WHY MIGHT SHARE PRICES FOLLOW A RANDOM WALK?

SAMUEL DUPERNEX

Senior Sophister

The Efficient Markets Hypothesis no longer holds the impervious position in finance it once did. Consequently the assumption that share prices follow a random walk is now uncertain. Samuel Dupernex defines and discusses the random walk model, outlining its relationship to the efficiency of markets. Empirical evidence is used to investigate the arguments for and against the model.

Introduction

As recent as 30 years ago, the efficient market hypothesis (EMH) was considered a central proposition in finance. By the mid-1970s there was such strong theoretical and empirical evidence supporting the EMH that it seemed untouchable. However, recently there has been an emergence of counter arguments refuting the EMH. The EMH is the underpinning of the theory that share prices could follow a random walk. Currently there is no real answer to whether stock prices follow a random walk, although there is increasing evidence they do not.

In this paper a random walk will be defined and some of the literature on the topic will be discussed, including how the random walk model is associated with the idea of market efficiency. Then the arguments for and against the random walk model will be presented. It will be shown that, in some cases, there is empirical evidence on the same issue that could be used to support or challenge the theory.

Random Walks and the Efficient Market Hypothesis

As mentioned above, the idea of stock prices following a random walk is connected to that of the EMH. The premise is that investors react instantaneously to any informational advantages they have thereby eliminating profit opportunities. Thus, prices always fully reflect the
information available and no profit can be made from information based trading (Lo and MacKinley, 1999). This leads to a random walk where the more efficient the market, the more random the sequence of price changes.

However, it should be noted that the EMH and random walks do not amount to the same thing. A random walk of stock prices does not imply that the stock market is efficient with rational investors. A random walk is defined by the fact that price changes are independent of each other (Brealey et al, 2005). For a more technical definition, Cuthbertson and Nitzsche (2004) define a random walk with a drift \( \delta \) as an individual stochastic series \( X_t \) that behaves as:

\[
X_t = \delta + X_{t-1} + \epsilon_{t+1} \quad \epsilon_{t+1} \sim iid(0, \sigma^2_{\epsilon})
\]

The drift is a simple idea. It is merely a weighted average of the probabilities of each price the stock price could possibly move to in the next period. For example, if we had €100 and this moved either 3.0% up or 2.5% down with \( P=0.5 \) for each case, then the drift would be 0.25%, calculated by (Brealey et al, 2005):

\[
0.5(0.03) + 0.5(-0.025) = 0.0025 = 0.25\%
\]

However, even though it is useful, the model is quite restrictive as it assumes that there is no probabilistic independence between consecutive price increments. Due to this, a more flexible model called the ‘martingale’ was devised. This improved on the random walk model as it can “be generated within a reasonably broad class of optimizing models” (LeRoy, 1989:1588).

A martingale is a stochastic variable \( X_t \) which has the property that given the information set \( \Omega_t \), there is no way an investor can use \( \Omega_t \) to profit beyond the level which is consistent with the risk inherent in the security (Elton et al, 2002).

The martingale is superior to the random walk because stock prices are known to go through periods of high and low turbulence. This behaviour could be represented by a model “in which successive conditional variances of stock prices (but not their successive levels) are positively autocorrelated” (LeRoy, 1989:1590). This could be done with a martingale, but not with a random walk.\(^1\)

Fama (1970) stated that there are three versions of efficient markets:

\(^1\) Samuelson (1965) proved this result.
1. **Weak-form**: $\Omega$ comprises of historical prices only, meaning that it is not possible to earn superior risk adjusted profits which are based on past prices (Shleifer, 2000). This leads to the random walk hypothesis.

2. **Semi-strong form**: $\Omega$ includes historical prices and all publicly available information as well.

3. **Strong form**: “$\Omega$ is broadened still further to include even insider information” (LeRoy, 1989:1592).

Each of these forms has been tested and some of the results of these studies will be discussed later in the paper. As the strong form is considered somewhat extreme, analysis focuses on the weak and semi-strong forms.

**Arguments against the Random Walk Model**

There has been myriad of empirical research done into whether there is predictability in stock prices. Below, a summary of the main theories will be presented.

**Short-Run and Long-Run Serial Correlations and Mean Reversion**

Lo and MacKinley (1999) suggest that stock price short-run serial correlations are not zero. They also propose that in the short-run stock prices can gain momentum due to investors ‘jumping on the bandwagon’ as they see several consecutive periods of same direction price movement with a particular stock. Shiller (2000) believes it was this effect that led to the irrational exuberance of the dot-com boom.

