

An Empirical Investigation into Speculative Bubbles in Ethereum's Price

Michaela Fricova, Senior Sophister
Jonathon McKeown, Senior Sophister

Cryptocurrencies have dramatically changed the way we think about money. With the rapid development of electronic payments around the globe, their popularity has skyrocketed over the past decade, and cryptocurrencies now occupy a considerable space in many investors' portfolios. With this increase in popularity has come a huge amount of price volatility, and consequently most of these currencies are prone to mildly explosive speculative bubbles. In this paper, Michaela Fricova and Jonathon McKeown seek to establish whether the Ethereum market reflects periodically collapsing speculation in its prices, using a recursive unit root procedure introduced by Phillips (2013a). Their results strongly support the hypothesis of multiple bubbles emerging in the series. They date-stamp 15 bubbles over the investigated time period, ranging from one day to 73 days in length. The longest bubble period, spanning between February 2, 2017 and April 17, 2017, is discussed with respect to price evolution in the Bitcoin market, as well as with regards to Ethereum software being adopted for commercial use.

I. Introduction

Since its introduction in early 2015, Ethereum has become the second largest cryptocurrency by total dollar market value (Beneki et al., 2019). By early 2017, market capitalization of the cryptocurrency surpassed \$69 billion (Liu & Serletis, 2019). Ethereum was developed as a decentralized network of applications, eliminating third party institutions that tend to control crucial data by granting users their control over information (Ethereum Foundation, 2016). Ethereum's price has displayed high volatility (Catania et al., 2018). As shown in Figure 1, the several thousand percent surge in Ethereum prices since the beginning of 2017 has been accompanied by an increasing return in dispersion. As Phillips et al. (2015) point out, the history of financial markets tends to repeat itself. Therefore, accurate ex post identification of speculative bubbles might provide cryptocurrency investors with warning mechanisms to prevent losses on their current positions (Houbner, 2018).

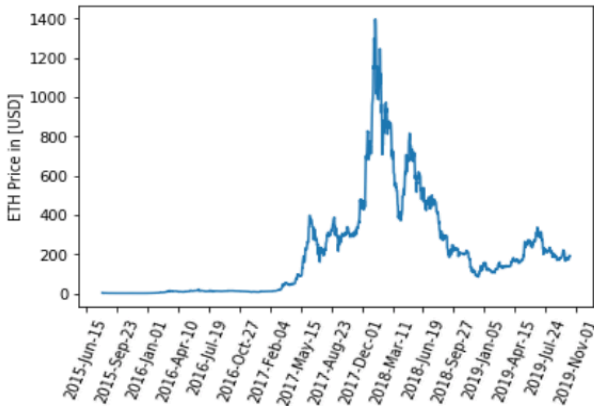


Figure 1. Crude USD Ethereum Prices between 8/2015 and 10/2019

II. Literature Review

The first attempt to describe a bubble in the financial markets can be attributed to Keynes (1973) who noted that “stock prices may not always be governed by an objective view of ‘fundamentals’ but by what average opinion expects the average opinion to be” (quoted in Cuthbertson & Nitzsche, 2004). However, such a definition makes it extreme-

ly difficult to quantify exuberance phenomena in the financial markets (Blanchard, 1982; Phillips, 2015).

A viable alternative has been proposed by Adam and Szafarz (1992) who define bubble as a scenario in which prices are driven up by the expectation of further growth. Consequently, market actors can be conceptualized as rational speculators¹ who bet on further price rises. The definition can be easily accommodated in the traditional Asset Pricing Approach to speculative bubbles, whereby exuberance constitutes the part of the market price which exceeds an asset's fundamental value. Detecting the existence of bubbles therefore entails determining the fundamental value of the underlying asset. This is usually performed by calculating the expected present value of the payoffs (including dividends) considering all relevant information and then subtracting this computed present value from the market price of the asset (Cuthbertson et al., 2004). A major problem with this approach for cryptocurrencies, however, is that they are hard to value as they do not have clearly identifiable cash flows (e.g. dividends) (Taipalu, 2012; Pesaran & Johnsson, 2018).

