A Game Theory Analysis of the Suspension of the INF Treaty

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On February 1st, Donald Trump confirmed that the U.S was suspending its obligations under the Intermediate-Range Nuclear Forces (INF) Treaty amidst allegations of repeated Russian breaches. Despite this, the U.S will not fully withdraw until August, giving Russia the opportunity to respond and attempt to salvage the treaty. Daire de Hora analyses this U.S-Russian interaction as an extensive form game with incomplete information, in an attempt to identify the possible equilibrium outcomes. In doing so, she provides an interesting analysis of the potential motivations of the two players concluding that Russian beliefs about American intentions could ultimately be the deciding factor for the future of the treaty.

INTRODUCTION

The INF treaty, signed in 1987 by the USSR and the US, required the elimination of all ground-launched missiles with a range between 500 and 5,500 km and prohibited future development of such missiles (nti.org, 2018). In October 2018, President Trump announced that the US planned to withdraw from the INF treaty, citing Russia's non-compliance with the treaty's provisions as the main reason (nti.org, 2018). In February 2019, it was announced that the US was suspending its compliance with the treaty and planned to withdraw in August (nti.org, 2018). The allegations have been denied by President Putin and he has stated that Russia is not in favour of the destruction of the treaty (rferl.org, 2018).

OUTLINE

This paper will model the situation between Russia and the US as an extensive form game with incomplete information. The US has two types: an independent type and a cooperative type. The independent type values its independence and receives a higher payoff from withdrawing from the treaty. The cooperative type values the security provided by the treaty, and receives the highest payoff when it remains in the treaty and Russia complies.

In order to solve the game, it is modelled as a game of imperfect information. Nature moves first and determines the US' type. The probability that the US is the independent type is denoted by α and the probability that the US is the cooperative type is denoted by 1-α and these probabilities are known to both players. After Nature moves, the US knows its type, but Russia does not.

The US moves after Nature and decides whether to announce that it plans to withdraw from the treaty or not. If the US does not announce that it will withdraw, the game ends and both players remain in the treaty. If the US announces its withdrawal, Russia can move. Russia observes the US' action and, if the US played announce, can then decide whether to stand firm and continue to claim that it is not in violation of the treaty, or to cave and admit to the alleged violation of the treaty and agree to stop. The US can then move again and either withdraws from the treaty or changes its mind and does not withdraw.

The payoffs to each outcome are given at each terminal node and the US' payoffs are always listed first. The payoffs to the different potential outcomes are ranked in the following way (where the outcome is the result of the players following the actions in parentheses):

Independent type of the US:

U(Announce, Cave, Withdraw)>U(Announce, Stand firm, Withdraw)>U(Announce, Cave, Change mind)>U(Don't announce)>(Announce, Stand firm, Change mind)

The independent type obtains the highest payoff when it withdraws from the treaty, since it values its independence, and the payoff is higher when Russia admits it has breached the treaty than when Russia plays stand firm. The independent type prefers to remain in the treaty when Russia admits to breaching the treaty than it does to not announce its plans to withdraw at all. Lastly, the worst option for the independent U.S type is to announce its withdrawal and then change its mind when Russia has played stand firm.

Cooperative type of the US:

U(Announce, Cave, Change mind)>U(Don't announce)>U(Announce, Cave, Withdraw)> U(Announce, Stand firm, Withdraw)>(Announce, Stand firm, Change mind)

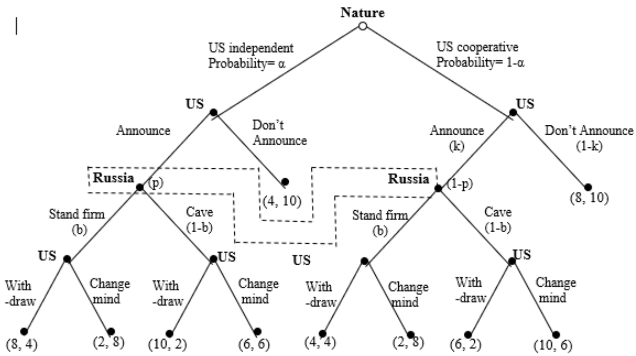
The cooperative type receives the highest payoff when it remains part of the treaty and Russia agrees to comply with the treaty. The cooperative type prefers not to announce its plans to withdraw from the treaty than to withdraw from the treaty. The lowest payoff to the cooperative type of the US is the same as that of the independent type, which is when it threatens to leave the treaty but changes its mind after Russia plays Stand firm.