However, in the long-run this does not continue and in fact we see evidence of negative autocorrelation. This has been dubbed ‘mean reversion’ and although some studies (e.g. Fama and French (1988)) found evidence of it, its existence is controversial as evidence has not been found in all research.

Chaudhuri and Wu (2003) used a Zivot-Andrews sequential test model to increase test power, thus decreasing the likelihood that previous results were a result of data-mining and obtained better results. To date, this method has not been widely adopted.

**Market Over- and Under-reaction**

Fama (1998) argues that investors initially over or under-react to the information and the serial correlation explained above is due to them fully
reacting to the information over time. The phenomenon has also been attributed to the `bandwagon effect`.

Hirshleifer discusses `conservatism’ and argues that “under appropriate circumstances individuals do not change their beliefs as much as would a rational Bayesian in the face of new evidence” (Hirschleifer, 2001:1533). He asserts that this could lead to over-reaction or under-reaction.

**Seasonal Trends**
Here, evidence is found of statistically significant differences in stock returns during particular months or days of the week. The ‘January effect’ is the most researched, but Bouman and Jacobsen (2002) also find evidence of lower market returns in the months between May and October compared with the rest of the year.

One problem with finding patterns in stock market movements is that once found, they soon disappear. This seems to have been the case with the January effect, as traders quickly eliminated any profitable opportunities present because of the effect.

**Size**
Fama and French (1993) found evidence of correlation between the size of a firm and its return. It appears that smaller, perhaps more liquid firms, garner a greater return than larger firms. Figure 1 shows the results:

**Figure 1. Average monthly returns for portfolios formed on the basis of size (1963-1990)**

![Graph showing average monthly returns for portfolios formed on the basis of size.](source: Malkiel, 2003)
However, it should be noted that the results may not accurately reflect reality, as this size trend has not been seen from the mid 1980’s onwards. In addition to this, the beta measure in the CAPM\(^2\) may be incorrect, as Fama and French (1993) point out. The market line was in fact flatter than the beta of the CAPM would have you believe. An illustration of this can be seen in Figure 2 below, where the market line should follow a fit of points 1-10.

**Figure 2. Average Premium Risk (1993-2002), %**

![Figure 2: Average Premium Risk (1993-2002), %](image)


**Dividend Yields**

Some research has been done on the ability of initial dividend yields to forecast future returns. As can be seen from the Figure 3, generally a higher rate of return is seen when investors purchase a market basket of equities with a higher initial dividend yield. It should be noted that this trend does not work dependably with individual stocks.

---

\(^2\) Capital Asset Pricing Model
However, Malkiel (2003) notes that as dividend yields are intrinsically linked with interest rates, this pattern could be due more to the general economic condition rather than just dividend yields. Also, dividends are becoming replaced by things such as share repurchase schemes, so this indicator may no longer be useful.

Shiller looked at how dividend present value was related to stock prices. There seemed to be very little correlation. For example, during the bull market of the 1920s, the S&P Composite Index (in real terms) rose by 415.4%, while the dividend present value increased by only 16.4% (Shiller, 2000). The results are seen in Figure 4 below:
Value vs. Growth Firms
It has been noted by many\(^3\) that in the long-term, value (low price to earnings (P/E) and price to book-value (P/BV) ratios) firms tend to generate larger returns than growth (high P/E and P/BV ratios) firms. In addition, Fama and French (1993) found there to be good explanatory power when the size and P/BV were used concurrently.

Fama and French (1995) then took this idea further and asserted that there are 3 main factors that affect a stock’s return\(^4\):

1. The return on the market portfolio less the risk-free rate of interest.
2. The difference between the return on small and large firm stocks.
3. The difference between the return on stocks with high book-to-market ratios and stocks with low book-to-market ratios (Brealey and Myers, 2005)

These arguments are powerful and could lead people to doubt the EMH and random walks, assuming that the CAPM is correct. However, as Malkiel

---

\(^3\) Hirshleifer (2001), Malkiel (2003) and Fama and French (1993), among others
\(^4\) This is part of the arbitrage pricing theory, which does not assume that markets are efficient. Instead it assumes that stocks returns are linearly related to a set of factors, and the sensitivity to each factor depends on the stock in question.
(2003) points out, it may be that the CAPM fails to take into account all the appropriate aspects of risk.