To circumvent the issue of fundamental value determination, Diba et al. (1988) devised a right-tailed unit root test, known as the conventional cointegration-based bubble test in the literature. However, simulations performed by Evans (1991) indicate that this technique produces a false positive result 25% of the time.

Phillips et al. (2011) build upon the idea developed by Diba et al. (1988), but instead of running a single test over the whole sample, they implement right-tailed Augmented Dickey-Fuller tests using subsets of the data incremented by one observation at each run. They name this method the Supremum Augmented Dickey Fuller test and show that it does not only result in much greater power - even in the presence of periodically collapsing bubbles - but also allows us to pinpoint the start and end date of the bubble with its backward date-stamping procedure. At its core, the technique assumes that the series satisfies the sub-martingale² property.

The sub-martingale definition of bubble time series, as outlined

1 Hence the terms "speculative" or "rational" bubble later in the text.

2 Asset prices adhere to martingale behaviour when successive price changes are unpredictable, although the variance of the price changes can be predicted from past variances under the martingale property (definition derived from Cuthbertson & Nitzsche, 2004). This is contrasted with a sub-martingale behaviour of a series whereby there is a widely anticipated direction of price-change in the asset market - the price changes have a strict lower bound during a run-up stage of a bubble. Refer to section "III. Methodology" for further details

in Phillips et al. (2011), has been more strongly supported in the research on financial market speculation (Biagini et al., 2013; Protter, 2013). Sub-martingale bubbles have also been advocated in the cryptocurrency literature - Cheung et al. (2015), for example, applied the Backward Supremum Augmented Dickey Fuller procedure to Mt. Gox Bitcoin prices. Their analysis discovered 33 time periods of exuberance in the Mt. Gox exchange between July 2010 and February 2014. To our knowledge, no paper has aimed at identifying speculative bubbles in Ethereum ex-post.

III. Methodology

This section introduces the Backward Supremum Augmented Dickey Fuller method³ (as defined by Phillips et al., 2013a) and outlines how the procedure will be utilized for the purposes of our analysis.

At its core, the BSADF methodology is based on the assumption that during its bubble run-up period, asset prices exhibit sub-martingale behaviour. It is assumed that there is a widely anticipated direction of price-change in the asset market as opposed to a martingale behaviour of asset prices whereby the best forecast of all future values of the bubble depends only on its current value (Cuthbertson et al., 2004).

The aforementioned definition of a price evolution in a market with bubbles be illustrated via a standard Asset Pricing Model, as specified in Equations (1) and (2).

$$P_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+r_f}\right)^i E_t(D_{t+i} + U_{t+i}) + B_t \quad (1)$$

$$E_t(B_{t+1}) = (1 + r_f)B_t \quad (2)$$

It is evident from (1) that if the bubble term B_t is equal to zero, then today's price of the asset is equal to the asset's discounted expected value, which can be further defined as a function of the expected dividend stream D_{t+i} , unobserved fundamentals U_{t+i} , and the risk-free interest rate r_f . In line with the martingale assumption, both D_{t+i} and U_{t+i} are assumed to be stationary or at most integrated to order one and, hence, the prices P_t follow a unit root process in the absence of a bubble. However, if the bubble term B_t deviates from zero, the process is not a unit root anymore, but instead exhibits explosiveness over time. Such a solu-

³Later referred to as the BSADF method in the text.

tion can be easily derived from (2), whereby the evolution of the bubble term is explosive (under the assumption that the risk-free rate has a zero lower-bound, i.e. $r_f > 0$). Clearly, the Asset Pricing Model outlined in (1) does not allow for the presence of run-down periods after the bubble has reached its peak price, and hence lacks the required complexity to allow for multiple and periodically collapsing bubble periods.

Our method enriches the specification by allowing for the possibility of a time-varying risk-free interest rate (i.e. $r_f = f(t)$) into an Asset Pricing model (in line with Phillips & Yu, 2011). With such a non-constant discount factor, bubble periods can be mildly explosive, meaning that they can temporarily divert from the fundamentals but subsequently return to the discounted expected value⁴.