Russia:

U(Don't announce)>U(Announce, Stand firm, Change mind)>U(Announce, Cave, Change mind)> U(Announce, Stand firm, Withdraw)>(Announce, Cave, Withdraw)

Russia's payoffs do not depend on the type of the US, only on the actions taken by the players. Russia's most preferred outcome is for the US not to announce its withdrawal from the treaty. Russia prefers to stay in the treaty, so if the US does announce its withdrawal, Russia receives a higher payoff when the US changes its mind than when it follows through with the withdrawal. Russia also receives a higher payoff from playing Stand firm than Cave, for a given action of the US at its second decision node.

Model



Assumptions

In order to formulate the payoffs, assumptions had to be made about how the players would rank the outcomes and also the values they might attach to them. The assumptions about the ranking of the preferences seem reasonable given the behaviour of the players in reality. For example, as mentioned in the introduction, President Putin has made it clear that Russia would prefer to remain part of the treaty, so it makes sense for Russia to receive a higher payoff when the treaty is not terminated.

There is no way to discern the exact payoffs the players would assign to each outcome, so for simplicity it has been assumed each player's most preferred outcome gives them a payoff of 10, and that each less preferred outcome gives a payoff that is lower by 2. Changing the value of the payoffs could change the equilibria of the game, but any values would be arbitrary, so it is assumed that these values are correct.

It is also assumed that Russia does not know the US' true preferences when the US announces its plans to withdraw from the treaty. This is modelled in the game as the US having two types and Russia not knowing which type it is playing against (shown by the information set in the decision tree).

To solve the game α must be given a value, so it will first be assumed that α is equal to 0.6. It seems probable that the US is more likely to be the independent type, because there has been speculation that the US is concerned about the build-up of the nuclear arms of other countries, China in particular (rferl.org, 2018). The US might thus prefer to withdraw from the treaty, regardless of whether Russia agrees to comply. This assumption will then be relaxed, to examine how changing this probability will change the predictions of the game.

Equilibria

When α=0.6, the game yields one perfect Bayesian equilibrium (PBE):

Separating equilibrium:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Don't announce; Withdraw; Change mind

Russia's strategy: Stand firm

Russia's beliefs: Prob(Independent|Announce)= 1

This is a separating equilibrium because the two types of the US play different actions at their first decision nodes. This allows Russia to update its beliefs after observing the US' first move.

The assumption that α=0.6 will now be relaxed to examine how the predictions of the game change as the value of α changes.

Increasing the value of α does not change the equilibria of the game. Any value of α greater than 0.6 yields only the separating equilibrium.

When α=0.5, the game yields two PBE:

Separating equilibrium:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Don't announce; Withdraw; Change mind

Russia's strategy: Stand firm

Russia's beliefs: Prob(Independent | Announce)= 1

Pooling equilibrium:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Announce; Withdraw; Change mind

Russia's strategy: Stand firm with probability<1/3; Cave with probability≥2/3

Russia's beliefs: Prob(Independent | Announce)= 1/2

There is now also a pooling equilibrium, in which no information is revealed to Russia when the US moves because both types play Announce.

When α<0.5, the game yields three PBE:

Separating equilibrium:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Don't announce; Withdraw; Change mind

Russia's strategy: Stand firm

Russia's beliefs: Prob(Independent | Announce)= 1

Pooling equilibrium:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Announce; Withdraw; Change mind

Russia's strategy: Cave

Russia's beliefs: Prob(Independent|Announce)= α

Semi-separating equilibrium:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Announce with probability k; Don't announce with probability 1-k; Withdraw; Change mind (where $1/9 \leq k \leq 2/3$, depending on the value of α)

Russia's strategy: Stand firm with probability 1/3; Cave with probability 2/3

Russia's beliefs: Prob(Independent|Announce)= 1/2

There is now a semi-separating equilibrium, in which both Russia and the cooperative type of the US are using mixed strategies.

ANALYSIS

This game highlights that there might be a situation in which the INF treaty remains in effect, even though the US has already announced its plans to withdraw from it. So far, Russia has denied the allegations of its non-compliance with the treaty's provisions. The pooling equilibrium shows that Russia will admit that it has violated the treaty, if it believes that the US is less likely to be the independent type than the cooperative type given that it played announce ($p \leq 1/2$). Whether Russia would admit to breaching the treaty in reality has yet to be seen. This result implies, however, that, if the assumption that Russia wants to stay in the treaty is correct and if Russia believes that the probability that the US has the preferences of the cooperative type in reality is sufficiently high, it might be willing to admit that it has breached the treaty in order to prevent its demise.

