THE VALIDITY OF MARSHALLIAN CONSUMERS' SURPLUS

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The concept of the Marshallian consumers' surplus is probably familiar to every economics student. However, not everyone is aware of the pitfalls associated with this seemingly 'harmless' tool of economic analysis. Building on strong theoretical grounds, this essay by Barry John Rafferty warns about the limitations of consumers' surplus, highlights situations when its use is justified and gives an overview of alternative measures of welfare change.

Introduction

Marshallian consumers' surplus (hereafter referred to as consumers' surplus) is one of the most controversial concepts in economics. It is the most widely used tool of welfare analysis, but it is based on debatable theoretical foundations. Consumers' surplus seeks to provide a cardinal measure of the surplus utility a group of individuals get, cumulatively, from consuming a quantity of a good at a given price (Currie et al, 1971). This is the absolute net consumers' surplus. In this essay, I will be primarily concerned with the change in consumers' surplus, for a group of individuals, as a result of a move from a base economic state (seen here as a set of prices and incomes) to another economic state, and the use of this measure to rank different states relative to the base state.

The change in the individual consumer's surplus from a base state to another state, seeks to provide a cardinal ranking of the two states for the individual. Aggregating across individuals, the change in consumers' surplus seeks to provide a cardinal ranking for many individuals together of different states. Aggregating across goods seeks to provide a cardinal ranking of states when multiple markets are affected by different states (i.e. if the prices of many goods change). Therefore, used this way consumers' surplus seeks to be a powerful tool of welfare analysis, enabling us to cardinally rank various states for policy purposes when many individuals and many markets are affected. In these circumstances, it seeks to indicate how much society is better off (in terms of utility) in one state relative to another.

In light of the controversy surrounding consumers' surplus, I will analyse the problems associated with its use for the role outlined above. I will seek to answer a number of pertinent questions. Can the change in consumers' surplus from the base state to alternative states be used to cardinally rank alternative states relative to the base state? If not, then can it be used as the correct money measure of the change in welfare relative to the base state? By this I mean that the difference in consumers' surplus between alternative states and the base state would be a correct measure of the amount of money that the group of individuals are better off/worse off in one state relative to the base state. We cannot evaluate the difference in utility between states, since we do not know how much each individual subjectively values the monetary figure of the change in individual consumer's surplus and therefore we do not know what the sum of their valuations is. The correct money measure of the change in welfare (of the group of individuals) relative to the base state correctly ranks states with higher values corresponding to more preferred states (for the group together). If the change in consumers' surplus relative to the base state is not a correct money measure of welfare change, then what are the alternatives? Furthermore, do we need a money measure of the change in welfare, or can we just use ordinal preferences to evaluate whether one state is preferred to another? If we do see fit to use a money measure of welfare change that correctly ranks states, then could and should consumers' surplus be used as a good approximation?

What is Marshallian Consumers' Surplus?

Marshall quoted in Hicks (1941), referred to individual consumer's surplus as ''the excess of the price which he would pay rather than go without the thing, over that which he actually does pay''. This is absolute net consumer's surplus. Aggregating across individuals gives the consumers' surplus for the good. When applied to a single good, we can obtain total consumers' surplus for the good from the area below the ordinary market demand curve (Marshallian) and above the price line. This area can be expressed formally, as the definite integral below, where x is the quantity demanded of the good in question and P^0 is its price.

$$\int_{p^\circ}^{\infty} x(t)dt$$

The change in consumers' surplus from one state to another (where only the price of the good in question is changing between states) can be derived from the difference in total consumers' surplus, for the good, between the two states. That is

consumers' surplus in the new state minus consumer's surplus in the base state. This can be derived from the definite integral below, where P' is the price in the new state.

$$\int_{p^\circ}^{p'} x(t)dt$$

For Marshall, the above definite integral would be a cardinal measure of the change in utility for the group of individuals affected, of a move from the state with P^0 to the state with P'. Aggregating across goods when multiple prices are affected involves summing the changes in consumers' surpluses in the various markets affected.