Repetitiveness aside, we anticipate the asset prices to follow a unit root process under the null hypothesis of no speculative bubbles in the market. In the BSADF testing literature, this unit root process is usually limited to a random walk (Phillips et al., 2011) or a random walk with an asymptotically negligible drift (Phillips et al., 2015). These two possible null hypotheses are outlined in Equations (3) and (4). It is clear from Equation (4) that the drift term d converges to zero as the sample size T approaches infinity under our assumption of $n > 0.5$. Furthermore, these two null hypotheses are tested against the competing alternative of $\theta > 1$. Hence, unlike the standard ADF unit root method with left-tailed alternative hypothesis of stationarity, we are testing for the right-hand alternative hypothesis of explosive series.

$$y_t = \theta y_{t-1} + \varepsilon_t \quad \text{with } \varepsilon_t \sim \text{iid}(0, \sigma^2), \theta=1 \quad (3)$$

$$y_t = dT^{-n} + \theta y_{t-1} + \varepsilon_t \quad \text{with } \varepsilon_t \sim \text{iid}(0, \sigma^2), \theta=1, n > 0.5 \quad (4)$$

In addition, the BSADF testing procedure involves a sliding window regression applied recursively throughout the series. This allows for detecting multiple structural breaks, i.e. time series periods indicative of a bubble starts or ends (Enders, 2014). To put it more simply, BSADF specification is outlined via the differenced equation (5).

$$\Delta y_t = \hat{\alpha}_{r_1, r_2} + \hat{\beta}_{r_1, r_2} y_{t-1} + \sum_{i=0}^k \hat{\phi}_{r_1, r_2}^i \Delta y_t + \hat{\varepsilon}_t \quad (5)$$

$$r_2 = r_\omega + r_l \quad \text{with } r_\omega > 0 \quad (6)$$

⁴ Phillips and Yu (2011) introduce the time-varying discount factor into the continuous time Gordon Growth Model [GGM].

In (5), the r_1 and r_2 correspond to the start and the end points of the sliding window regression, respectively. It must be pointed out that r_1 and r_2 are fractions of the overall dataset T and, therefore, the regression is performed on the data ranging from the r_1^{th} fraction of the sample till the r_2^{th} fraction of the sample. Furthermore, the fractional window size r_ω of the regressions is specified in Equation (6). It is evident from (6) that the higher the r_ω the lower the overall number of recursions performed. Consequently, the coefficient ω also determines the sample size of each regression, denoted by T_ω . The ADF statistic based on this sliding window regression can be specified as $ADF_{r_1}^{r_2}$

$$SADF(r_0) = \sup ADF_{r_1}^{r_2} \text{ with } r_2 [r_0, 1] \quad (7)$$

The Supremum in the BSADF stands for the $ADF_{r_1}^{r_2}$ statistic with a flexible window size. More specifically, in the traditional SADF test (depicted in Equation (7)), we allow for the fraction parameter r_2 to vary, while the parameter r_1 is still assumed to be fixed at $r_1=0$. As a result, the sample sequence r_ω varies in size, with the ADF statistic for the longest possible sample size being denoted by $ADF_0^{r_2}$

In line with Phillips et al. (2015) who performed a simulation study on the optimal minimum window size selection, we select r_0 based on the lower bound of 1% of the full sample. The precise specification of r_0 for the purpose of our analysis is depicted in Equation (8).

$$r_0 = 0.01 + 1.8/\sqrt{T} \quad (8)$$

Performing BSADF comprises of computing SADF on a backward expanding sample sequence. In such a case, the fraction parameter r_2 is fixed to 0, while r_1 is allowed to vary. As outlined in Equation (9), we let the r_1 parameter to take on any value between zero and r_2-r_0 .

$$BSADF(r_0) = \sup ADF_{r_1}^{r_2} \text{ with } r_1 [0, r_2 - r_0] \quad (9)$$

Having acquired the $BSADF(r_0)$ statistic, we compute critical

values using Monte Carlo simulations⁵. During this stage, we follow Phillips et al. (2015) by setting the number of bootstrap replications to 200 and by not permitting multi-core computations. Finally, for inferring significance, we rely on the 99% confidence interval. Such decision boundary is in line with Cheung et al. (2015) who applied the same methodology to testing multiple bubbles in the Bitcoin cryptocurrency market.