**Arguments for the Random Walk Model**

Shleifer (2000) identified three main arguments for EMH:

1. Investors are rational and hence value securities rationally.
2. Some investors are irrational but their trades are random and cancel each other out.
3. Some investors are irrational but rational arbitrageurs eliminate their influence on prices.

If all these exist, then both efficient markets and stock prices would be very unpredictable and thus would follow a random walk.

Brealy and Myers (2005) employed a statistical test to assess the EMH by looking for patterns in the return in successive weeks of several stock market indices.

**Figure 5. Scatter diagrams showing the return in successive weeks on two stock market indices between May 1984 and May 2004**

![Scatter diagrams showing the return in successive weeks on two stock market indices between May 1984 and May 2004](source: Brealey, R. A., Myers, S. C. and Allen, F. (2006:338))

Some of the results appear in Figure 5 and show almost no correlation in the returns.

**Event Studies**

Event studies help test the semi-strong form of the EMH. One such study examined how the release of news regarding possible takeover attempts affected abnormal returns. The results, illustrated below in Figure 6, showed that:

- Share prices rose prior to announcement as information is leaked.
- Share prices jump on the day of announcement.
- Share prices steadied after the takeover, showing that news affects prices immediately.

![Figure 6. Cumulative abnormal returns of shareholders of targets of takeover attempts around the announcement date](source: Shleifer, A. (2000:8))

In another study, Scholes (1972) observed how prices reacted to non-information by seeing how share prices reacted to large share sales by large
Why Might Share Prices Follow a Random Walk?

Investors. This study is important as it directly deals with the issue of the availability of close substitutes for individual securities.

Scholes finds they lead to small price changes and that this could be due to negative news regarding the share sale. Thus, the results support the random walk theory.

Predictability of Technical Trading Strategies

Fama (1965) found evidence that there was no long-term profitability to be found in technical trading strategies. Malikiel (2003) supports this view and provides us with evidence, such as Figure 7, that more often than not traders find it difficult to perform better than the benchmark indices. When they do, their success is often not repeated in the long-run.

Figure 7. Percentage of Various Actively Managed Funds Outperformed by Benchmark Index 10 Years to 12/31/01

![Chart showing percentage of various actively managed funds outperformed by benchmark index](source: Malkiel, B. G. (2003:79))

*This is central to the arguments of arbitrage in the EMH, as the theory states that ‘a security’s price is determined by its value relative to that of its close substitutes and not on market supply’ (Shleifer, 2000)*

176
On the other hand, why are there investors with sophisticated tools if their efforts are futile? This does seem to be the problem, as clearly rational investors would not invest if they could not ‘beat the market’. Indeed there is evidence to support this point of view. Lo, Mamaysky and Wang (2000) found that “through the use of sophisticated nonparametric statistical techniques… [analysts] may have some modest predictive power” (Malkiel, 2003:61)

**Mis-pricing**

There are many theories that assume mis-pricing. Mis-pricing does not affect our belief in the EMH or random walks so long as the profitable opportunities are small or they are the result of public information being misunderstood or misused by everyone.

**Conclusion**

As many of the results have contradictory evidence, it is very difficult to come to a conclusion. Data mining is certainly a problem, as one can manipulate data to support their findings. Also, many of the results could be due to chance.

It has also been suggested by Conrad that the evidence on cross-sectional predictability could be due to “missing risk factors in a multifactor model … [and conclude that the] pricing errors are persuasive evidence against linear multifactor model and therefore for other types of models, or they are evidence of data-snooping biases, significant market frictions, or market inefficiencies” (Conrad, 2000:516).

However, evidence suggests that markets are to a certain extent predictable. This does not mean that there are opportunities for arbitrage though, because these would soon be exploited and then vanish. In the real world (with taxes, transaction costs etc.) you can have some predictability without there being profitable opportunities.

It seems that stocks do approximately follow a random walk, but there are other factors, such as those discussed by Fama and French (1995), which appear to affect stock prices as well.

Studies on random walks and the EMH are important, as they can give us some information on the relative efficiency of markets. The EMH can be used as a benchmark for measuring the efficiency of markets, and
from this we have at least a rough idea as to whether the stocks are likely to follow a random walk.

**Bibliography**


