DO SHARE PRICES FOLLOW A RANDOM WALK?

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Ever since it was proposed in the early 1960s, the Efficient Market Hypothesis has come to occupy a sacred position within the belief system of modern finance. In fact modern finance can trace its origins to the universal acclaim accorded to the Efficient Market Hypothesis. One of its key corollaries suggests that shares should follow what has been succinctly referred to as a 'a random walk'. In this essay, Michael Sherlock uses the techniques of econometrics to empirically test the validity of this theory. Interestingly, his results differ from the conclusions of the previous essay in this section, highlighting the equivocal nature of many answers to economics' questions.

Introduction

In 1978, Jensen (1978: 95) declared that 'there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Market Hypothesis' (EMH). Subsequent empirical testing has led experts to question the validity of this statement. Lo and MacKinlay (1999: 52) best summarised the debate when they stated that 'even after three decades of research and literally thousands of journal articles, economists have not yet reached a consensus about whether markets, particularly financial markets are efficient or not'. This essay will outline the theoretical underpinning of this thesis and in particular how the random walk model is associated with the idea of market efficiency. The empirical approach will then be presented along with any adjustments made to it. A detailed description of the empirical results will be provided to aid analysis. Finally, the results will be discussed and conclusions made based on the findings.

Background and literature review

The EMH concerns prices and information: it contends that prices reflect all known information and that the only driver of changing prices is 'news', i.e. new information (Cuthbertson and Nitzsche, 2001). This implies that stock returns are entirely unpredictable based on past returns, as past information is already incorporated into prices. Therefore, the only determinant of price movements is new information and as this is unforecastable, and hence unpredictable, share prices themselves are unpredictable and their behaviour is said to exhibit a random walk (Shleifer, 1999). Fama's early research supported this. He found that on any given day the price of a stock was just as likely to rise after a previous day's increase as after a previous day's decline (Fama, 1965).

A closely related concept and one which will be used to empirically test this contention is that of volatility, which refers to patterns of price movements. Volatility itself can be interpreted as an indicator of efficiency, being a measure of the speed at which securities are able to incorporate new information (Frewen, 2004). Essentially, if markets are efficient and prices unpredictable, price movements are driven by news (as shares adjust to fundamentals). Such price adjustments should give rise to normal volatility patterns. Once volatility changes over time in a predictable fashion -, i.e. there is a statistically significant relationship between volatility in one period and volatility in the next - the pattern of volatility clusters can be examined to determine whether excess or normal volatility occurs (Koop, 2005). If markets are not efficient, there is excess volatility which reflects the fact that prices may deviate from fundamentals for long periods (Cuthbertson and Nitzsche, 2001). Early volatility studies pioneered by Shiller (1981) provided statistical evidence against the Efficient Market Hypothesis by finding excess volatility. Shillers' results suggested that stock market prices are far more volatile than could be justified by the incorporation of news alone. However Cuthbertson and Nitzsche (2001) note that such variance-bound tests are plagued by statistical problems (for example, the peso problem). Consequently, a different methodology (based on Koop's approach) will be tested in this project.

Empirical Approach

To model random walk behaviour using financial time series, the following AR(1) model was used, as suggested by Koop (2005):

$$\Delta y^2_t = \alpha + \Phi\Delta y^2_{t-I} + e_t$$

The dependent variable is volatility at time t and the independent variable is the volatility lagged one period. α is the drift term and e_t is the error term. The drift term captures the fact that in reality shares generally display a slight upward trend over time; the inclusion of α accounts for this observation. The error term models the unexpected events that continually influence prices (and which are now being controlled for). Assuming rational expectations and orthogonality conditions, this implies that the error term is independent and identically distributed. This model, known as the random walk with drift model, implies that shares on average increase by α per period but are otherwise unpredictable. Note this regression equation is a modified model of the standard random walk with drift model used in financial time series analysis ($\Delta Y_t = \alpha + e_t$).

Certain adjustments were necessary to capture the relationship under examination. Incorporating differenced variables eliminates nonstationarity, thus avoiding the generation of a spurious regression (Gujarati, 2002). Because squaring amplifies changes, allowing one to detect patterns more easily, this was incorporated into the model. Cumulatively, this means that Δy^2_t is the estimate of volatility at time t . Overall, specifying the model as above allowed one to test for volatility clusters and thereby provided a means to test the random walk hypothesis.

Description of Data Set

Autoregressive models such as that specified above essentially model clustering. This model as Koop (2005: 182) points out 'has volatility in one period depending on volatility in a previous period'. Once a statistically significant relationship is found among clusters, one can reliably examine the pattern of clusters to determine whether this data set exhibits excess volatility. The data set consists of the stock price of a company collected on a weekly basis for a four year period, giving 208 observations.[1] The data has been logged for ease of use and manipulated according to the following algebraic operations to yield the chosen measure of volatility. The stock price data was differenced, deviations from the mean were taken and the result squared. In mathematical terms:

$\Delta y_t = \Delta Y_t - \Delta \hat{Y}$ where $\Delta \hat{Y} = \sum \Delta Y_t / N$

This procedure was necessary for a variety of reasons. Firstly, an unadjusted measure of variance could not be used as the measure of volatility, since a variable that would model *the change* in the volatility of an asset over time is needed. Specifying the model in this form also had certain statistical benefits, as the dependent variable was regressed against a lagged deviation from its mean as opposed to simply a pure lag of itself. According to Koop (2005), this allows the coefficient of correlation to be estimated using standard statistical tests, i.e. OLS (otherwise a tau(τ) test statistic would have been required to estimate the coefficient). Likewise, the fact that a differencing procedure was involved took care of the problem of autocorrelation, as differenced time-series can be assumed not to be autocorrelated (Gujarati, 2002). The econometric software package Microfit was used to carry out all the necessary adjustments and subsequent statistical tests.