IV. Data Set Overview

In our analysis, we consider daily closing prices of Ethereum over the time period 7 August 2015 to 11 October 2019. We only use the log value of weighted prices for further computations.

The dynamics of our data are outlined in Table 1. It is evident from the summary statistics that the series exhibits high deviations from the long-run mean. We also detect an upward trend in the data, with prices ranging between 0.416 USD in 2015 and 1397 USD during its all-time high in January 2018. The interquartile range of approximately 283 USD indicates that the data exhibits fat tails.

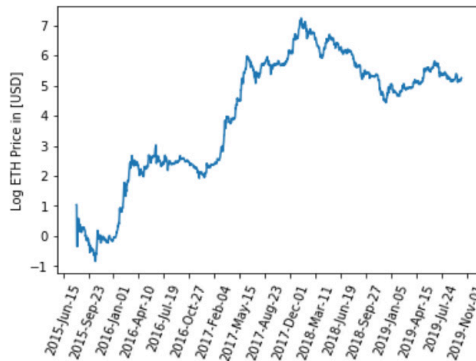


Figure 2. USD Ethereum Prices in log scale over the Sample Period.

Table 1. Summary Statistics

Variable	Obs.	Mean	SD	Min	Max	IQR	Skewness	Kurtosis
Ethereum P	1527	206.3718	249.5032	0.4316	1397.4800	283.0773	1.731618	6.145035

⁵ The asymptotic critical values are tabulated but in small samples these computations require Monte Carlo simulations as specified in Phillips et al. (2011).

Figure 2 also supports the possibility of non-constant volatility in the series, which could be indicative of conditional heteroskedasticity in returns. However, visual inspection of squared returns (procedure outlined in Diebold et al., 2019) does not support the conditional heteroskedasticity hypothesis. In addition, Monte Carlo simulations by Pedersen and Schütte (2017) assert minimal impact of heteroscedasticity on the BSADF procedure (i.e. no critical test statistic distortions). We, therefore, do not perform any additional diagnostic checks on the variance dynamics.

As for serial correlation in the data, the time series is highly persistent because its autocorrelation is close to 1 and significantly different from 0 for lags 1-100. Figure 3 depicts the autocorrelation for lags 0 through 20. As for the partial autocorrelation function, we see both the values lagged 1 and 2 periods to be significant in explaining today's Ethereum prices. Since our autocorrelation function results imply high persistence in the time series, it is important to determine whether the observed trend is deterministic or stochastic. To do so, we used the left-tailed Augmented Dickey-Fuller test. We applied the test to three specifications: (1) model without a constant; (2) model with a constant; and (3) model with a trend. In all cases, we failed to reject the null hypothesis of a unit root in the series⁶. This result supports a stochastic trend, i.e. the series to be integrated of order one. However, the test's critical region is limited to the left-tail and, hence, does not incorporate the possibility of explosive series. We consequently apply the BSADF test to investigate the presence of explosiveness in the prices.