It is also interesting to consider the impact that the incomplete information has on the outcomes for the two players, especially when the US is the cooperative type. The separating equilibrium, which exists for all values of α, results in a payoff of 10 to Russia and a payoff of 8 to the US if Nature determines that the

US is the cooperative type. If Russia were playing against the cooperative type and there was complete information, the only subgame perfect equilibrium would be for the US' strategy to be announced, withdraw, change mind and for Russia's to be cave. Russia and the US would therefore obtain payoffs of 6 and 10, respectively. In this case, complete information would make Russia worse off and the US better off. It is as if Russia's lack of information gives credibility to its threat of playing stand firm. With perfect information, it would not be optimal for Russia to play stand firm, because it would receive a higher payoff from playing cave. When Russia does not know the US' type, however, it can credibly claim that it will play stand firm and the cooperative type optimally responds by playing don't announce.

Extensions

President Putin has claimed the Russia will retaliate and develop weapons that were prohibited under the treaty should the US withdraw(Johnson, 2018). A possible extension to this game would be to incorporate this development by allowing Russia to have an additional move at the end of the game, where it can decide to retaliate or not. Alternatively, an option to threaten retaliation could be included at Russia's first decision node. It would be interesting to examine whether expanding Russia's set of available actions would change the equilibria of the game.

Reference List:

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Appendix

Solving for the PBE:

Starting at the bottom of the game, at the independent type of the US' second decision node, the independent type should choose Withdraw when Russia chooses Stand firm and when Russia chooses Cave. At the independent type's first decision node, it will choose Announce, because it strictly dominates Don't announce.

At the second decision node of the cooperative type of the US, when Russia plays Stand firm, the cooperative type should play Withdraw. When Russia plays Cave, the cooperative type should play Change mind. At its first decision node, the cooperative type should play Don't announce when Russia plays Stand firm and it should play Announce when Russia plays Cave.

Russia's optimal move, given its belief, p:

$EU_R(SF|p)= (p)4+(1-p)4= 4$

$EU_R(C|p)= (p)2+(1-p)6= 6-4p$

Russia prefers to play Stand firm when $EU_R(SF|p)> EU_R(C|p) = 4>$

$6-4p = p>1/2$

When $p>1/2$, Russia will play Stand firm

When $p<1/2$, Russia will play Cave

When $p=1/2$, Russia is indifferent between playing Stand firm and Cave

P=Prob(Independent|Announce)= (prob(Announce|Independent).prob(Independent))/(prob(Independent).prob(Indepedent)+ prob(Cooperative).prob(Cooperative))

$= (1(\alpha))/(1(\alpha)+ k(1-\alpha))$

We know that the independent type will always choose Announce, so the probability that the US will choose Announce when it is the independent type is 1.

Let k denote the probability that the US will choose Announce when it is the cooperative type.

The value of p thus depends on the value of α and the value of k.

What value of k leads to p being less than, greater than, and equal to 1/2?

p<1/2 when $(1(\alpha))/(1(\alpha)+k(1-\alpha)) < 1/2 = 2\alpha < \alpha+k(1-\alpha) =$

$k > \alpha/((1-\alpha))$

p>1/2 when $k < \alpha/((1-\alpha))$

p=1/2 when $k = \alpha((1-\alpha))$

$\alpha=0.1: \alpha/((1-\alpha)) = 0.1/((1-0.1)) = 1/9$

$\alpha=0.2: \alpha/((1-\alpha)) = 0.2/((1-0.2)) = 1/4$

$\alpha=0.3: \alpha/((1-\alpha)) = 0.3/((1-0.3)) = 3/7$

$\alpha=0.4: \alpha/((1-\alpha)) = 0.4/((1-0.4)) = 2/3$

All of these values of k lie between 0 and 1 (which is required since k is a probability).

$0.1 \leq \alpha \leq 0.4$

Case 1: p<1/2

If p<1/2, Russia prefers to play Cave.

If Russia plays Cave, the cooperative type of the US will play Announce at the first decision node, so k=1.

To have p<1/2, we must have k>1/9 for $\alpha=0.1$; k>1/4 for $\alpha=0.2$; k>3/7 for $\alpha=0.3$ and k>2/3 for $\alpha=0.4$.

A value of k=1 is thus consistent with p<1/2 for all values of α between 0.1 and 0.4, so beliefs are consistent.

The independent type of the US will always play Announce at the first decision node and Withdraw at both of its following decision nodes.

Since both types of the US are playing Announce at the first decision node, no information is revealed through this action, so Russia's posterior beliefs are equal to its prior beliefs.

PBE:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Announce; Withdraw; Change mind

Russia's strategy: Cave

Russia's beliefs: Prob(Independent | Announce)= α

Case 2: $p>1/2$

If $p>1/2$, Russia prefers to play Stand firm.

If Russia plays Stand firm, the cooperative type of the US will play Don't announce at the first decision node, so $k=0$.

To have $p>1/2$, we must have $k<1/9$ for $\alpha=0.1$; $k<1/4$ for $\alpha=0.2$; $k<3/7$ for $\alpha=0.3$ and $k<2/3$ for $\alpha=0.4$.