Marshallian Consumers' Surplus as a cardinal measure of utility and use of changes in Consumers' Surplus from a base state to alternative states to cardinally rank states:

The main problem with consumers' surplus is that it purports to be a cardinal measure of utility. Changes in consumers' surplus from the base state are used to cardinally rank different states. This means that we can say by how much more a state is preferred to another state. We can get the difference in aggregate utility between two states. Utility theory nowadays, however, is purely ordinal. It is claimed that we cannot get a unique cardinal measure of utility or changes in utility. Samuelson claims that without ''introspective information'', obtained under interrogation, we cannot know about the intensity of an individual's preferences (Morey, 1984). Hence, we can know nothing of the intensity of aggregate preferences between states. We can only ordinally rank states. We can say that one state is preferred to another state, but not by how much. In light of this, the ability of consumers' surplus to measure cardinal utility and changes in cardinal utility becomes extremely tenuous. I will now examine the special case where consumers' surplus can be used to measure cardinal utility and changes in cardinal utility.

For consumers' surplus to be an exact cardinal measure of utility and for the change in consumers' surplus of a move from the base state to another state, to be used to construct a meaningful cardinal preference ordering between different states[1] then two things must hold.

Firstly, the marginal utility of money must be constant. This must be the case since there must be a constant conversion factor between changes in money and changes in utility. Therefore, the marginal utility of money must be constant

[1] i.e. we can say by how much utility the group of individuals are better off in one state relative to another

regardless of how much income the group of individuals have to spend, the amount of other goods they have, and the size of consumers' surplus. Every single monetary unit of consumers' surplus must be converted to utility at the same marginal rate.

Secondly, preferences for each good x, affected by a change in price must be represented by a quasilinear utility function. This would mean that all individuals face this utility function and therefore, we would be able to use it to represent aggregate preferences. Such a function would be of the form:

$$U(x, y) = v(x) + y$$

in the case where y refers to a composite good comprising all goods other than x (Dooley, 1983). Such a utility function is linear in y and (possibly) non-linear in x. For Marshall, such a function implied decreasing marginal utility in the first good and constant marginal utility in the second good (Ibid). Where y was money to spend on other goods, this implied a constant marginal utility of money. Such a utility function has the implication that the demand for x is independent of the level of income, implying that there is no income effect and that x has a vertical Engel curve. It is important to note that in this instance the ordinary (Marshallian) market demand curve is identical to the Hicksian compensated market demand curve.

The use of consumers' surplus as a cardinal measure of utility and therefore, changes in consumers' surplus to cardinally rank states, has faced a lot of criticism. The problems associated with the use of consumers' surplus as a cardinal measure of utility and utility change stems mainly from the validity (or lack of validity) of the assumptions required for consumers' surplus to be a cardinal measure. Dooley (1983) describes four main criticisms of consumer's surplus, and by extension consumers' surplus (since consumers' surplus is the sum of individual consumer's surpluses), which I will now outline.

Firstly, whether an additive utility function (with independent utilities for each good) adequately explains consumer behaviour. Marshall uses an additive utility function (the quasilinear utility function) to justify the use of consumer's surplus as a correct cardinal measure of utility. However, this was criticised heavily by Patten (Ibid) who argued that the utility an individual derived from consuming a quantity of a good was not independent of the quantity of other goods, which he could also consume. He argued that the utility of the good also depended on the amount of other goods that the individual could consume.

Secondly, whether the marginal utility of income could be treated as a constant. This is undoubtedly the most controversial assumption used by Marshall to allow consumer's surplus (and therefore consumers' surplus) to be a cardinal measure of utility. However, rigorous analysis has been conducted to show the implications of the constancy assumption. The best-known analysis of the constancy

assumption was by Samuelson (1966). He proved that the marginal utility of income could not be constant with respect to both price changes and to changes in income. Analysing the implications first of the marginal utility of money being constant with respect to price changes, Samuelson showed that the income elasticity of all goods must be unity. Adding the assumption of independent and additive utility, the price elasticity of demand would also have to be unity to ensure constancy of the marginal utility of income. Analysing the implications of the marginal utility of money being constant with respect to changes in income; Samuelson showed that all income would be spent on a single good only, with the income elasticity of demand for all other goods being zero, which implies all other goods having vertical Engel curves. All of these implications are completely at variance with empirical evidence. It is clearly the case that the constancy of the marginal utility of income is not a valid assumption.