⁶ We did not utilize the Bayesian information criterion (BIC) to decide on one of the three specifications, i.e. (1) model without a constant, (2) model with a constant and (3) model with a trend, as such an approach was criticized by Phillips et al. (2015)

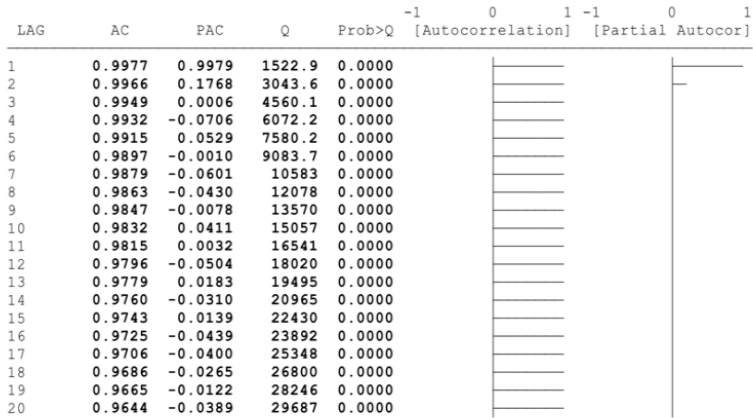


Figure 3. Autocorrelation Plot

Table 2: Augmented Dickey-Fuller test for unit root

Underlying DPS	# of lags included (p)	Test statistic	10% Significance Critical Values	5% Significance Critical Values	1% Significance Critical Values
ADF without a constant (Model 1)	0	0.585	-1.620	-1.950	-2.580
ADF with drift (Model 2)	0	-2.101	-2.570	-2.860	-3.430
ADF with drift (Model 2)	1	-1.415	-2.570	-2.860	-3.430
ADF with drift (Model 2)	2	-1.819	-2.570	-2.860	-3.430
ADF with trend (Model 3)	0	-1.185	-3.120	-3.410	-3.960
ADF with trend (Model 3)	1	-0.442	-3.120	-3.410	-3.960
ADF with trend (Model 3)	2	-0.655	-3.120	-3.410	-3.960

V. Results

This section reports the results of our analysis of whether the market prices of Ethereum exhibited speculative bubble behaviour between 7th August 2015 and 11th October 2019. As outlined in Figure 4, we find multiple bubble periods using the BSADF testing procedure. The test statistic, represented by the solid blue line, clearly shows that it exceeds its corresponding 99% critical value (denoted by the purple dotted line) 15 times over the sample period, which corresponds to the identification of 15 episodes of speculative bubbles in the data.

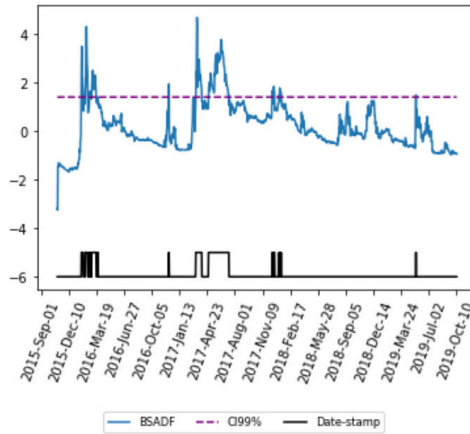


Figure 4. Backward Supremum Augmented Dickey Fuller Test.

In line with Katsiampa (2018), it might be argued that the ubiquity of bubble formation throughout 2016 be linked to DAO hacker attacks. This event undoubtedly would have spooked investors. Likewise, there were notable reasons for the prolific bubble periods of 2017 with the increasing media attention and legitimacy Ethereum was receiving from investors and politicians alike. For example, when Vitalik Buterin (Ethereum's Founder) described the opportunities for using the technologies he developed in Russia, a statement released by the Kremlin in June 2017 stated that President Putin supported the idea of securing further Russian investment in Ethereum. Furthermore, eToro added Ethereum to its listings on February 23, 2017 when the cost of one coin was only \$23. In May 2017 AVAT Trade added Ethereum to its listings at a time when one Ether coin was trading at \$100. Moreover, in February 2017 large institutions, such as J.P Morgan Chase, Intel and Microsoft, began to use Ethereum's software (Crosby et al., 2016), and this credibility might have continued to fuel the bullish market.