A value of $k=0$ is thus consistent with $p>1/2$ for all values of α between 0.1 and 0.4, so beliefs are consistent.

The independent type will always play Announce at the first decision node and Withdraw at both of its following decision nodes.

Since each type of the US is playing different actions at the first decision node, Russia can update its beliefs using Bayes' rule after observing the US' action.

PBE:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Don't announce; Withdraw; Change mind

Russia's strategy: Stand firm

Russia's beliefs: Prob(Independent | Announce)= 1

Case 3: $p=1/2$

If $p=1/2$, Russia is indifferent between playing Stand firm and Cave.

If $p=1/2$, k is equal to a particular value less than 1 when α takes these values, so the cooperative type is mixing between Announce and Don't announce.

In order for the cooperative type to be willing to mix, it must be indifferent between paying Announce and Don't announce. This is only possible if Russia is also mixing.

The cooperative type of the US is indifferent between Announce and Don't announce when EUUSC(Announce|b)= EUUSC(Don't announce|b) =

(b)4+(1-b)10 = 8 =

b=1/3

where b denotes the probability with which Russia will play Stand firm.

PBE:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Announce with probability k; Don't announce with probability 1-k; Withdraw; Change mind

Russia's strategy: Stand firm with probability 1/3; Cave with probability 2/3

Russia's beliefs: Prob(Independent|Announce)= 1/2

where k=1/9 when α=0.1; k=1/4 when α=0.2; k=3/7 when α=0.3 and k=2/3 when α=0.4

α=0.5

$\alpha/((1-\alpha)) = 0.5/((1-0.5)) = 1$

For p to take a value less than 1/2 would require k to be greater than 1. This is not possible, since k is a probability and must lie between 0 and 1, so there cannot exist an equilibrium in which p<1/2.

Case 1: p>1/2

If p>1/2, Russia prefers to play Stand firm.

If Russia plays Stand firm, the cooperative type of the US will play Don't announce at the first decision node, so k=0.

To have p>1/2, we must have k<1. A value of k=0 is thus consistent with p<1/2, so beliefs are consistent.

The independent type of the US will always play Announce at the first decision node and Withdraw at both of its following decision nodes.

Since each type of the US is playing different actions at the first decision node, Russia can update its beliefs using Bayes' rule after observing the US' action.

PBE:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Don't announce; Withdraw; Change mind

Russia's strategy: Stand firm

Russia's beliefs: Prob(Independent | Announce)= 1

Case 2: $p=1/2$

If $p=1/2$, Russia is indifferent between playing Stand firm and Cave.

If $p=1/2$, $k=1$ when $\alpha=0.5$

The cooperative type of the US is willing to play Announce with probability 1 ($k=1$) when EUUSC(Announce | b)>EUUSC(Don't announce | b)=

$(b)4+(1-b)10 > 8 =$

$b<1/3$

PBE:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Announce; Withdraw; Change mind

Russia's strategy: Stand firm with probability<1/3; Cave with probability≥2/3

Russia's beliefs: Prob(Independent | Announce)= 1/2

$0.5 < \alpha \leq 1$

$\alpha=0.6$: $\alpha/((1-\alpha)) = 0.6/((1-0.6)) = 3/2$

$\alpha=0.7$: $\alpha/((1-\alpha)) = 0.7/((1-0.7)) = 7/3$

$\alpha=0.8$: $\alpha/((1-\alpha)) = 0.8/((1-0.8)) = 4$

α=0.9: $\alpha/((1-\alpha)) = 0.9/((1-0.9)) = 9$

For all of these values of α, there only can exist an equilibrium in which p>1/2. For p to be equal to or less than 1/2 would require k to be greater than 1, which is not possible since k is a probability.

Case 1: p>1/2

If p>1/2, Russia prefers to play Stand firm.

If Russia plays Stand firm, the cooperative type will play Don't announce at the first decision node, so k=0.

To have p>1/2, we must have k<3/2 for α=0.6; k<7/3 for α=0.7; k<4 for α=0.8 and k<9 for α=0.9

A value of k=0 is thus consistent with p>1/2 for all values of α between 0.6 and 0.9, so beliefs are consistent.

The independent type of the US will always play Announce at the first de cision node and Withdraw at both of its following decision nodes.

Since each type of the US is playing different actions at the first decision node, Russia can update its beliefs using Bayes' rule after observing the US' action.

PBE:

Independent type's strategy: Announce; Withdraw; Withdraw

Cooperative type's strategy: Don't announce; Withdraw; Change mind

Russia's strategy: Stand firm

Russia's beliefs: Prob(Independent|Announce)= 1