[1] Data set source: www.wileyeurope.com/go/Koopdata2ed.

Empirical Results

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
INTER:	.2416E-5	.1486E-5	1.6259[.106]
VOLLAG :	.73618	.047368	15.5419[.000]

R-Squared: .54214 R-Bar-Squared: .53990

DW-statistic: 1.9471

Serial Correlation*CHSQ(1)= .26308[.608]*F(1, 203)= .25958[.611]*

Heteroscedasticity*CHSQ(1)= 84.5770[.000]*F(1, 204)= 142.0959[.000]*

White's Heteroscedasticity adjusted S.E.'s:
INTER T-Ratio[Prob]= 1.7206[.087], VOLLAG= 2.8309[.005]

ARCH F(1,202)= 245.5902[.000]

Unit root tests for residuals:

DF: -13.8764, ADF(1): -7.8959

95% critical value for the Dickey-Fuller statistic = -3.3672

AR(2):
NVOLLAG Coeff (.22914), T-Ratio[Prob]: 3.3512[.001]
R-Squared: .089354.

Regression: VOL = .2416E-5 + .73618VOLLAG + e_t

Note: VOL= Δy^2_t and VOLLAG= Δy^2_{t-I}

Comment

The value 0.73618 is the regression coefficient and tells us the percentage change (the original data was logged) in the dependent variable from a 1% increase in the independent variable. The magnitude of both t and p statistics confirm that it is statistically significant at the 5% significance level. This can be interpreted as meaning that a statistically significant relationship exists, i.e. a causal relationship exists, in which this week's volatility is dependent on last week's volatility. The positive value is indicative of clustering. In addition, an R^2 value of 0.54 suggests that 54% of the variation in this week's volatility can be explained by last week's volatility. The interval term is statistically

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insignificant; as this does not affect the primary contention of the study, it can be ignored.

Although heteroscedasticity is not usually a statistical problem with time series data, a test of heteroscedasticity was automatically included in this regression. This showed the presence of heteroscedasticity. In fact, this was to be expected as the underlying model is an autoregressive conditional heteroscedasticity model (this was confirmed by running an ARCH test - see table). This means that the interpretative value of the initial test statistics was not compromised. A heteroscedasticity adjusted standard error test was performed which confirmed the validity of the OLS results (both t and p values confirmed statistical significance at 5% level). Similarly the problem of autocorrelation was taken care of by subjecting all the data to a differencing procedure (as mentioned earlier). The D-W value reported (1.9471) was close to 2 and would normally be interpreted as confirmation of no autocorrelation. But in this case no interpretative weight can be attached to the value as an autoregressive process was involved; this violates one of the key conditions of the test (Gujarati, 2002).

As Microfit provides the facility to test for unit roots, this test was run to confirm the assumption of stationarity and no autocorrelation. Since both the Dickey-Fuller test and Augmented Dickey-Fuller test statistics were more negative than the provided critical value, the unit root hypothesis was rejected in each case; the unadjusted test confirming stationarity and the augmented test confirming no autocorrelation. Cumulatively, all the statistical evidence can be interpreted as confirming the presence of volatility clusters, i.e. one can be reasonably sure that periods of high volatility are followed by periods of high volatility (and similarly for periods of low volatility).

Is this confirmed graphically? Given that a statistically significant relationship exists, one can examine the graph for clustering; upon doing so it becomes apparent that clustering patterns are evident i.e. periods of high volatility are followed by periods of high volatility and periods of low (normal) volatility are followed by periods of low volatility. What is of interest in this study is the pattern of occurrence. If it is assumed that periods of high volatility can be interpreted as excess and low volatility as normal, there are periods of excess volatility in weeks 90-97 and, to a lesser extent ,weeks 4-8 and 101-107. But in general for most of the period under examination, normal volatility prevails (as Figure 1 indicates).

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Figure 1: Volatility Clustering

Conclusion

The establishment of a statistically significant relationship between regressor and regressand demonstrated the presence of volatility clusters in the data set. Examining this diagrammatically allows one to conclude that in general stock prices seem to adhere to the Efficient Market Hypothesis and exhibit random behaviour, as the general pattern exhibited was one of normal volatility. Although there were one or two periods of excess volatility, these were rare occurrences and were generally time-limited. However, it should be noted that the Efficient Market Hypothesis is robust enough to accommodate such anomalies. Indeed, many proponents argue that limited periods of excess volatility are essentially rational and are more accurately described as high volatility (Scheifer, 1999). At certain periods in the business cycle, news may be highly variable, causing a significant deviation from normal levels as prices and fundamentals efficiently adjust. This may explain the patterns observed; the fact that they are time-limited suggests this is a plausible explanation. As the dates of the data were not supplied nor the type of business the company was in, it is impossible to speculate as to what macroeconomic event could have been responsible.

Although Schiller's volatility study provides evidence against the Efficient Market Hypothesis, this does not invalididate the findings of this project, as a different methodology was used. A possible extension could be to use a variance bound test, as Schiller did, to test for excess volatility and compare results. As dividend data was not available in the data set this was not empirically possible. Another possible extension could be to test successive lags to see if inclusion improves on initial results. This was tested by running an AR(2) regression. Although statistically significant, inclusion did not improve the model's explanatory power.

To conclude: based on the methodology used, the results of this project seem to suggest that markets are efficient and stock prices do follow a random walk.

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