VI. Limitations and Possible Extensions

Despite the clear-cut findings of our analysis, some caution is required with respect to the interpretation of the discovered bubble periods. Primarily, our results stem from the assumption that the underlying price series exhibits explosive behaviour during periods of speculative

bubbles. However, other specifications of speculative excess have also been outlined in the literature. For example, Monschang et al. (2019) define a rational bubble which follows a random walk. Therefore, more research and discussion are recommended as to which specification of cryptocurrency bubble is more suitable. This also brings up the problem of the very definition of speculative behaviour in the cryptocurrency market. As stressed in Pesaran et al. (2018), the BSADF test does not allow for the possibility that the detected periods of exuberance are not in fact bubbles but are rather signs of rapidly changing fundamentals in the cryptocurrency.

Another caveat of our research concerns findings recently published by Phillips et al. (2015). According to Monte Carlo simulations, the BSADF test has much lower detective capacity than a recently introduced generalized version of the test – the GSADF. The superior power of GSADF is particularly evident when multiple periods of market exuberance are present in the data. This suggests the direction of future research in Ethereum should include the usage of the GSADF instead.

VII. Conclusion

We found 15 periods of speculation within the price of Ethereum, with the most notable bubble period lasting from 3rd February 2017 to 17th April 2017. Thus, our results did support the hypothesis that Ethereum was in fact in a bubble for a significant period of time during its price highs. Our findings might be of interest to investors who are considering Ethereum's place within an investment portfolio. Also, the discovered exuberance might hold the attention of policy makers, particularly with respect to creating cryptocurrency legislation. While this paper focused on Ethereum, it is important to outline that there are numerous other cryptocurrencies, such as Bitcoin, Ripple and Litecoin, that have all experienced a similar price evolution to Ethereum and therefore, it does pose the question as to whether or not the market is a bubble in its entirety.

VIII. References

1. Adam, M., and Szafarz, A. (1992). Speculative Bubbles and Financial Markets. Oxford Economic Papers, 44(4).

2. Adämmer, P., and Bohl, M. (2015). Speculative bubbles in agricultural prices. *The Quarterly Review of Economics and Finance*. Vol. 55.
3. Baeck, C., and Elbeck, M. (2015). Bitcoins as an investment or speculative vehicle? A first look. *Applied Economics Letters*, 22, 30-34.
4. Beneki, C., Koulis, A., Kyriazis, N., Papadamou, S. (2019). Investigating volatility transmission and hedging properties between Bitcoin and Ethereum. *Research in International Business and Finance*, Vol. 48.
5. Blanchard, O., Watson, M. (1982). Bubbles, Rational Expectations and Financial Markets. NEBR Working Paper Series, No. 945.
6. Catania, L., Grassi, S. and Ravazzolo F. (2018). Predicting the Volatility of Cryptocurrency Time-Series. CAMP Working Paper Series, No 3/2018.
7. Cheah, E. T., & Fry, J. (2015). Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economics Letters*, 130, 32-36.
8. Cheung, A., Roca, E., Su, J. J. (2015). Crypto-currency bubbles: an application of the Phillips-Shi-Yu (2013) methodology on Mt. Gox bitcoin prices. *Applied Economics*, 47, 2348-2358.
9. Chen, M., Narwal, N. and Schultz, M. (2019). Predicting Price Changes in Ethereum. Stanford, CA: Stanford University Press.
10. Crosby M., Pattanayak, P., Verma S., and Kalyanaraman, V. (2016). Blockchain technology: beyond Bitcoin. *Applied Innovations*. 2: 6–10.
11. Cuthbertson, K., and Nitzsche D. (2004). *Quantitative Financial Economics: Stocks, Bonds and Foreign Exchange*. Chichester, England: Wiley.
12. Corbet, S., Lucey, A., and Yarovaya, L. (2018). Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters*, Vol. 165.
12. Cuñado, J., Gil-Alana, L. (2005). A test for rational bubbles in the NASDAQ stock index: A fractionally integrated approach. *Journal of Banking and Finance*, Vol. 29.

