A_{jt}^*$ is the same as for $\dot{A}_{jt}/A_{jt}$, which was given in (26a)–(26b). Hence, $L_{st}^*(A) > L_{st}(A)$ implies that $\dot{A}_{jt}^*(A)/A_{jt}^*(A) < \dot{A}_{jt}(A)/A_{jt}(A)$ for all $A \in (\underline{A}, \infty)$.