Thirdly, whether the quantity demanded of a good can be treated as a function of its price alone. Walras criticised Marshall's assumption of partial equilibrium analysis claiming that one could not vary the price of a good, while holding constant the prices of all other goods and productive services (Dooley, 1983). In particular, he argued that ‘‘the selling prices of products and the prices of productive services are mutually interrelated’’(Ibid).

Fourthly, whether it is possible to make interpersonal comparisons. To get an aggregate consumers’ surplus, it is necessary to make interpersonal comparisons. Marshall assumed that ‘‘a shilling’s worth of gratification to one Englishman might be taken as equivalent with a shilling’s worth to another’’ (Ibid). This assumption would enable us to use the market demand curve to get the aggregate consumers’ surplus. This assumption however implies that each consumer has the same utility function and the same level of income. Also with regard to changes in economic state, Marshall claimed that ‘‘it happens that by far the greater number of events with which economics deals, affect in about equal proportions all the different classes of society’’ (Ibid). These two interpersonal assumptions do not hold since we do not know how each individual subjectively values their respective consumer’s surpluses. We do not know how a change in aggregate consumers’ surplus will affect individuals. Therefore, there is no way therefore that we can arrive at a precise cardinal value for the change in utility for a group of individuals, as a result of a change in state.

It is evident therefore that consumers’ surplus cannot be used to obtain a cardinal measure of utility and that use of the change in consumers’ surplus from the base state to cardinally rank states, for the group of individuals is not permissible.

Circumstances under which the change in Consumers' Surplus is the correct money measure of the change in welfare between two states:

Having established that the change in consumers' surplus from the base state cannot be used to cardinally rank and compare states; can the change in consumers' surplus from the base state be used to provide the correct money measure of the change in welfare? The correct money measure takes into account and compensates for income effects. It correctly ranks states, in terms of ordinal preferences, with higher values of the money measure representing more preferred states. The correct money measure is therefore a cardinally scaled monotonic transformation of the ordinal utility function (Morey, 1984). It also provides an indication in monetary terms of how much the group of individuals is better off/worse off, relative to the base state. In addition, the correct money measure gives the same money figure when multiple prices change, regardless of the sequence of the price changes (Johansson, 1991).

For the change in consumers' surplus to provide the correct money measure of the change in welfare, we may drop the assumption of the marginal utility of money being constant. However, aggregate preferences must still be represented by a quasilinear utility function. There must still be no income effect, with the ordinary market demand curves being identical to the Hicksian market demand curves. However this is extremely restrictive, with the income effect for most goods rarely if ever being zero. Therefore, the change in the consumers' surplus will be inaccurate as a money measure of welfare change the higher the income effect. It will fail to give the correct money measure of welfare change and may rank states incorrectly. Also, when multiple prices change, consumers' surplus may be affected by the path dependency problem, i.e. the sequence in which prices change.

Alternatives to Marshallian Consumers' Surplus:

Seeing as consumers' surplus and the change in consumers' surplus from a base state may fail to provide a money index that correctly ranks different states, but what are the alternatives? There are two options available. We can get a correct money measure of the welfare change or we could simply use the ordinal preference field described by Samuelson (Bergson, 1975) to ordinally rank states.

If we choose to use the correct money measure of welfare change relative to a base state to rank states, there are two alternatives. These are the compensating and equivalent variations developed by Hicks. According to Johansson (1991), the