13. Diebold, F. (1988). Conditional Heteroskedasticity in Economic Time Series. In: *Empirical Modeling of Exchange Rate Dynamics*. Lecture Notes in Economics and Mathematical Systems, 303. Berlin: Heidelberg.
14. Dowd, K. (2014). *New Private Monies: A Bit-Part Player*. Institute of Economic Affairs Monographs.
15. Enders, W. (2014). *Applied Econometric Time Series*. Chichester, England: Wiley
16. Engsted, T., Hviid, S., and Pedersen, T. (2016). Explosive bubbles in house prices? Evidence from the OECD countries. *Journal of International Financial Markets, Institutions and Money*, 40.
17. Evans, G. (1991). Pitfalls in Testing for Explosive Bubbles in Asset Prices. *American Economic Review*, 18(4), 922-930.
18. Escobari, D., Garcia, S. and Mellado, C. (2017). Identifying Bubbles in Latin American Equity Markets: Phillips-Perron-based Tests and Linkages. *Emerging Markets Review* [forthcoming].
19. Fry, J., Cheah, E-T. (2016). Negative bubbles and shocks in cryptocurrency markets. *International Review of Financial Analysis* [forthcoming].
20. Houbner, R., (2018). *Cryptocurrencies and blockchain*. Department for Economic, Scientific and Quality of Life Policies.
21. Keynes, J. M. (1973). *The Collected Writings of John Maynard Keynes*, Volume 7. Cambridge, UK: Cambridge University Press.
22. Kindleberger, C. P. (2000). *Manias, Panics, and Crashes: A History of Financial Crisis*. 4th ed. New York: John Wiley & Sons.
23. Katsiampa, P. (2018). *An Empirical Investigation of Volatility Dynamics in the Cryptocurrency Market*. *Research in International Business and Finance*.
24. Monschang, V. and Wilfling, B. (2019). Sup-ADF-style bubble detection methods under test. *The Centre for Quantitative Economics*, Vol. 78.
25. Nakamoto, S. (2008). *Bitcoin: A Peer-to-Peer Electronic Cash System*. Available at <https://bitcoin.org/bitcoin.pdf> [Accessed 31 September 2019].

26. Pedersen, T. Q. and Montes Schütte, E. C. (2017). Testing for Explosive Bubbles in the Presence of Autocorrelated Innovations. SSRN Working Papers. No 2916616.
27. Phillips, P. C. B and Yu, J. (2011). Dating the Timeline of Financial Bubbles During the Subprime Crisis. *Quantitative Economics*, Vol. 2. 455-491.
28. Phillips, P. C. B., Shi, S. and Yu, J. (2013a). Testing for multiple bubbles: historical episodes of exuberance and collapse in the S&P 500. Available at <http://ssrn.com/abstract=2327609> [Accessed 11 October 2019].
29. Phillips, P. C. B., Shi, S. and Yu, J. (2013b). Technical supplement to the paper: testing for multiple bubbles 2: limit theory of real time detectors. Manuscript.
30. Psaradakis Z., Sola M., Spagnolo F. (2001). A Simple Procedure for Detecting Periodically Collapsing Rational Bubbles. *Economics Letters*, Vol 24: 317-323.
31. Raskin, M. and Yermack, D. (2016). Digital currencies, decentralized ledgers and the future of central banking. NBER Working Papers, no. 22238.
32. Roche, M. (2005). The rise in house prices in Dublin: bubble, fad or just fundamentals. *The Journal of Economic Modelling*, Vol. 18.
33. Rotterman, B. and Wilfling, B. (2017). A new stock-price bubble with stochastically deflating trajectories. *Journal of Applied Economics Letters*, Vol. 25. No.15.
34. Shiller, R. (2019). *Narrative Economics: How Stories Go Viral and Drive Major Economic Events*. Princeton, NJ: Princeton University Press.
35. Shiller, R. (2010). *Irrational Exuberance*. Princeton, NJ: Princeton University Press.
36. Taipalus, K. (2012). Detecting asset price bubbles with time-series methods. Scientific monographs.
37. White, E. (1990). Bubbles and Busts: The 1990s In the Mirror of the 1920s. NEBR Working Series, paper no. 12138.