XSCH3011

Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
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FACULTY OF ARTS, HUMANITIES AND SOCIAL SCIENCES

SCHOOL OF SOCIAL SCIENCES AND PHILOSOPHY

DEPARTMENT OF ECONOMICS

Senior Freshman
BESS, TSM, PPES

Hilary Term 2018

EC: QUANTITATIVE METHODS

FOUNDATION SCHOLARSHIP EXAMINATION

Tuesday 9th January 2018 GMB 14.00 – 16.15

Professor Andrea Guariso

Instructions to Candidates:

Please answer any 3 of the 4 questions Each question carries equal weight

Materials permitted for this examination: Standard calculator

You may not start this examination until you are instructed to do so by the Invigilator.

Please answer 3 of the 4 questions

Question 1

(a) Consider the following simple IS/LM economic model

$$Y = C + I + G_0$$

$$C = 200 + 0.8Y$$

$$I = 1000 - 2000r$$

$$M_0 = -1000r + 0.1Y$$

Where Y is national income, C is consumption, I is investment, G_0 is government spending, r is the interest rate, and M_0 is the money supply. The endogenous variable are Y, C, I, and r.

(i) Set up the model in matrix form.

[20 marks]

(ii) Explain what Cramer's rule is and how, in general, it is important for the analysis of economic models.

[20 marks]

(iii) Use Cramer's rule to determine the equilibrium level of income Y^*.

[20 marks]

(iv) What is the impact of an increase in government spending on Y^*? And of an increase in Money supply? Interpret your findings.

[20 marks]

Question continues on next page.

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(b) Compute the following definite integral

$$\int_{1}^{2} \frac{2x^3}{(2+x^2)^3}\,dx$$

[20 marks]

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Question 2

(a) Consider an unconstrained optimization problem in which you want to maximize the function $U = f(x, y, \alpha)$ where x and y are the choice variables and α is a parameter.

(i) Write down the First Order Conditions. What are the solutions x^* and y^* going to depend on?

[10 marks]

(ii) Explain what the Value Function represents. Write down the Value Function for this problem.

[10 marks]

Now consider a new function that captures the difference between the objective function and the value function: $h(x, y, \alpha) = f(x, y, \alpha) - V(\alpha)$. This can now be considered a function of three variables x, y, and α.

(iii) You now want to maximize this new function $h(x, y, \alpha)$. Write down the First Order Conditions. Comment on the conditions and their solutions, comparing them to the ones you obtained at point (a).

[20 marks]

(iv) Write down the Hessian matrix and the second-order sufficient conditions required to obtain a maximum.

[20 marks]

Question continues on next page

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(v) Now consider the value function $V(\alpha)$ and write down the expression for V_α and $V_{\alpha\alpha}$. Assume that α only enters the first order condition relating to x and suppose that $f_{\alpha\alpha} - V_{\alpha\alpha} < 0$ and $f_{x\alpha} > 0$. Can you say anything about the sign of the derivative $\frac{\partial x^*}{\partial \alpha}$?

[20 marks]

(b) The revenues generated by a continuous revenue stream of D dollars per year over the time interval $[t, t + dt]$ is given by $D\,dt$. Suppose now that the stream is continuously discounted at the rate of r per year (i.e. the discount rate is e^{-rt}). What is the present value of the continuous revenue stream, if it lasts for n years? And what if it lasts forever? Provide an interpretation of the present value.

[20 marks]

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Question 3

(a) Andrea collects two things: mathematical economics books and soccer cards. Andrea's utility function is $U(b, c) = b + 100c - c^2$, where b represents the number of econ books on his shelves, and c is the number of boxes of soccer cards on the shelves. In his office, Andrea has a total of 500cm of shelf space to allocate between these two collections, and he is trying to figure out what the utility maximizing display arrangement is. Books take up just 1cm apiece, while the boxes of cards take up 4cm.

(i) Write down the constrained optimization problem for Andrea and the corresponding Lagrangian function.

[20 marks]

(ii) Solve for optimal values of b, c and the Lagrange muliplier λ.

[20 marks]

(iii) Give an interpretation of the value of the Lagrange muliplier λ.

[20 marks]

(iv) Construct the bordered Hessian matrix and check whether your solution is indeed a maximum.

[20 marks]

(b) Provide an economic example of a concave function, show that it is concave, and discuss its economic intuition. Why are concave functions important and very much used in economics?

[20 marks]

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Question 4

(a) Consider a firm that operates in a perfectly competitive market with the following marginal cost function: $MC(y) = 3y^2 + 4y + 2$

(i) Find the total variable costs $TVC(y)$ for the firm. Does the expression from which you extract the total variable cost function include an additional term? If so, what is its economic interpretation? Justify this interpretation.

[20 marks]

(ii) Find the shut-down price for the firm. What does the shut-down price tell us? What would be the shut-down price if instead $MC(y) = 3y^2 - 4y + 2$? Explain.

[20 marks]

(iii) Evaluate the producer surplus at $p = 9$.

[20 marks]

(iv) Suppose the price increases to $p = 41$. Evaluate the change in producer surplus. There are at least two ways of tackling this question. Indicate (without implementing) an alternative procedure to the one you used.

[20 marks]

(b) Suppose the marginal propensity to save (MPS) is the following function of income: $MPS(Y) = 0.3 - 0.1Y^{-\frac{1}{2}}$. First, comment on how the MPS varies with income. Second, supposing that the aggregate saving S is nil when income Y is 81, find the saving function $S(Y)$.

[20 marks]

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