compensating variation for a household[2] gives ‘‘the maximum (minimum) amount of money that can be taken from (must be given to) a household to make it just as well off as it was before a fall (rise) in prices”. Meanwhile, the equivalent variation for a household gives ‘‘the minimum (maximum) amount of money that must be given to (taken away from) a household to make it as well off as it would have been after a fall (rise) in price’’ (Ibid). The compensating variation returns the agent to the original indifference curve and is measured at the new prices. The equivalent variation is measured at the original prices and takes the agent to the new indifference curve. Summing both of these measures over individuals will yield the aggregate compensating and equivalent variations (compensating and equivalent variations will refer hereafter to the aggregated variations, unless otherwise stated). The two money measures will differ in all cases except quasilinear utility[3], since they are measured at different prices, with the value of a dollar depending on what the relevant prices are (Varian, 1992). Unlike the case of consumers’ surplus, the compensating and equivalent variations will give the correct money figure of the change in welfare when multiple prices change regardless of the order of the price changes.[4] Both measures will correctly rank any two states. Whilst the compensating variation cannot rank more than two states (since we would not be using the same prices), the equivalent variation can rank all states since it is measured relative to the base prices (Johansson, 1991; Morey, 1984).Therefore, all states will be ranked correctly in terms of ordinal preferences if the equivalent variation is used. Compensating and equivalent variations serve distinct purposes with compensating variation more useful for arranging compensation at the new prices (Varian, 1992) and the equivalent variation more useful for ranking states at the base prices.

The other alternative is to use the ordinal preference field spoken about by Samuelson to ordinally rank states, (Bergson, 1975) obviating the need for a money measure of welfare change between states. Bergson (Ibid) indicates how knowledge of the market demand equations[5] is sufficient information to determine the ordinal preference ordering and to derive an ordinal indifference curve map. Using this map, we can evaluate the effect on the group of individuals of the different states. We can only indicate a preference ordering, as we have no capacity to know the intensity of these preferences. In line with this view, all that is needed for decision-making is that different options or policies are ranked correctly, in so far as the different states corresponding to different policies are ranked correctly.

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[2] Households here are equivalent to individual agents

[3] Where they will also be the same as the consumers' surplus measure of welfare change

[4] The compensating and equivalent variations are path independent.

[5] Relative prices indicate the marginal rates of substitution between goods.

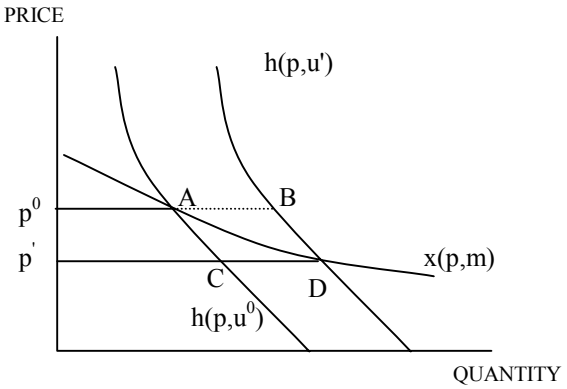
Which option is chosen will depend on whether one views correct money measures of welfare change to be useful for more than just their ordering of states. That is whether one sees any worth in using the value of money measure as some rough indication of the intensity of preferences. Bergson (Ibid) argues that often politicians need a money figure of the benefit or cost of a proposal, against which they can compare political costs or benefits and other factors unaccounted for in the economic evaluation process, e.g. externalities. A simple ordinal ranking may not suffice. As Morey interprets Bergson:

> ''The politician needs cardinally scaled measures of the economic benefits of each alternative to compare them with the political costs of each alternative so as to determine his ordinal ranking of the projects'' (Morey, 1984).

As Bhagwati points out ''whether we like it or not, this is what the policy makers do want'' (Bergson, 1975).

Having established that a correct money measure of welfare change is desirable for decision-makers, how can we calculate the compensating and equivalent variations of changes in states? This will depend on whether we can derive the Hicksian market demand curve, or not. If we can solve the integrability problem, then knowledge of demand will enable us to calculate a money metric utility function that is a monotonic transformation of the ordinal utility function. This in turn will enable us to calculate an expenditure function and consequently a Hicksian market demand curve. We will be able to get a Hicksian market demand curve for the original utility level, and a Hicksian market demand curve for the new utility level. Seen below, we have Hicksian market demand curves in the case where x is a normal good.

Figure 1: Hicksian market demand curves in the case where x is a normal good



For a normal good, the Hicksian market demand curves are h (p,u^0) for the original utility level and h (p,u') for the new utility level. Both of these curves are steeper than the Marshallian market demand curve x (p, m) when x is a normal good.[6] This is due to the fact that the Hicksian demand curves are compensated demand curves along which real income is constant, as opposed to the Marshallian demand curve which holds money income constant but fails to take account of the change in real income as price changes. The compensating variation (CV) between two states, with P' being the new state and P^0 being the original state, can be measured by the definite integral:

$$\int_{p'}^{p^\circ} h(p,u^\circ)dp$$

The equivalent variation (EV) between two states, with P' being the new state and P^0 being the original state, can be measured by the definite integral:

$$\int_{p'}^{p^\circ} h(p,u^\circ)dp$$

In the diagram above we can see that for a price fall from p^0 to p' the compensating variation is given by the area P^0ACP'. The equivalent variation is given by the area P^0BDP'. The change in consumers' surplus (CS) meanwhile is given by P^0ADP'. In this case the identity CV<CS<EV holds. This will be reversed if we are increasing price with the identity CV>CS>EV holding.

Therefore, we can see how to solve for the compensating and equivalent variations, when we can derive the Hicksian market demand curves. However, the problem is that we usually can not derive the Hicksian demand curves. Therefore, we have to estimate the two measures. How can this be achieved? We could use a method developed by McKenzie and Pearce to estimate the equivalent variation. This involves a Taylor series expansion from the base state (P^0, M^0) using the first and higher-order partial derivatives of the demand equations evaluated at (P^0, M^0) to determine the coefficients (Morey, 1984).

However, could we alternatively use the change in consumers' surplus to approximate the compensating and equivalent variations?

[6] If x is an inferior good, the Hicksian demand curves will be flatter than the Marshallian demand curve.

Use of Marshallian consumers' surplus to approximate the unknown compensating and equivalent variations:

Willig (1976) showed that the use of individual consumer's surplus gives a good approximation of the unknown compensating and equivalent variations for individuals when the income elasticity of demand is low or expenditure on the good is a small share of an individual's total expenditure. In his 1976 paper, he established precise bounds for the errors of estimating an individual's compensating and equivalent variations using the consumer's surplus measure of welfare change (i.e. the change in consumer's surplus between two states). From Johansson (1991) these error bounds are as follows:

$$(CV - CS) / CS \approx -\eta (CS / 2y)$$
$$(CV - CS) / CS \approx -\eta (CS / 2y)$$

Where η refers to the income elasticity of demand for the good, and y refers to total income available to spend on goods. According to Willig, if the absolute value of the terms on the right hand side of the equations above is less than 0.05, then these equations accurately reflect the errors of using consumers' surplus to measure the individual's compensating and equivalent variations. Willig (1976) concludes by saying:

> ''...at the level of the individual consumer, cost-benefit welfare analysis can be performed rigorously and unapologetically by means of consumer's surplus''.

This can be extended to justify using consumers' surplus to approximate the compensating and equivalent variations. The necessary criteria are now a low income elasticity of demand for all the goods affected, or that total expenditure on each of the goods affected represents only a small share of the total expenditure by the group of individuals.

Conclusion:

Therefore, consumers' surplus cannot be used as a cardinal measure of utility and the change in consumers' surplus cannot be used to cardinally rank alternative states. However, the change in consumers' surplus does provide a useful approximation to the unknown compensating and equivalent variations. This is providing that income elasticity of demand is low for the goods whose markets are affected by the change, or total expenditure on the goods affected constitutes a small

proportion of total expenditure by the group. However, as we consider changes in multiple prices and the path dependency problem[7] and also as we aggregate over more and more goods (the more and more prices change), errors begin to mount and accumulate. The change in consumers' surplus becomes less valid as a measure of welfare change. However, the money measure will give a rough indication of the benefit of a policy against, which policy makers can compare political and other unaccounted costs. In particularly with large projects, it may be more appropriate to use the McKenzie Pearse technique to estimate the equivalent variations. However, in the case of purely partial welfare analysis, where there are not that many prices changing, and such price changes do not unduly affect other prices, then consumers' surplus would appear to be justified as a useful money measure approximation of welfare change.

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[7] The overall change in consumer's surplus depends on the sequence of price changes.

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